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Monetary-Fiscal Theory of Prices in Modern DSGE Models

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Symbols

Notes The list is ordered alphabetically with abstract symbols preceding first the Latin and then the Greek alphabet. Pairs of lower- and upper-case letters denote real and nominal variables, respectively—except for interest and inflation, in which case they denote net and gross rates, respectively. Some variables are stated including time indices in order to clarify notation. For combined agent-related superscripts (e.g., \square^{TH} or \square^{DF}) please turn to the explanations given in the respective chapters.

\square_{\bigcirc}	partial derivative of \square with respect to \bigcirc
$\hat{\square}$	hypothetical deviation of \square from a reference value (Chapter 5)
$\bar{\square}$	upper limit to \square
$\check{\square}$	difference of \square from steady state (Chapter 7, see Equation $\langle 7.31 \rangle$)
$\hat{\square}$	percentage deviation of \square from steady state (Chapter 7, see Equation $\langle 7.29 \rangle$)
\square^*	target value for \square
a	technology (Chapter 7)
α	adaptive-expectations coefficient (Chapter 3.6)
b, B	bonds
c, C	consumption
c', C'	'given' level of total consumption in consumption allocation on different varieties (Chapter 7/Appendix F.1)
\square^C	superscript connected to the central bank (Chapter 3)
\square^{cpn}	\square in a model setup with coupon (instead of discount) bonds
\square^D	superscript connected to Domestic (Chapter 6)
d, D	budget deficit of the treasury
\square^{dis}	\square in a model setup with discount (instead of coupon) bonds
$\mathbb{E}_t \square_{t+1}$	expectation operator: expectation formed in t of \square_{t+1}
\square^F	superscript connected to Foreign (Chapter 6)
g, G	treasury expenditure
g^{fix}	constant element in the treasury-expenditure rule (Chapter 7)
\square^H	superscript connected to the household (Chapter 3)
i_{t+1}, I_{t+1}	nominal interest rate between periods t and $t + 1$
\dot{I}	opportunity cost of holding money (shorthand defined in Equation $\langle 2.23 \rangle$)

j	index used in sums, products, and integrals
J	specific (typically final) value of index j
k	index used in sums, products, and integrals
ℓ	labor
L	liquidity preference
\mathbb{L}	Lagrange function
L_{SW81}	liquidity preference in the model of Sargent and Wallace (1981) (Chapter 3.5)
m, M	money balances
$m_{\text{C56},t}$	money demand in the model of Cagan (1956) (Chapter 3.6)
mc, MC	marginal costs (Chapter 7)
\square^{\max}	maximal value of \square
$O_{\text{I}}\text{--}O_{\text{VI}}$	miscellaneous coefficients (Chapter 3, see Table A.1)
\dot{o}	coefficient (hypothetical percentage deviation from reference value, Chapter 5)
\square°	optimal value of \square
P	price level
Q	bond price
r_{t+1}, R_{t+1}	real interest rate between periods t and $t + 1$
s, S	budget surplus of the treasury
s^{fix}	constant element in the surplus rule (Chapter 7)
\dot{s}	surprise inflation (shorthand defined in Equation (3.6))
\square_{SS}	steady-state value of \square
\square_t	time index: \square in period t
T	time index: specific point in time (mostly/unless indicated otherwise)
\square^T	superscript connected to the treasury (Chapter 3)
t^{TH}, T^{TH}	net transfers from the treasury (T) to the household (H)
u	utility
\square^U	superscript connected to the monetary union (Chapter 6)
v, V	stochastic discount factor
w	wage
y, Y	output
\tilde{y}	output gap (log-linearized, Chapter 7, see Equation (7.30))
$y^{\text{nat}}, Y^{\text{nat}}$	natural output (Chapter 7)
Z_t	nominal consolidated-government liabilities outstanding at the end of period t
z^*	steady-state/target real consolidated-government liabilities
z', Z'	alternative definition of consolidated-government liabilities (Chapter 8)

α	Calvo parameter (share of firms unable to adjust optimally, Chapter 7)
β	discount factor
γ_ε^C	autoregressive parameter in a central-bank policy shock process (Chapter 7)
γ_π^C	Taylor coefficient
γ_a	autoregression coefficient in the (log) technology process (Chapter 7)
γ_y^C	central-bank reaction parameter for output (Chapter 7)
γ_b^T, γ_B^T	treasury reaction parameter related to debt
γ_ε^T	autoregressive parameter in a treasury shock process (Chapter 7)
γ_m^T	treasury policy parameter relating to a seigniorage rebate (Chapter 7)
γ_s^T	autoregressive parameter in a treasury surplus rule (Chapter 8)
γ_y^T	treasury reaction parameter for output (Chapter 7)
γ_Z^T	treasury reaction parameter for consolidated-government liabilities
$\dot{\gamma}_Z^T$	alternative treasury reaction parameter for consolidated-government liabilities (Chapter 5.2.1)
Γ	public-spending ratio (relative to private consumption, Chapter 7)
$\Delta \square_t$	difference operator (first difference of \square_t)
ε^a	technology shock process (Chapter 7)
ε^i	interest-rate shock process
ϵ^i	innovation in the interest-rate policy shock process (Chapter 7)
ε^s	surplus shock process
e^s	innovation in the surplus policy shock process (Chapter 7)
ζ	(part of the) exponent on labor in production (Chapter 7)
η	(part of the) exponent on labor in CES utility (Chapter 7)
θ	(part of the) exponent in the Dixit and Stiglitz consumption bundler (Chapter 7)
Θ	shorthand defined for the New-Keynesian Phillips Curve in Equation (7.36) (Chapter 7)
Θ'	shorthand defined for the New-Keynesian Phillips Curve in Equation (F.46) (Chapter 7)
Θ''	shorthand defined for the New-Keynesian Phillips Curve in Equation (F.50) (Chapter 7)
κ	slope of the Phillips curve (shorthand defined in Equation (7.38), Chapter 7)
λ	parameter on the respective constraint in a Lagrange function
μ_\square	growth rate of \square
μ'_\square	growth reate of \square (alternative definition, Chapter 3.6)
ν	(part of the) exponent on real money in CES utility (Chapter 7)
ξ_\square	weight of \square in the utility function (Chapter 7)

Ξ_a	shorthand for the dynamic IS curve defined in Equation $\langle 7.34 \rangle$ (Chapter 7)
Ξ_g	shorthand for the dynamic IS curve defined in Equation $\langle 7.35 \rangle$ (Chapter 7)
Ξ_r	shorthand for the dynamic IS curve defined in Equation $\langle 7.33 \rangle$ (Chapter 7)
π_t, Π_t	net/gross inflation rate between periods $t - 1$ and t
π'	net inflation rate (alternative definition, Chapter 3.6)
π^{ZLB}	critical inflation value with a nominal-interest zero lower bound (Chapter 5.2.1)
ρ	(part of the) exponent on consumption in CES utility (Chapter 7)
ϱ	desired (flexible-price) markup (Chapter 7)
ς^i	interest-saving seigniorage (Chapter 3)
ς^m	real-money seigniorage (Chapter 3)
τ^{TH}	transfer rate
$\varphi' - \varphi'''$	miscellaneous coefficients (Appendix A.1.3, see Table A.1)
ψ, Ψ	firm profits

1. Introduction

Background At least since the Euro crisis starting in 2010, fiscal variables such as budget deficits and debt levels of member countries as well as the construction of the Eurozone as a whole have received renewed attention. Furthermore, in the wake of the preceding global financial crisis, the economic profession faced accusations of not being able to generate satisfactory answers to pressing questions from the models commonly used in public institutions and academic circles.

Motivation These issues arise from and drive different strands of the macroeconomic literature. The intersection of topics studied in the present thesis is motivated by personal experience: Towards the end of my studies, I happened to stumble upon a rather verbal textbook rendition of the ‘fiscal theory of the price level’ (in Görgens, Ruckriegel, and Seitz 2008) which was equally baffling and intriguing. Then, as a teaching assistant at the Chair of Economic Policy, I had to familiarize myself with the ‘New-Keynesian’ model now taught in graduate classes at the University of Hohenheim. To keep it short, when the fiscal theory reappeared on my horizon (after having fallen into oblivion for a while) and was met by the ambition to master the literature on dynamic stochastic general-equilibrium (DSGE) models, my dissertation project was born.

Research Goals Against this backdrop, the present thesis can be summarized in two main research questions:

1. How do common modern DSGE models work, that is, how do they arrive at unambiguous descriptions of macroeconomic events as equilibrium outcomes?
2. What is the role of the two main fields of macroeconomic policy, monetary and fiscal policy, and the interrelations between them, in these descriptions?

Rather than simply bundling a number of isolated and possibly incompatible approaches, I want to build and maintain a unified framework throughout the dissertation to tackle these questions. The development of this framework should proceed as clearly and comprehensibly as possible.

Plan of the Book While I hope it has enough structure to be ‘easily’ read from cover to cover without repeatedly circling back to the introduction for guidance, I want to give a brief outline of this dissertation as a golden thread.

Laying the groundwork for most of the thesis, Chapter 2 develops a baseline micro-founded DSGE model and takes first steps in the determination of equilibrium. After a brief excursus into the interactions arising from the consolidated-government budget equation and some traditional approaches to their description in Chapter 3, Chapter 4 returns to the baseline model (without money) and closes it by defining simple policy rules. It is shown that, depending on the respective constellation of policy parameters, several regimes emerge which are discussed with regard to stability and determinacy.

Chapter 5 makes three additions to the model used hitherto—money, the zero lower bound on nominal interest rates, as well as debt limits—and examines their implications for determinacy of equilibrium. The extension of the theory to monetary unions is studied in Chapter 6. The final and most demanding advancement is the New-Keynesian model variant expounded in Chapter 7, a linearized version of which is simulated to generate graphical impulse responses to technology as well as policy shocks.

Chapter 8 hints at empirical issues in the distinction between policy regimes. Finally, Chapter 9 provides a discussion of the results obtained in the course of the thesis. Chapter 10 concludes.

* * *

Notation & Stylistic Matters Symbols are explained (sufficiently, I hope) in the symbol list at the beginning and upon their introduction in the main text. I managed to avoid nonstandard abbreviations almost entirely, which is why there is no respective list. External URLs and internal cross-references are clickable throughout the PDF, often indicated by the ‘link symbol’ ↗. The only notational issue which may not be straightforward are underlined equation numbers, they indicate that the respective equation is explicitly derived in the appendix.

Files This PDF file as well as the Matlab/Dynare files necessary to produce the impulse-response functions in Chapter 7/Appendix F are available upon request.

2. Baseline Optimizing Model

The model laid out in this chapter is the foundation for all formal analysis in the present thesis. In an infinite-horizon representative-household setup, I derive optimality conditions, construct the present-value budget equation that is at the heart of the ‘fiscal theory,’ present a simplified constant-endowment version of the model, and start up the discussion about equilibrium determination.

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2.1. The Model

Utility An infinitely-lived representative household maximizes per-period utility

$$u(c_t, m_t) \tag{2.1}$$

by choosing real sequences of consumption c , money m , and bonds b . The utility function satisfies the usual properties, that is, marginal utility of either argument is positive but decreasing:

$$\begin{aligned} u'(\cdot) &> 0 \\ u''(\cdot) &< 0 \end{aligned} \tag{2.2}$$

It is often assumed that utility $\langle 2.1 \rangle$ is additively separable in its arguments c_t and m_t , where the properties $\langle 2.2 \rangle$ apply to each summand individually. In the context of the baseline model, I generally adopt this simplification (at the cost of some generality).

Future utility is discounted by the discount factor $\beta \in (0, 1)$.

Budget Constraint The household has to respect the flow budget constraint

$$\begin{aligned} C_t + M_t + Q_t B_t &= Y_t + T_t^{TH} + B_{t-1} + M_{t-1} \\ \Leftrightarrow P_t c_t + M_t + Q_t B_t &= P_t y_t + P_t t_t^{TH} + B_{t-1} + M_{t-1} \end{aligned} \quad \langle 2.3 \rangle$$

$$\Leftrightarrow c_t + m_t + Q_t b_t = y_t + t_t^{TH} + \frac{b_{t-1} + m_{t-1}}{\Pi_t}, \quad \langle 2.4 \rangle$$

in each period. P_t denotes the time- t goods price level in terms of money M_t . In most cases, lower-case symbols denote real variables (an obvious exemption are interest rates), where identities analogous to $C_t \equiv P_t c_t$ hold. B_t is the amount of discount bonds issued in period t at the bond price Q_t (see also *Bond Prices* on p. 25 below).

Fisher Equation The equivalence of Equations $\langle 2.3 \rangle$ and $\langle 2.4 \rangle$ follows from the Fisher equation

$$1 + r_t = \frac{1 + i_t}{\Pi_t}, \quad \langle 2.5 \rangle$$

which describes the relationship between the real interest rate, nominal interest rate, and inflation rate

$$\Pi_t \equiv \frac{P_t}{P_{t-1}} \quad \langle 2.6 \rangle$$

in equilibrium. All three variables are dated t , and they are *all* meant to connect periods $t - 1$ and t (which is a somewhat unconventional, yet more consistent, notation in the case of interest rates).

The Fisher equation $\langle 2.5 \rangle$ is necessary to prevent arbitrage between real and nominal assets (it could be formally derived in a model that includes real capital, cf. Walsh 2010, pp. 35-39, for instance). In models with a constant real interest rate (see Section 2.4), it is often—but not exclusively—understood to determine the nominal interest rate between t and $t + 1$, i_{t+1} , as a function of expected inflation:¹

$$1 + i_{t+1} = (1 + \mathbb{E}_t r_{t+1}) \mathbb{E}_t \Pi_{t+1} \quad \langle 2.7 \rangle$$

¹ An alternative interpretation is that it can also determine (expected) inflation since the nominal interest rate is usually set by the central bank (cf. Section 4.2.1.1) and real interest rates are determined by other factors such as time preference and productivity. Some references on this ‘Neo-Fisherian’ view, ordered by length, are Williamson (2016), Uribe (2017), and Cochrane (2016).

i_{t+1} is not preceded by the time- t expectation operator \mathbb{E}_t because, while it is contractually *set* based on expectations (in period t), it does not change *ex post* if the underlying variables deviate from these expectations (in period $t + 1$).

Bond Prices In period t , B_t nominal discount bonds are issued at the price Q_t . In period $t + 1$, they are redeemed at par, paying out B_t units of money. No-arbitrage requires that the implied yield is equal to the nominal interest rate (cf. Woodford 2003b, p. 66):

$$Q_t = \frac{1}{1 + i_{t+1}} \quad \langle 2.8 \rangle$$

Bond Types As a side note, there is an alternative to the present specification in that debt could also be issued in the form of coupon bonds. Which type of bond is chosen is mostly a matter of notation (see also Section 2.3.2); once the first-order conditions are combined into the standard equilibrium conditions, the contractual form of bonds is immaterial in this model. For the sake of clarity: The flow budget constraint $\langle 2.3 \rangle$ uses discount bonds. If, alternatively, coupon bonds are used, it reads

$$P_t c_t + B_t + M_t = P_t y_t + P_t t_t^{TH} + (1 + i_t) B_{t-1} + M_{t-1}. \quad \langle 2.9 \rangle$$

The same no-arbitrage argument as above requires that coupon bonds also pay a yield equal to the nominal interest rate; discount and coupon bonds are equivalent in this respect.

Endowments The household is endowed with a real income stream y in each period which is typically assumed constant for simplicity (cf. Section 2.4).

Treasury Policy The treasury decides on spending G_t and net transfers to the household T^{TH} . If T^{TH} is negative, the household effectively pays taxes. The treasury's budget surplus S is defined as

$$S_t \equiv - \left(G_t + T_t^{TH} \right). \quad \langle 2.10 \rangle$$

2.2. Maximization Problem & First-Order Conditions

Lagrangian The maximization problem can be expressed by the Lagrangian

$$\max_{\{c_j, m_j, b_j\}_{j=t}^{\infty}} \mathbb{L}_t = \mathbb{E}_t \sum_{j=t}^{\infty} \left\{ \beta^{j-t} u(c_j, m_j) \right.$$

$$-\lambda_j \left[c_j + m_j + b_j - y_j - t_j^{TH} - (1 + r_j) b_{j-1} - \frac{m_{j-1}}{\Pi_j} \right] \Big\}, \quad \langle 2.11 \rangle$$

where λ is the Lagrange coefficient on the budget constraint (2.4). (I use coupon bonds and omit discount bonds here for brevity; as already stated, this does not affect the resulting first-order conditions below.)

First-Order Conditions The first-order conditions with respect to real consumption, real money, and real treasury-bond holdings, respectively, are

$$\frac{\partial \mathbb{L}_t}{\partial c_t} = u_c(c_t, m_t) - \lambda_t \stackrel{!}{=} 0 \quad \Leftrightarrow \quad \lambda_t = u_c(c_t, m_t) \quad \langle 2.12 \rangle$$

$$\frac{\partial \mathbb{L}_t}{\partial c_{t+1}} = \beta \mathbb{E}_t u_c(c_{t+1}, m_{t+1}) - \mathbb{E}_t \lambda_{t+1} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad \mathbb{E}_t \lambda_{t+1} = \beta \mathbb{E}_t u_c(c_{t+1}, m_{t+1}) \quad \langle 2.13 \rangle$$

$$\frac{\partial \mathbb{L}_t}{\partial m_t} = u_m(c_t, m_t) - \lambda_t + \frac{\mathbb{E}_t \lambda_{t+1}}{\mathbb{E}_t \Pi_{t+1}} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad u_m(c_t, m_t) = \lambda_t - \frac{\mathbb{E}_t \lambda_{t+1}}{\mathbb{E}_t \Pi_{t+1}} \quad \langle 2.14 \rangle$$

$$\frac{\partial \mathbb{L}_t}{\partial b_t} = -\lambda_t + \mathbb{E}_t \lambda_{t+1} (1 + \mathbb{E}_t r_{t+1}) \stackrel{!}{=} 0 \quad \Leftrightarrow \quad 1 + \mathbb{E}_t r_{t+1} = \frac{\lambda_t}{\mathbb{E}_t \lambda_{t+1}} \quad \langle 2.15 \rangle$$

(Equation (2.13) does not constitute a first-order condition in its own right as it only repeats Equation (2.12) one period ahead; it is stated for convenience).

Stochastic Discount Factors In the DSGE literature, reference is often made to the stochastic discount factor. It is equal to the intertemporal marginal rate of substitution and sometimes also called pricing kernel (predominantly in the context of asset pricing; cf. Cochrane 2005a, p. 7). When dealing with real values, one uses the real stochastic discount factor

$$\mathbb{E}_t v_{t,t+1} \equiv \beta \frac{\mathbb{E}_t u_c(c_{t+1}, \cdot)}{u_c(c_t, \cdot)}; \quad \langle 2.16 \rangle$$

accordingly, the nominal stochastic discount factor

$$\mathbb{E}_t V_{t,t+1} \equiv \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1}, \cdot)}{u_c(c_t, \cdot)} \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1}, \cdot)}{u_c(c_t, \cdot)} \frac{1}{\Pi_{t+1}} \right] = \frac{\mathbb{E}_t v_{t,t+1}}{\Pi_{t+1}} \quad \langle 2.17 \rangle$$

is used in combination with nominal values (cf. Cochrane 2005a, pp. 6-10). Usually, $u_c(c_{t+1}, \cdot)$ (just as P_{t+1}) is preceded by the expectation operator \mathbb{E}_t because it is not known with certainty in t , so the same applies to the stochastic discount factors.

Using Equations (2.12), (2.13), and (2.15), it can be shown that the real stochastic

discount factor is equal to the inverse of the gross real interest rate in equilibrium:

$$\mathbb{E}_t v_{t,t+1} = \frac{1}{1 + \mathbb{E}_t r_{t+1}} \quad \langle 2.18 \rangle$$

A similar relationship holds for their nominal counterparts if maximization is performed using nominal equations (that is, using the budget constraint $\langle 2.3 \rangle$ instead of its real counterpart $\langle 2.4 \rangle$ in the maximization problem $\langle 2.11 \rangle$). Adding the fact that the inverse of the gross nominal interest rate is equal to the bond price here (Equation $\langle 2.8 \rangle$), we have

$$\mathbb{E}_t V_{t,t+1} = \frac{1}{1 + i_{t+1}} = Q_t. \quad \langle 2.19 \rangle$$

Both the nominal and the real stochastic discount factor can be ‘linked’ such that, for example,

$$\mathbb{E}_t V_{t,t+1} \cdot \mathbb{E}_t V_{t+1,t+2} \cdot \mathbb{E}_t V_{t+2,t+3} = \mathbb{E}_t V_{t,t+3} = \prod_{j=0}^2 \frac{1}{1 + \mathbb{E}_t i_{t+1+j}}. \quad \langle 2.20 \rangle$$

Furthermore, $v_{t,t} = V_{t,t} = 1$.

Consumption Euler Equation Combining Equations $\langle 2.12 \rangle$ and $\langle 2.15 \rangle$ gives the consumption Euler equation

$$u_c(c_t, m_t) = \beta (1 + \mathbb{E}_t r_{t+1}) \mathbb{E}_t u_c(c_{t+1}, m_{t+1}). \quad \langle 2.21 \rangle$$

(Implicit) Money Demand Further, plugging the first-order conditions $\langle 2.12 \rangle$ and $\langle 2.15 \rangle$ as well as the Fisher equation $\langle 2.7 \rangle$ into the first-order condition with respect to money $\langle 2.14 \rangle$ implies that money demand is a function of the nominal interest rate and (the marginal utility of) consumption:

$$\begin{aligned} u_m(c_t, m_t) &= u_c(c_t, m_t) - \frac{u_c(c_t, m_t)}{(1 + \mathbb{E}_t r_{t+1}) \mathbb{E}_t \Pi_{t+1}} \\ &= \left[1 - \frac{1}{1 + i_{t+1}} \right] u_c(c_t, m_t) = \dot{I}_{t+1} u_c(c_t, m_t). \end{aligned} \quad \langle 2.22 \rangle$$

At this, the shorthand

$$\dot{I}_{t+1} \equiv \frac{i_{t+1}}{1 + i_{t+1}}. \quad \langle 2.23 \rangle$$

represents the opportunity cost of holding money (cf. Woodford 2003b, p. 67).

Alternatively, solving money demand $\langle 2.22 \rangle$ for the nominal interest rate

$$i_{t+1} = \frac{u_m(c_t, m_t)}{u_c(c_t, m_t) - u_m(c_t, m_t)} \quad \langle 2.24 \rangle$$

shows that it is a decreasing function of real money M_t/P_t : If real money increases, its marginal utility goes down, which decreases the right-hand side of the equation via both the numerator and the denominator.

Liquidity Preference Given the standard properties of utility $\langle 2.2 \rangle$, marginal utility of consumption decreases as consumption increases. By Equation $\langle 2.22 \rangle$, *ceteris paribus*, marginal utility of money has to decrease as well—in short, real money demand increases in the level of consumption. Similarly, real money demand depends negatively on the nominal interest rate since \dot{I}_{t+1} increases in i_{t+1} . Consequently, one can restate implicit money demand $\langle 2.22 \rangle$ to isolate m_t by using a liquidity preference function $L(c_t, i_{t+1})$,

$$\frac{M_t}{P_t} = L(c_t, i_{t+1}), \quad \langle 2.25 \rangle$$

with

$$\begin{aligned} L_c(\cdot) &> 0 \\ L_i(\cdot) &< 0. \end{aligned} \quad \langle 2.26 \rangle$$

Of course, $L(\cdot)$ in Equation $\langle 2.25 \rangle$ is just as general as $u_m(\cdot)$ and $u_c(\cdot)$ in Equation $\langle 2.22 \rangle$ unless a particular utility function is specified, meaning that it only allows to make basic qualitative assertions such as $\langle 2.26 \rangle$.

Goods Market Clearing In equilibrium, goods markets clear so that

$$y_t = c_t + g_t. \quad \langle 2.27 \rangle$$

Borrowing & Ponzi Games Another important condition for equilibrium is that the household can not use debt to attain infinite utility by engaging in ‘Ponzi games.’ I discuss the formal aspects of this requirement in Section 2.3.1 below.

2.3. Budget Constraints and Infinite Time Horizons

2.3.1. Household Borrowing, Ponzi Games, and the Transversality Condition

Ponzi Games Without further constraint, a household facing the flow budget constraint $\langle 2.3 \rangle / \langle 2.9 \rangle$ has an incentive to play a ‘Ponzi game,’ that is, finance the nominal of an original loan plus interest on it by taking up more debt, postponing ultimate repayment indefinitely to a nonexistent last period. This way, it could attract infinite resources and thus derive infinite utility.

Borrowing Limit Of course, a hypothetical lender would not accept this and instead impose a borrowing limit, the least constraining form of which holds that the household can at most borrow what it is able to pay back, with interest, over the course of its entire (infinite) lifetime.

Define (predetermined) nominal wealth of the household in period t as

$$Z_{t-1}^{\text{dis}} \equiv B_{t-1}^{\text{dis}} + M_{t-1} \quad \langle 2.28 \rangle$$

$$Z_{t-1}^{\text{cpn}} \equiv (1 + i_t) B_{t-1}^{\text{cpn}} + M_{t-1}, \quad \langle 2.29 \rangle$$

where the ‘dis’ and ‘cpn’ superscripts indicate use of discount bonds (budget constraint $\langle 2.3 \rangle$) or coupon bonds ($\langle 2.9 \rangle$), respectively. Then, using either notation, the formal representation of the borrowing limit reads

$$Z_t \geq -\mathbb{E}_t \sum_{j=0}^{\infty} V_{t+1,t+1+j} P_{t+1+j} \left(y_{t+1+j} + t_{t+1+j}^{TH} \right), \quad \langle 2.30 \rangle$$

which implies that the household’s nominal wealth position can in fact be negative, but not less than what it expects to have disposable to make good on its debt. Put differently, the borrowing limit determines the value of liabilities Z_t that the household can carry over from period t , which, in period $t + 1$, must be at least equal to the present value of the stream of available resources. In order to lead to a meaningful equilibrium, the sum on the right-hand side must be finite,

$$\mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} P_{t+j} \left(y_{t+j} + t_{t+j}^{TH} \right) < \infty, \quad \langle 2.31 \rangle$$

or else the household would still be able to attain infinite utility even though the described financial-market ‘loophole’ is closed (Equation $\langle 2.31 \rangle$ in fact goes beyond this requirement by positing finiteness in the current period t as well).

Present-Value Budget Constraint of the Household Either household budget constraint $\langle 2.3 \rangle / \langle 2.9 \rangle$ can be rearranged to read

$$Z_{t-1} = \mathbb{E}_t V_{t,t+1} Z_t + P_t c_t + \dot{I}_{t+1} M_t - P_t (y_t + t_t^{TH}), \quad \langle 2.32 \rangle$$

which, upon repeatedly iterating, yields

$$Z_{t-1} = \mathbb{E}_t (V_{t,t+J+1} Z_{t+J}) + \mathbb{E}_t \sum_{j=0}^J V_{t,t+j} \left[P_{t+j} c_{t+j} + \dot{I}_{t+j+1} M_{t+j} - P_{t+j} (y_{t+j} + t_{t+j}^{TH}) \right]. \quad \langle 2.33 \rangle$$

If the transversality condition

$$\lim_{J \rightarrow \infty} \mathbb{E}_t (V_{t,t+J+1} Z_{t+J}) = 0 \quad \langle 2.34 \rangle$$

is satisfied, the equation above can accordingly be rearranged to the present-value budget constraint of the household:

$$\mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} (P_{t+j} c_{t+j} + \dot{I}_{t+j+1} M_{t+j}) = \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} P_{t+j} (y_{t+j} + t_{t+j}^{TH}) + Z_{t-1} \quad \langle 2.35 \rangle$$

(I go into a little more detail on the formal aspects in Section 2.3.2 below.) A borrowing limit such as $\langle 2.30 \rangle$ also holds for current outstanding nominal wealth Z_{t-1} (however, it is predetermined in period t). The smallest value it can have taken would make the right-hand side equal exactly zero, implying that all current and future disposable income is pledged for repayment so that the household will never be able to consume or hold positive money balances again.² Otherwise, for nominal wealth Z_{t-1} larger than this, positive consumption levels and money balances are feasible. Note that the present-value budget constraint $\langle 2.35 \rangle$ is equivalent to the combination of the flow budget constraint $\langle 2.3 \rangle / \langle 2.9 \rangle$ and the transversality condition $\langle 2.34 \rangle$ (cf. Woodford 2003b, p. 70).

² This is an unrealistic case if, as is often the case, the utility function is assumed to satisfy the ‘Inada conditions’ (cf. Inada 1963) because marginal utility approaches infinity as the respective argument of the utility function goes towards zero. In short, an optimizing household would never choose such a consumption profile (for example).

2.3.2. Present-Value Budget Equation of the Consolidated Government

Using Discount Bonds Imposing no-arbitrage (2.8) and goods-market clearing (2.27) on the household budget constraint (2.3) implies the consolidated-government budget equation

$$\frac{1}{1+i_{t+1}}B_t^{\text{dis}} + M_t = P_t(g_t + t_t^{TH}) + B_{t-1}^{\text{dis}} + M_{t-1}. \quad (2.36)$$

(Note that I do not call it a consolidated-government budget *constraint*; the reason is discussed in Section 9.2.) With only two sectors, nominal wealth of the private sector Z equals nominal liabilities of the public sector, allowing continued use of Z_{t-1}^{dis} as defined in Equation (2.28). Combining this with Equations (2.10), (2.23), and (2.36) gives

$$\begin{aligned} Z_{t-1}^{\text{dis}} &= Z_t^{\text{dis}} - \frac{i_{t+1}}{1+i_{t+1}}B_t^{\text{dis}} + S_t \\ &= Z_t^{\text{dis}} - \frac{i_{t+1}}{1+i_{t+1}}(Z_t^{\text{dis}} - M_t) + S_t \\ &= \frac{Z_t^{\text{dis}}}{1+i_{t+1}} + i_{t+1}M_t + S_t. \end{aligned} \quad (2.37)$$

Using Coupon Bonds Using budget constraint (2.9) instead of (2.3), the budget equation of the consolidated government becomes

$$B_t^{\text{cpn}} + M_t = P_t(g_t + t_t^{TH}) + (1+i_t)B_{t-1}^{\text{cpn}} + M_{t-1}. \quad (2.38)$$

Accordingly, definition (2.29) takes into account that the structure of the bond contract is different from the discount-bond case. The consolidated-government budget equation (2.38) can now be written as

$$Z_{t-1}^{\text{cpn}} = Z_t^{\text{cpn}} - i_{t+1}B_t^{\text{cpn}} + S_t.$$

Like this, the flow budget equation (2.38) can be rearranged into

$$\begin{aligned} Z_{t-1}^{\text{cpn}} &= Z_t^{\text{cpn}} - i_{t+1}\left(\frac{Z_t^{\text{cpn}} - M_t}{1+i_{t+1}}\right) + S_t \\ &= \frac{Z_t^{\text{cpn}}}{1+i_{t+1}} + i_{t+1}M_t + S_t, \end{aligned} \quad (2.39)$$

which looks equivalent to Equation (2.37), at least on the face of it (see \nearrow Comparison below).

The Actual Present-Value Budget Equation Dividing through by P_t , dismantling the nominal interest rate in either Equation (2.37) or (2.39) using the Fisher equation (2.5) / (2.7) as well as Equation (2.18), and dropping the ‘cpn’ / ‘dis’ superscripts, we have

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \left(v_{t,t+1} \frac{Z_t}{P_{t+1}} \right) + \dot{I}_{t+1} m_t + s_t. \quad (2.40)$$

Iterating forward,

$$\begin{aligned} \frac{Z_{t-1}}{P_t} &= \mathbb{E}_t \left[v_{t,t+1} \left(v_{t+1,t+2} \frac{Z_{t+1}}{P_{t+2}} + \dot{I}_{t+2} m_{t+1} + s_{t+1} \right) \right] + \dot{I}_{t+1} m_t + s_t \\ &\vdots \\ &= \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}}{P_{t+J+1}} \right) + \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}) \end{aligned} \quad (2.41)$$

The first term on the right-hand side represents the possibility to postpone the redemption of open contractual positions to the far-distant (indefinite) future; it is excluded by imposing

$$\lim_{J \rightarrow \infty} \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}}{P_{t+J+1}} \right) = 0, \quad (2.42)$$

the transversality condition for equilibrium.³

³ Some remarks are in order. First, Equation (2.42) is derived from a budget equation in real terms (2.40), whereas the household’s transversality condition (2.34) stems from a nominal budget constraint (2.32). With this in mind, Equations (2.16)-(2.17) imply that the two versions are equivalent.

Second, Equation (2.42) actually follows from two separate arguments. Keeping wealth indefinitely instead of consuming it would offer room for improvement, so positive wealth levels in the ‘final’ period can not be optimal. Hence, if and to the extent that it is possible, optimizing households would prefer non-positive lifetime wealth (initial wealth plus the expected stream of available income). However, it is not possible: As explained in the context of the no-Ponzi argument and borrowing limit (2.30), households are required by lenders to maintain non-negative lifetime wealth. The obvious compromise and only solution is the transversality condition’s actual form (2.42). (Cf. Obstfeld and Rogoff 1996, pp. 64-65; Bergin 2000, p. 40, for example. Earlier, and somewhat more technical, treatments include Gray and Salant 1981; Gray 1984; Dixit 1990.)

Finally, McCallum (2001, p. 21) splits it into two separate conditions for real money m and treasury debt b on the grounds that a joint transversality condition can be satisfied for $m \rightarrow \infty$ and $b \rightarrow -\infty$, which is deemed implausible. Whether one considers this as a good (enough) reason or an unnecessary

The remainder of Equation (2.41),

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}), \quad (2.43)$$

is called *intertemporal* or *present-value budget equation* of the consolidated government. I mostly use the latter expression because *flow* budget equations such as (3.1), (4.2), or (2.3) are also intertemporal in that they link two adjacent periods whereas present-value budget equations like (2.43) contain *stocks* (outstanding liabilities on the left-hand side to discounted sums of flows on the right-hand side).

If endowments as well as treasury expenditure are fixed, the real interest rate is constant (cf. Section 2.4) and Equation (2.43) simplifies further to

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}). \quad (2.44)$$

Comparison There is indeed a difference between the coupon-bond and the discount-bond version of the present-value budget equation: Outstanding government liabilities Z were defined differently in Equations (2.28) and (2.29), so that the respective present-value budget equations also differ:

$$\left. \begin{array}{l} \frac{(1 + i_t) B_{t-1}^{\text{cpn}} + M_{t-1}}{P_t} \\ \frac{B_{t-1}^{\text{dis}} + M_{t-1}}{P_t} \end{array} \right\} = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}) \quad (2.45)$$

Is it a problem that the cases differ on the left-hand side? While it would certainly be convenient to get *exactly* the same results (as is the case with the first-order conditions), the case differentiation (2.45) poses no fundamental—that is, economic—problem. In simple models, it is a mere modeling choice without severe consequences. Discount bonds yield interest implicitly by redeeming more than what was initially paid out to the debtor whereas coupon bonds make this interest payment explicitly. In actual numbers, $(1 + i_t) B_{t-1}^{\text{cpn}}$ would be equal to B_{t-1}^{dis} if one switched from one bond type to the other.⁴

ad-hoc restriction is certainly a matter of opinion.

⁴ To give an example: At a market interest rate of 100%, a treasury in need of one billion Euros will either issue coupon bonds with a face value of 1 bn. € in period t and pay back 2 bn. € in period $t + 1$ or, alternatively, issue discount bonds with a nominal of 2 bn. € which are to be redeemed in full in period $t + 1$ but only equip the treasury with 1 bn. € in period t . As a means to finance the same project (costing 1 bn. €), both types of bonds lead to exactly the same payment streams in both periods.

In richer models, the treasury could be modeled to use both kinds of contracts, leading to outstanding liabilities of the form $[(1 + i_t) B_{t-1}^{\text{cpn}} + B_{t-1}^{\text{dis}} + M_{t-1}] / P_t$.

Finally, note that the appearance of the nominal interest rate on the left-hand side does not cancel out P_t in the denominator. By Equation (2.7), the gross nominal interest rate is given by $1 + i_t = \beta^{-1} \mathbb{E}_{t-1} \Pi_t = \beta^{-1} \mathbb{E}_{t-1} P_t / P_{t-1}$ (assuming constant endowments and therefore a constant real interest rate, cf. Section 2.4). Since $\mathbb{E}_{t-1} P_t$ is not necessarily equal to P_t , these two terms do not cancel out. The nominal interest rate i_t is completely predetermined in period t .

2.4. Equilibrium Determination in the Constant-Endowment Economy

Since a nontrivial part of this thesis is devoted to the question whether certain variants of the model are determinate, it is probably helpful to summarize some common elements already at this point.

Constant Real Interest Rate To make the model as simple as possible, real income y is often assumed to be an exogenous endowment. An even stronger assumption is that it is constant over time. If, in addition, the treasury keeps real spending g constant, goods-market clearing (2.27) implies that consumption c is also invariant. Furthermore, if utility (2.1) is additively separable, the consumption Euler equation (2.21) uniquely determines the gross real interest rate $(1 + r)$ —which is constant and equal to the inverse of the household's discount factor β :⁵

$$1 + r_t = \beta^{-1} \quad \forall t \quad (2.46)$$

As a consequence, the Fisher equation (2.7) reduces to

$$1 + i_{t+1} = \beta^{-1} \mathbb{E}_t \Pi_{t+1}. \quad (2.47)$$

Counting Variables and Equations Tables 2.1b and 2.1a list all available equations and variables of the model, respectively. The paragraph on the *Constant Real Interest Rate*

⁵ Another way to achieve the simplification of a constant real interest rate would be to include treasury expenditure in the household's utility function so that $u(c_t + g_t, \cdot) = u(y_t, \cdot)$, which never changes in its first argument if the endowment y is assumed to be constant. Woodford (1996; 2001) uses this approach, for instance.

no.	variable	determination	description
<i>'truly' exogenous</i>			
1	y	exogenous	endowment
<i>quasi-exogenous in the constant-endowment economy</i>			
2	g	policy	treasury expenditure
3	c	$\langle 2.27 \rangle$	consumption
4	λ	$\langle 2.12 \rangle$	Lagrange parameter for the budget constraint
5	r	$\langle 2.15 \rangle$	real interest rate
<i>endogenous by definition</i>			
6	Π	$\langle 2.6 \rangle$	inflation rate
7	s	$\langle 2.10 \rangle$	budget surplus of the treasury
8	v	$\langle 2.16 \rangle$	real stochastic discount factor
9	V	$\langle 2.17 \rangle$	nominal stochastic discount factor
10	\dot{i}	$\langle 2.23 \rangle$	opportunity cost of holding money
11	Z	$\langle 2.28 \rangle / \langle 2.29 \rangle$	nominal wealth (household)/liabilities (cons. gov.)
<i>to be determined by equations or set exogenously by policy</i>			
12	M		money supply
13	t^{TH}		net transfers from the treasury to the household
14	B		bonds
15	i		nominal interest rate
16	P		price level
17	Q		bond price

Table 2.1a: Overview of Variables in the Baseline Optimizing Model. \diamond Explanations: See \nearrow Counting Variables and Equations on p. 34.

no.	Equation	description
<i>'normal' equations</i>		
1	$\langle 2.3 \rangle / \langle 2.9 \rangle$	flow budget constraint of the household
2	$\langle 2.5 \rangle / \langle 2.7 \rangle$	Fisher equation
3	$\langle 2.8 \rangle$	no-arbitrage condition for bond prices
4	$\langle 2.12 \rangle$	first-order condition with respect to consumption
5	$\langle 2.14 \rangle$	first-order condition with respect to money
6	$\langle 2.15 \rangle$	first-order condition with respect to bonds
7	$\langle 2.27 \rangle$	goods-market clearing
<i>'mere definitions'</i>		
8	$\langle 2.6 \rangle$	inflation rate
9	$\langle 2.10 \rangle$	budget surplus of the treasury
10	$\langle 2.16 \rangle$	real stochastic discount factor
11	$\langle 2.17 \rangle$	nominal stochastic discount factor
12	$\langle 2.23 \rangle$	opportunity cost of holding money
13	$\langle 2.28 \rangle / \langle 2.29 \rangle$	nominal wealth (household)/liabilities (cons. gov.)
<i>'everything else'</i>		
14	exogenous variable	
15-17	set by policy	

Table 2.1b: *Equations Available for Equilibrium Determination in the Baseline Optimizing Model.* \diamond Explanations: See \nearrow Counting Variables and Equations on p. 34.

already explains how the first five variables in Table 2.1a are determined (the determination of λ is implicit in this). Moving on, six variables are ‘mere definitions.’ One slight exception might be the definition of inflation $\langle 2.6 \rangle$: It could be argued that the equation explains either inflation or the price level given that the respective other variable is determined by another equation, depending on the policy regime. However, the relationship between inflation and price levels seems too basic and the actual determination of both variables too obvious to warrant (even more) lengthy discussions. As a final note, the bond price Q depends solely on the nominal interest rate i because of the no-arbitrage condition $\langle 2.8 \rangle$.

Since three equations ($\langle 2.3 \rangle$, $\langle 2.5 \rangle$, $\langle 2.14 \rangle$) remain to determine the final five variables (M , t^{TH} , B , i , P), two of them have to be set exogenously by policy.

Sequential vs. Simultaneous Equilibrium Determination It has to be noted that this approach to equilibrium determination—solving for variables ‘one after another’—is sequential in spirit whereas it is an important property of equation systems that they can often only be solved *simultaneously*. However, as long and insofar as it is possible to pair individual variables with certain equations, I do so because I believe it helps explain the underlying economics.

Infinite Recursion So far, it might seem as if the number of equations matched the number of endogenous variables, but this is not the case. In intertemporal optimization problems like the present one, expectations about the future play a crucial role. Even if one argues that the expected marginal utility of consumption $\mathbb{E}_t u_c(c_{t+1}, \cdot)$ in the Euler equation (2.21) is invariant in the constant-endowment case, the model still features expected inflation $\mathbb{E}_t \Pi_{t+1}$ and thus adds the expected price level $\mathbb{E}_t P_{t+1}$ to the list of variables that have to be determined. Since the latter is determined in period $t + 1$ in the same way P_t is determined in the current period t (introducing $\mathbb{E}_t P_{t+2}$, and so on), finding equilibrium ultimately amounts to solving the same problem infinitely often (cf. Blanchard 1979, p. 115).

Arbitrary expectations can lead to viable equilibria; the model is prone to indeterminacy. Of course, the solution lies in the adequate design of policy. While there already exists a provision related to infinite recursion—namely, the transversality condition—it might not be able to restrict the set of possible solutions to a single path as there may be many equilibria that satisfy Equations (2.34)/(2.42). Therefore, it is necessary to find a policy constellation that leads to a unique equilibrium path on which the transversality condition is satisfied (whereas it is not on all other paths). Chapter 4 does this.

Key Takeaways from Chapter 2

The present-value budget equation of the consolidated government emerges as a combination of the household budget constraint and goods-market clearing. In principle, even if all contemporaneous variables are determined, the presence of expectations leads to infinite recursion and indeterminacy of the model.

3. Interactions Arising from the Consolidated-Government Budget Equation

I hint at the difficulties in clearly separating between fiscal and monetary policy and at the formal interconnection of the two policymakers, the central bank and the treasury, in particular with regard to debt, money, inflation, and seigniorage. Towards the end, I present two traditional analyses that discuss the “unpleasant monetarist arithmetic” of fiscal dominance over monetary policy and unearth the fiscal roots of high (or even hyper-) inflation, respectively. The present chapter represents a digression from the main path of the thesis insofar as the approaches described here are not entirely compatible with the model developed in Chapters 2 and 4.

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Derivations for this chapter can be found in Appendix A.

3.1. What Is Fiscal, What Is Monetary Policy?

Who Does It? A typical classification often found in textbooks or dictionaries is based on *who* carries out policy measures: Fiscal policy is understood as action taken by elected government bodies, i.e., spending and taxation at the local, state, national, or possibly even supranational level while monetary policy is carried out by a central bank that is independent from the former entities (cf. Abel, Bernanke, and Croushore 2014, p. 34; Kocherlakota 2008). In a formal representation, the respective entities would be

equipped with separate budget equations; however, there can still be a link in the form of remittances (see Section 3.2 and ↗*More Applications* at the end of Chapter 9).

One example demonstrating the pitfalls of this ‘player-based’ approach is the fact that setting short-term interest rates has not always been a task of the central bank (and does not necessarily need to be, cf. Goodhart 2011, pp. 141-142).⁶ This way, the same policy measure could be considered monetary policy at one point in time and fiscal policy at another. Therefore, while being easily understood, a classification of policies according to ‘who does it’ is not consummate. Monetary policy is not simply what the central bank does, to draw on a proverbial definition of art, neither is fiscal policy merely what the treasury does.

Intended Effects On a more practical note, since the beginning of the Eurozone crisis in 2010, it has often been stated that monetary policy is overreaching because it *in effect* assumes *responsibilities* that should belong to fiscal policy (see, for instance, Fuest 2011, The Guardian 2012, Sinn 2014b, Legrain 2015, Stark 2016). This line of argument implies that rather than the executing entity, it is the responsibility for a certain outcome or intended effect which makes a policy measure monetary or fiscal. Unfortunately, this approach is not perfectly clear-cut either.

I want to start with an intuitive argument which relies on macroeconomic common sense: Typically, levying taxes, not least with the intention of redistribution between certain groups, is thought to be a fiscal responsibility. However, it is also frequently said that “[i]nflation is a tax” (Walsh starts his 2010 textbook chapter on “Money and Public Finance” with this quote; see also ↗*Optimal Policy*, p. 182). More broadly, monetary policy often develops redistributive ‘side effects’ which may be unintended but blur the lines nonetheless.

It is not cast in stone that certain objectives must be reached by one of the two branches. Kocherlakota (2008, Lesson 1) goes even further, stating that “monetary policy is merely fiscal policy by another name.” He gives three examples: The first is that monetary and fiscal policy are linked via budget and resource constraints (see ↗*Who Does It?* above as well as Section 3.2). Second, aiming at the *effects* of policy, he likens the interest rate to a sales tax in that an increase in both would deter the purchase of goods. Finally, monetary policy affects relative prices if some goods’ prices are more flexible than those of others; again, the same effect could also be achieved by fiscal policy, namely if it introduced a good-specific tax (cf. Correia, Nicolini, and Teles 2008).

⁶ In a discussion about budget equations, such as in Section 3.2 below, one typically argues in terms of the monetary base instead of a short-term interest rate, but this would not pose a problem since money supply can also be set (by adjusting it along a money demand curve) such that a desired level for the interest rate is achieved. Interestingly, in his seminal article on the choice between money supply and the interest rate as monetary policy instrument, Poole (1970) concludes by suggesting that fiscal policy is faced with a similar problem, namely, the choice between tax rates and tax volumes.

No Satisfaction All in all, a distinction between monetary and fiscal policy is always blurry on closer inspection. Because they are inseparably linked in the end, the two macroeconomic policy branches can not be sufficiently separated by either the player-based or the intended-effects approach. Any dividing line is drawn arbitrarily so that discussions about, for instance, monetary interfering with fiscal policy are always in danger of being impaired by (or even reducing to nothing but) semantics.

Working Titles To leave it at that and stop using the terms monetary and fiscal policy altogether is of course not an option. Going forward, I typically separate between the two using the player-based approach: In the following section, I introduce the consolidated-government as well as the treasury's and central bank's individual budget equations, which naturally lends itself to a distinction based on entities.

Furthermore, the clarity of exposition is probably greater with the terms 'monetary' and 'fiscal' tied to specific instruments (or, put formally, variables) than to more or less well-defined desired results that are supposed to obtain after possibly complicated policy transmission. To give an example: Defining an increase in transfers (to certain groups of society) as fiscal policy seems more clear-cut than 'relaxing (certain) household-sector budget constraints.' This is exactly because the latter could be achieved by transfer payments just as well as interest-rate decreases—or suitable (relative-) price movements which, in turn, could be brought about by yet other instruments.

3.2. Separating the Consolidated-Government Budget Equation

The Consolidated Government A flow budget equation like

$$G_t + i_t B_{t-1}^H + T_t^{TH} = (B_t^H - B_{t-1}^H) + (M_t - M_{t-1}) \quad \langle 3.1 \rangle$$

is part of many models in monetary macroeconomics. The left-hand side of the equation starts with government consumption G_t and ends with net-of-tax transfers to households T_t^{TH} (cf. also Footnote 7). In between are the interest payments on the part of treasury debt that is held by the private (household) sector B_{t-1}^H . Recall that i_t is defined to be the net nominal interest rate between periods $t - 1$ (when the contract is closed) and t (when the interest payment is due; see *Fisher Equation*, p. 24). On the right-hand side, the consolidated government acquires means by either increasing the outstanding stock of debt ($B_t^H - B_{t-1}^H$) or money supply ($M_t - M_{t-1}$).

Branches The consolidated-government budget equation can be split up into two separate parts:

$$G_t + i_t B_{t-1}^T + T_t^{TH} = (B_t^T - B_{t-1}^T) + T_t^{CT} \quad \langle 3.2 \rangle$$

$$(B_t^C - B_{t-1}^C) + T_t^{CT} = i_t B_{t-1}^C + (M_t - M_{t-1}) \quad \langle 3.3 \rangle$$

Equation $\langle 3.2 \rangle$ is attributed to a treasury branch and Equation $\langle 3.3 \rangle$ to a monetary branch of the consolidated government or, in short, the central bank. Treasury debt is a liability to the treasury (B^T) and an asset to the central bank if it holds it (B^C). Further,

$$B^T = B^H + B^C \quad \Leftrightarrow \quad B^H \equiv B^T - B^C \quad \langle 3.4 \rangle$$

which represents two interpretations of the same fact: Treasury debt can be held either by the central bank or by households (on the left) and the private sector is counterparty to a net debt position of the consolidated government sector (on the right, cf. Walsh 2010, Ch. 4, for example). For the sake of clarity, I will not use ‘treasury’ and ‘government’ synonymously when discussing such separations.⁷ Finally, T^{CT} are remittances from the central bank to the treasury.

What is the point of this exercise? Equation $\langle 3.1 \rangle$ is the pivotal element in macroeconomic policy. It can be expanded to allow for greater detail, as in Equations $\langle 3.2 \rangle$ and $\langle 3.3 \rangle$ or in a more in-depth analysis of the interplay between the treasury and the central bank. Benigno and Nisticò (2015; 2017) deliver such an analysis and show that macroeconomic outcomes can critically depend on the interplay between these two institutions and their balance sheets, where central-bank remittances to (and possible ‘backing’ of central-bank losses by) the treasury play a crucial role.⁸

This interplay also has serious implications for real-world policy. To give an example, central-bank independence might be upheld in fair weather and in times of modest tension, but when push comes to shove, incentives to ‘adjust’ the institutional setup might grow stronger (wars are a typical example, but also the Eurozone crisis starting in 2010).

⁷ The pitfalls of notation: In a trade-off between simplicity and consistency throughout this thesis, I have decided to use the *symbol* T for transfers and the *superscript* T for the treasury. To denote treasury consumption, I therefore ‘have to’ fall back on the common use of G (‘government expenditure’).

⁸ Their approach would make for an interesting extension of the sticky-price model developed in Chapter 7. I abstain from this possibility for three reasons: First, they consider *Optimal Policy* (p. 182), which involves a fair amount of additional formalism and discussion of consistency. Second, first-order Taylor expansions are not sufficient to capture the effects of risk (cf. Isoré and Szczerbowski 2017, p. 101), implying even an even greater formal extent if handled correctly. The final and most important argument is also related to risk characteristics: I am unsure about the equality of risk-free and risky rates in a model with risk-averse households and a ‘one-sided’ (i.e., strictly positive) shock variable as implied by Benigno and Nisticò 2017, pp. 6–10.

The change in circumstances and the transitions between the resulting regimes are seldomly part of the literature. Canzoneri, Cumby, and Diba (2011, p. 941) note that a possible “war of attrition” between monetary and fiscal authorities has not been formally modeled yet: “How would financial markets react? Would they limit the government’s purchasing power [...]? Would they impose a risk premium on government debt or an inflation premium on all nominal assets? Who would give in first?”

In the subsequent sections, I follow Walsh (2010, Ch. 4) in broad strokes and describe the interactions between monetary and fiscal policy that are well-established, but it does not hurt to explain notation and do some groundwork in the pertinent issues.

3.3. Treasury Debt, Monetization, and Unexpected Inflation

Monetization Monetization describes central-bank purchases of treasury debt in exchange for newly created money. As Equations (3.1)-(3.4) show, monetization is mainly a compositional issue in the consolidated-government budget equation; however, its effect on inflation and the interest-rate savings it allows might make it interesting for policymakers (cf. Section 3.4).

Debt Debasement Via Unexpected Inflation More relevant to this section is the share of treasury debt that is held by the private sector. Dividing Equation (3.1) by the price level P_t leads to the *real* consolidated-government budget equation

$$g_t + r_t b_{t-1}^H + t_t^{TH} = (b_t^H - b_{t-1}^H) + \left(m_t - \frac{m_{t-1}}{\Pi_t} \right). \quad (3.5)$$

The difference between the ex-ante and the ex-post real interest rate can be used to define a ‘surprise inflation’ term \dot{s}_t :

$$\mathbb{E}_{t-1} r_t - r_t = \left[\frac{(\pi_t - \mathbb{E}_{t-1} \pi_t) (1 + \mathbb{E}_{t-1} r_t)}{\Pi_t} \right] \equiv \dot{s}_t \quad (3.6)$$

Adding the difference between what the private sector expected to earn and what it actually earns on its public-debt holdings in real terms, i.e., $(\mathbb{E}_{t-1} r_t - r_t) b_{t-1}^H = \dot{s}_t \cdot b_{t-1}^H$, to both sides of Equation (3.5) then yields

$$g_t + \mathbb{E}_{t-1} r_t \cdot b_{t-1}^H + t_t^{TH} = (b_t^H - b_{t-1}^H) + \dot{s}_t \cdot b_{t-1}^H + \left(m_t - \frac{m_{t-1}}{\Pi_t} \right). \quad (3.7)$$

In comparison, the latter equation features the ex-ante instead of the ex-post real interest rate on the left-hand side and an additional term on the right-hand side (more on this

in Section 3.4). To make it more intuitive, add and subtract unity to turn net into gross inflation rates in the surprise-inflation term:

$$s_t b_{t-1}^H = \left\{ \frac{[(1 + \pi_t) - (1 + \mathbb{E}_{t-1} \pi_t)] (1 + \mathbb{E}_{t-1} r_t)}{\Pi_t} \right\} b_{t-1}^H.$$

The numerator of the fraction is the difference between the gross nominal interest rate that would completely insulate creditors given *actual* inflation and the gross nominal interest rate that is actually paid because of *expected* inflation. Made real by the denominator and multiplied by the real notional amount b_{t-1}^H , this is the consolidated government's yield from unexpected inflation.

3.4. Seigniorage

3.4.1. Seigniorage Measures

Real-Money Seigniorage After adding as well as subtracting m_{t-1} in (3.7) and rearranging, real-money-related seigniorage can be written as

$$\zeta_t^m \equiv (m_t - m_{t-1}) + \frac{\pi_t}{\Pi_t} m_{t-1}. \quad (3.8)$$

One source is represented by the bracket on the right-hand side: If the private sector wants to hold higher amounts of real money ($m_t - m_{t-1}$ is positive), the consolidated government as its monopoly supplier can acquire real resources in exchange for newly created balances. The second term describes another source: Households adjust their nominal money holdings to a rising price level in order to maintain a constant real money stock. (Cf. also Friedman 1971, p. 847.) As long and insofar as the private sector simply adjusts nominal money holdings at the rate of inflation, the government can obtain real resources even if no new real money is created. The real value of this effect is the rate of adjustment, i.e., π_t , times the existing real money stock m_{t-1} , devalued by the gross inflation rate $(1 + \pi_t)$.⁹

Interest-Saving Seigniorage Another definition shows that seigniorage can also be positive if the inflation rate is zero. To see this, define total consolidated-government debt

⁹ Consider the following example: Let $M_1 = 100$ and $P_1 = 1$ so that $m_1 = 100$. If the rate of inflation is 5%, $P_2 = 1.05$. In order to maintain $m_2 = m_1 = 100$, the consolidated government must set $M_2 = 105$, i.e., it must create 5 additional units of nominal money. Their real value is $5/1.05 \approx 4.76$, which the consolidated government now has at its disposal.

as

$$Z_t \equiv B_t^H + M_t \quad \langle 3.9 \rangle$$

and add $\mathbb{E}_{t-1} r_t \cdot m_{t-1}$ on both sides of Equation $\langle 3.7 \rangle$; after several rearrangements, the result is

$$g_t + (1 + \mathbb{E}_{t-1} r_t) z_{t-1} + t_t^{TH} = z_t + s_t z_{t-1} + \left(\frac{i_t}{1 + \pi_t} \right) m_{t-1}. \quad \langle 3.10 \rangle$$

Using this form, an alternative seigniorage measure is defined as

$$\varsigma_t^i \equiv \left(\frac{i_t}{\Pi_t} \right) m_{t-1} \quad \langle 3.11 \rangle$$

and describes the real value of interest expenses that the consolidated government saves by printing money instead of issuing bonds. A simple one-for-one exchange between the two keeps total consolidated-government debt Z constant, but the share of interest-bearing debt changes. Note that this measure (following Walsh 2010, p. 140) does not include the effects of surprise inflation $s_t z_{t-1}$, which is an arbitrary definition.

More Measuring *Somewhat* similarly, but in a different formal representation, Drazen (1985) also distinguishes two money-related sources of revenue for the consolidated government, one from an expansion of money supply (cf. his p. 327) and another from an interest-rate differential relating to the stock of existing money (p. 328), subsuming previously suggested measures under his approach as special cases.

King and Plosser (1985, pp. 149-150) define six measures of seigniorage for the United States, among which are also the aforementioned. Further, they relate the Fed's total earnings (i.e., nominal interest on its portfolio), its net interest earnings (i.e., net of operating costs), remittances to the treasury, as well as a hybrid income variable relative to GNP to get four additional seigniorage measures in terms of real GNP.

3.4.2. Stylized Facts

In industrialized countries, seigniorage is typically in the low single digits as a percentage of GDP:

- Interest-saving seigniorage amounted to 0.30% of GDP in the United States in 2007 and 0.27% of GDP in the Eurozone in 2009 (cf. Bénassy-Quéré et al. 2010, p. 308).
- Defining seigniorage as the mere change in base money, Grilli, Masciandaro, and Tabellini (1991, pp. 360-361) find the average ratio of seigniorage to GNP has been

relatively constant at around 1% on average while tax revenues increased significantly. (King 1995 complements their data with a study by Hudson and Nolan 1995 which is undiscoverable unfortunately.)

- The six measures of King and Plosser (1985) for the United States range between 0.02% and 1.37% of real GNP before World War II and between 0.25% and 0.47% after (1953 until 1982).
- An appended table in Aisen and Veiga (2008) shows that Israel has the highest mean real-money seigniorage earnings with 8.6% of GDP; the median of these time-dimension means is 1.4% of GDP. Note that the underlying IMF dataset carries information on 119 countries, that is, not only industrialized countries.
- As a share of (consolidated-) government revenues, seigniorage can play a significantly larger role: While the average figure in industrialized countries is still low at 1.64%, it amounts to 14.65% on average in developing countries (Aisen and Veiga 2008, p. 30).

3.4.3. The Seigniorage Laffer Curve

A Minimalistic Model This section rests on the baseline optimizing model of Section 2, adding the specific utility function

$$u(c_t, m_t) = \ln c_t + m_t (O_I - O_{II} \ln m_t) \quad \langle 3.12 \rangle$$

taken from Calvo and Leiderman (1992). O_I and O_{II} denote some variables not specified in more detail, I assume them to be time-invariant for simplicity (Table A.1 at the end of Appendix A.1.2 lists all such ‘minor’ coefficients used in this chapter). Consequently, money demand $\langle 2.22 \rangle$ takes the specific form

$$m_t = O_{III} \exp\left(-\frac{1}{O_{II}c_t} \cdot \frac{i_{t+1}}{1+i_{t+1}}\right), \quad \langle 3.13 \rangle$$

where $O_{III} \equiv \exp[(O_I/O_{II}) - 1]$ (see Appendix A.1.1 for derivations of the equations in this section). Plugging this into the definition of interest-saving seigniorage $\langle 3.11 \rangle$,

$$\varsigma_t^i = \left(\frac{i_t}{1+\pi_t}\right) O_{III} \exp\left(-\frac{1}{O_{II}c_{t-1}} \cdot \frac{i_t}{1+i_t}\right), \quad \langle 3.14 \rangle$$

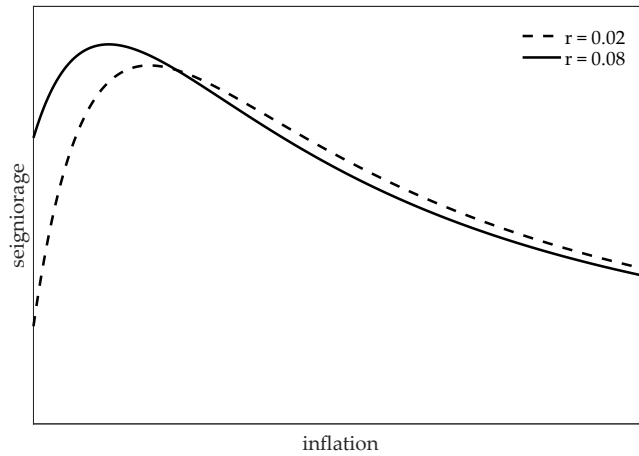


Figure 3.1: *The Microfounded Seigniorage Laffer Curve.* ♦ Source: Own illustration replicating Figure 4.1 in Walsh (2010, p. 155). ♦ *Explanations:* Sketch for illustrative purposes based on Equation $\langle 3.14 \rangle$ (inflation is an argument of the nominal interest rate).

and rearranging, it is possible to trace the seigniorage-maximizing inflation rate:

$$\pi_t^{\max} = \frac{1}{(1 + r_t)(1 - O_{\Pi}c_{t-1})} - 1 \quad \langle 3.15 \rangle$$

This corresponds to the maximum of the seigniorage Laffer curve in Figure 3.1. Below π_t^{\max} , the government can increase seigniorage by pushing up inflation; on the contrary, inflation rates above π_t^{\max} curb money demand so strongly that seigniorage falls again (via higher nominal interest rates in Equation $\langle 3.13 \rangle$). Note that Equation $\langle 3.15 \rangle$ implicitly makes this a steady-state representation as the expectation operator is left out, that is, expected equals actual inflation (cf. Walsh 2010, pp. 154-155).

Terminology: Laffer vs. Bailey Of course, the term ‘Laffer curve’ originated only later and put a name on the long-known fact that increasing tax rates not always leads to rising tax volumes (cf. Laffer 2004, Wanniski 1978). Some authors (McCandless 2008, p. 217, for instance) would call Figure 3.1 a ‘Bailey curve’ after Bailey (1956), who applies this notion to the study of inflation (tax) rates and seigniorage revenues directly.

Traditional Literature vs. Microfoundations The above microfounded result is not completely ‘backward compatible’ with the traditional literature. Cagan (1956) is a classic reference in the discussion of seigniorage and hyperinflations. There, money demand

takes the form

$$m_{C56,t} \equiv \exp(-O_{IV}\mathbb{E}_t\pi_{t+1}), \quad \langle 3.16 \rangle$$

where O_{IV} is a coefficient. By comparison, money demand from Calvo and Leiderman (1992) $\langle 3.13 \rangle$ can be rearranged to read

$$m_t = O_{VI} \exp\left(O_V \frac{1}{\mathbb{E}_t\Pi_{t+1}}\right), \quad \langle 3.17 \rangle$$

where O_V and O_{VI} are shorthands for more complicated terms (see Appendix A.1.2). Building a money demand equation like $\langle 3.16 \rangle$ on microfoundations is not possible, however. Figure 3.2 sketches both types: the solid line corresponds to the microfounded

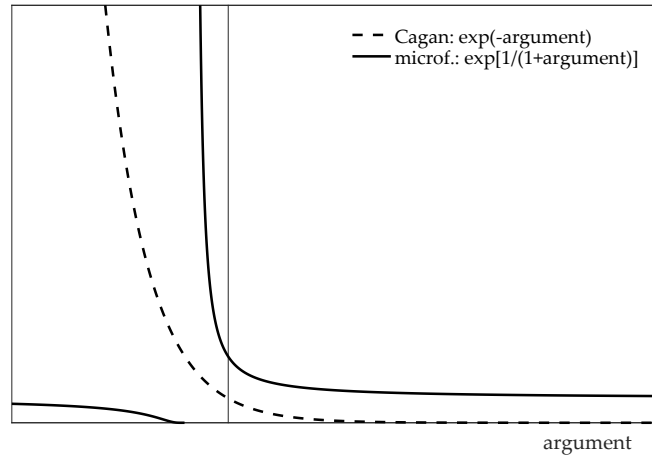


Figure 3.2: *Similarity of Certain Exponential Functions Which Are Part of Money Demand Functions.* ♦ Source: Own illustration. ♦ Explanations: The dashed and solid lines would correspond to the money-demand specifications of Equation $\langle 3.16 \rangle$ (Cagan 1956) and Equation $\langle 3.17 \rangle$ (microfoundations), respectively.

money demand in Equation $\langle 3.17 \rangle$ and the dashed line to the traditional form introduced by Cagan (1956) in Equation $\langle 3.16 \rangle$. It is obvious that the curves behave somewhat similarly for positive inflation rates (cf. Walsh 2010, p. 154; Appendix A.1.2 provides a more in-depth discussion), which is why I take the liberty of making some subsequent arguments (in Sections 3.5 and 3.6) based mostly on the traditional approach.

Different Functional Forms As a final technical note, the utility function $\langle 3.12 \rangle$ is not an arbitrary choice or a *pars pro toto* for a whole class of similar utility functions, other microfounded setups simply do not produce Laffer-type curves for seigniorage (see Ap-

pendix A.1.3 for two examples). Depending on how much value one places on micro-foundations, this poses a severe detriment to the generality of the Laffer-curve result.

3.5. Unpleasant Monetarist Arithmetic

Overview In their 1981 paper, Sargent and Wallace challenge the monetarist dictum that “inflation is always and everywhere a monetary phenomenon” (Friedman 1963) by varying the balance of power between the treasury and the central bank—in a monetarist setting. As already laid out in Sections 3.1-3.2, the two policymakers are connected through the consolidated-government budget equation. Cooperative coordination schemes aside, one of both assumes a dominant position which allows it to decide on its policy stance first whereas the other can only go second and has to adjust policy to ensure the budget equation is satisfied. The unpleasantness of the underlying arithmetic derives from the possibility of a dominant treasury that sooner or later rids the central bank of its power to control inflation.

Assumptions While Sargent and Wallace (1981) employ an overlapping-generations model with growth, the main points of their analysis can be transferred to the representative-agent model without growth as laid out in Section 2.¹⁰ Importantly, the results obtained in the following hinge on the assumption that the real interest rate on debt is greater than the (zero) growth rate of the economy. Also, it is assumed that the ability (or willingness) of the private sector to hold consolidated-government debt is limited to a certain maximal real value. Since their model assumes perfect foresight, the expectation operator is dropped in this section.

Money Demand Money demand proportionately depends on real income but not on expected inflation, at least in its basic variant. Further, since endowments y as well as treasury expenditures g and thus (via goods-market clearing (2.27), see Section 2.4) also consumption c are fixed by assumption here, $L(\cdot)$ can be replaced by the constant L_{SW81} in Equation (2.25), so that the price level varies with money supply:

$$P_t = \frac{M_t}{L(c_t, i_{t+1})} = \frac{M_t}{L_{SW81}} \quad \langle 3.18 \rangle$$

It should be noted that this assumption about money demand and the resulting Equation (3.18), like Equation (3.16) in Section 3.4.3, represent a departure from pure micro-foundations.

¹⁰ The 1981 paper is reprinted in Sargent (2013, Chapter 5) with the formulae typeset more neatly but also several typing errors.

Policy Initial outstanding real debt b_{t-1} as well as money balances M_{t-1} are predetermined. The novelty in Sargent and Wallace (1981) is that they assume the treasury to be dominant: In t , it exogenously decides upon and announces a path for real future budget deficits d_t ; the usual assumption is that they are positive.

By contrast, the central bank's policy is two-tiered: At first, it chooses a net growth rate for money supply μ_M so that

$$M_{t+1} = (1 + \mu_M) M_t. \quad \langle 3.19 \rangle$$

Because of Equation $\langle 3.18 \rangle$, this determines the inflation rate, which is also equal to $(1 + \mu_M)$. Still considering the case in which the combination of both policies with debt dynamics (a real interest rate greater than the growth rate) increases real consolidated-government debt b , the 'debt limit' assumption implies a second stage: Starting with some future period T , real debt must remain constant, which requires the central bank as the passive policymaker to set money supply in a way that generates sufficient seigniorage. Defaulting is not an option—not even in real terms, which is to say that treasury debt b is in effect indexed (Sargent and Wallace 1981, p. 4, argue that “such a default option [inflating the debt away] is not available as a policy to which a government can plan to resort persistently”).

The course of action in the following is to demonstrate how the growth rate of money affects the real value of treasury bonds outstanding in T , given the path of budget deficits set by the treasury, and how this amount of bonds affects inflation thereafter. Since there are no stochastic elements or 'surprises' of any kind, the credible policy announcements made in t are enough to determine a perfect-foresight equilibrium path.

How Money Growth Drives Debt Until T The consolidated-government budget equation with coupon bonds $\langle 2.38 \rangle$ can be rearranged to read

$$b_t = (1 + r_t) b_{t-1} + d_t - \frac{M_t - M_{t-1}}{P_t} \quad \text{for } t \leq T, \quad \langle 3.20 \rangle$$

which becomes

$$b_t = (1 + r_t) b_{t-1} + d_t - \frac{\mu_M}{1 + \mu_M} L_{\text{SW81}} \quad \text{for } t \leq T$$

when combined with Equations (3.18) and (3.19). Shifting this expression forward to time T and expressing it in terms of variables known in t by repeated substitution,

$$b_T = \prod_{j=t}^T (1 + r_j) b_{t-1} + \sum_{j=t}^T \left[\prod_{k=j}^T (1 + r_k) \right] \left(d_j - \frac{\mu_M}{1 + \mu_M} L_{\text{SW81}} \right),$$

reveals that the lower is money growth μ_M the higher is real debt in T . (Do not confuse this with the present-value budget equation (2.43)—the latter features a forward-looking sum term, ranging from t into the indefinite future, while the equation above is ‘backward-looking in the future,’ ranging from a future period T back to the present period t .)

Inflation After T In the second stage, real debt is to be held constant at b_T so that the budget equation (3.20) (in combination with Equation (3.18) solved for M) becomes

$$b_T = (1 + r_t) b_T + d_t - \frac{M_t - M_{t-1}}{P_t} \Leftrightarrow \Pi_t = \frac{L_{\text{SW81}}}{L_{\text{SW81}} - r_t b_T - d_t} \quad \text{for } t > T. \quad (3.21)$$

Inflation after T increases in b_T . (The right half of Equation (3.21) also implies the assumed debt limit: An unboundedly large b_T would make the right-hand side and thus also gross inflation on the left-hand side negative, which would in turn imply a negative price level. Cf. Sargent and Wallace 1981, p. 4.)

Tighter Money Now, Higher Inflation Later To sum up, if restrictive monetary policy (in the form of lower money growth μ_M) drives up real consolidated-government debt until a certain point in time and if, afterwards, this higher amount of debt is to be stabilized via endogenous monetary policy, the result is that “[t]ighter money now can mean higher inflation eventually” (Sargent and Wallace 1981, p. 2).

To add a qualification, this is true only if

$$\frac{L_{\text{SW81}}}{L_{\text{SW81}} - r_t b_T - d_t} > 1 + \mu_M,$$

that is, if the inflation rate after T (on the left-hand side, from Equation (3.21)) actually turns out to be greater than that before (on the right-hand side, implied by Equations (3.18) and (3.19)). In very restrictive domains, this is obvious: Given nonnegative budget deficits, lowering money growth μ_M from an already low value to zero (implying a gross inflation rate of unity) further increases real debt b_T , which then serves to decrease the denominator in the above inequality and thus yields a gross inflation rate greater

than unity.

A bit more drastic is the “inflation juggernaut” of Canzoneri, Cumby, and Diba (2011, p. 940): In a representative-agent setup, they show that lowering inflation by $\Delta\pi$ in period t leads to an increase by $(1+r)^j \Delta\pi$ in period $t+j$ because the treasury is forced to incur (real) debt in this case that is compounded at the real interest rate—“[a]n inflation hawk at the central bank can look good during his term in office, but only at the expense of his successors.”

Tighter Money Now, Higher Inflation Now The analysis carried out so far is of course very simple. Mostly in an appendix, Sargent and Wallace (1981, pp. 5-6, 10-15) study a richer model in which, similar to Cagan (1956), money demand depends not only on consumption but on expected inflation as well (cf. also Section 3.6). Equation (2.25), $M_t/P_t = L(c_t, i_{t+1})$, applies ‘fully’ again (however, the specific function used by Sargent and Wallace 1981, p. 10, can not be derived using microfoundations either). This complicates matters so much that the results reported eventually are only numerical and exemplary rather than analytical and general—but still “spectacular” (pp. 6, 14-15): Given certain parameter constellations, a tighter monetary policy can lead to higher inflation *instantaneously*, not only after T . The reason is that the expectation of faster money creation after T associated with tighter monetary policy beforehand raises inflation expectations and thus lowers real money demand. With the money growth rate given before T , only prices remain to adjust to lower $L(\cdot)$, which overcompensates the downward effect of slower initial money creation on inflation.

It should be noted, however, that Carlstrom and Fuerst (2000, p. 25) consider this outcome a “theoretical curiosity” because the interest elasticity of money demand (as a proxy for the influence of inflation expectations) would have to be greater than unity, which it is not empirically.

3.6. Fiscally Induced High and Hyperinflations

3.6.1. The Model of Bruno and Fischer (1990)

Preliminaries In the present subsection, I abandon the path of pure microfoundations and loosely follow Bruno and Fischer (1990) in adopting a Cagan (1956) money demand function to study how an economy can slide off into high or even hyperinflation.

I abstract from debt issuance, so the budget equation (3.5) reduces to

$$g_t + t_t^{TH} = m_t - \frac{m_{t-1}}{1 + \pi_t}.$$

Recalling the definition of real-money seigniorage (3.8) and denoting the treasury’s

nominal and real primary budget deficit by

$$D_t \equiv G_t + T_t^{TH} \Leftrightarrow d_t \equiv g_t + t_t^{TH}, \quad \langle 3.22 \rangle$$

respectively, the budget equation becomes

$$d_t = \zeta_t^m; \quad \langle 3.23 \rangle$$

the primary budget deficit is solely financed by seigniorage.

A peculiarity of the translation from continuous into discrete time (Cagan 1956 as well as Bruno and Fischer 1990 use the former) is that growth rates need to be defined slightly differently from usual, that is, with time- t instead of time- $(t-1)$ variables in the denominator:

$$\mu'_{M,t} \equiv \frac{\Delta M_t}{M_t}, \quad \pi'_t \equiv \frac{\Delta P_t}{P_t}, \quad \langle 3.24 \rangle$$

This adjustment is also made in the money demand function of Cagan (1956) $\langle 3.16 \rangle$:

$$m'_{C56,t} \equiv \exp(-O_{IV} \mathbb{E}_t \pi'_{t+1}) \quad \langle 3.25 \rangle$$

With real-money seigniorage $\langle 3.8 \rangle$ rearranged to $\zeta_t^m = \Delta M_t / P_t$, the combination of Equations $\langle 3.23 \rangle$, $\langle 3.24 \rangle$, and $\langle 3.25 \rangle$ yields

$$d_t = \frac{\Delta M_t}{P_t} = \frac{\Delta M_t}{M_t} \cdot \frac{M_t}{P_t} = \mu'_{M,t} m_t = \mu'_{M,t} \exp(-O_{IV} \mathbb{E}_t \pi'_{t+1}). \quad \langle 3.26 \rangle$$

In line with previous results, there is a limit to what the consolidated government can finance via seigniorage (see \nearrow *Steady-State Equilibria* below).

Steady-State Condition Aiming for a maximal sustainable (read: steady-state) level of seigniorage revenues means that $m_t = M_t / P_t$ does not change anymore:

$$\Delta m_t = \frac{\partial m_t}{\partial M_t} \Delta M_t + \frac{\partial m_t}{\partial P_t} \Delta P_t = \frac{1}{P_t} \Delta M_t - \frac{M_t}{P_t^2} \Delta P_t \stackrel{!}{=} 0 \Leftrightarrow \mu'_{M,t} = \pi'_t \quad \langle 3.27 \rangle$$

(The equation is written with normal time indices instead of \square_{SS} subscripts because it is used again later.)

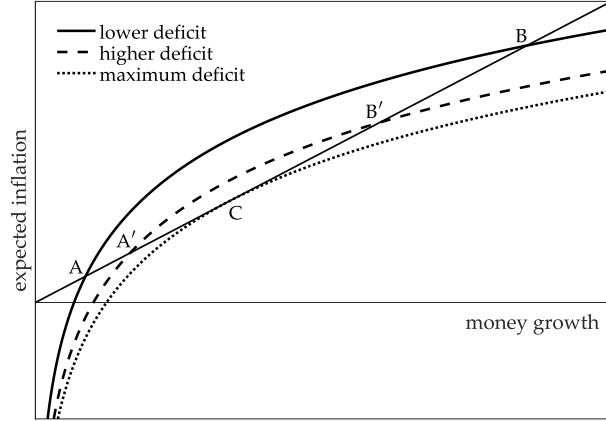


Figure 3.3: *Inflation as a Function of Money Growth.* \diamond Source: Own illustration based on Bruno and Fischer (1990, p. 355) and Walsh (2010, p. 157). \diamond Explanations: Plot of Equation (3.29) (the positively sloped straight line is the steady-state condition (3.27)). The budget deficit enters as a negative intercept.

Steady-State Equilibria Continuing in this fashion, inflation (and, since this is a steady-state deliberation, also expected inflation) can be substituted by the net rate of money growth in Equation (3.26), which then allows to find the seigniorage-maximizing rate of the latter:

$$\begin{aligned} \max_{\mu'_M} \mu'_M \exp(-O_{IV} \mu'_M) &\Rightarrow \exp(-O_{IV} \mu'_M) - O_{IV} \exp(-O_{IV} \mu'_M) \mu'_M \stackrel{!}{=} 0 \\ \Leftrightarrow \mu'^{\max}_M &= \frac{1}{O_{IV}} \end{aligned} \quad (3.28)$$

Plugging this into Equation (3.26) yields the maximal amount of seigniorage and, thus, the maximal budget deficit:

$$\zeta^{m,\max} = d^{\max} = [O_{IV} \exp(1)]^{-1}$$

In a graphical analysis, solving Equation (3.26) for the expected inflation rate,

$$\mathbb{E}_t \pi'_{t+1} = \frac{1}{O_{IV}} \ln \left(\frac{\mu'_{M,t}}{d_t} \right) = \frac{\ln \mu'_{M,t}}{O_{IV}} - \frac{\ln d_t}{O_{IV}}, \quad (3.29)$$

yields the curved lines in Figure 3.3; the straight (45°) line represents the steady-state condition (3.27) (see also Equation (3.31) and the surrounding discussion below). The

solid and the dashed lines differ insofar as they are associated with different budget deficits; as can be seen from Equation (3.29), a higher deficit pushes the graph down. For d^{\max} , the dotted graph intersects the 45° line at a single point (C), meaning there is a unique steady state. Higher deficits are not associated with steady-state equilibrium since the seigniorage equation (3.29) (or (3.26)) and the steady-state-condition line (3.27) do not intersect anymore. For deficits below d^{\max} , the two resulting intersection points (A and B or A' and B', respectively) indicate that there are two steady states, one with low and one with high inflation rates. (The discussion of balanced budgets and budget surpluses is more complicated due to the logarithmic form, see Appendix A.1.4.1.)

Adaptive Expectations Assume inflation expectations change adaptively following

$$\Delta \mathbb{E}_t \pi'_{t+1} \equiv \mathbb{E}_t \pi'_{t+1} - \mathbb{E}_{t-1} \pi'_t = \alpha (\pi'_t - \mathbb{E}_{t-1} \pi'_t), \quad (3.30)$$

where α is the adaptive-expectations coefficient. At time t , $\mathbb{E}_{t-1} \pi'_t$ is predetermined, the consolidated government can influence π'_t , and will thereby also influence $\mathbb{E}_t \pi'_{t+1}$ which is, in turn, relevant for money demand (3.25) and thus for seigniorage. How is actual inflation determined? Equation (3.27) implies that

$$\frac{\Delta m_t}{m_t} = \mu'_{M,t} - \pi'_t.$$

To get Δm_t , we use the total differential of money demand (3.25),

$$\begin{aligned} \Delta m_t &\equiv \frac{\partial m_t}{\partial t} = \frac{\partial m_t}{\partial \mathbb{E}_t \pi'_{t+1}} \cdot \Delta \mathbb{E}_t \pi'_{t+1} = -O_{IV} \exp(-O_{IV} \mathbb{E}_t \pi'_{t+1}) \Delta \mathbb{E}_t \pi'_{t+1} \\ &\Rightarrow \frac{\Delta m_t}{m_t} = -O_{IV} \Delta \mathbb{E}_t \pi'_{t+1}, \end{aligned}$$

so that

$$\pi'_t = \mu'_{M,t} + O_{IV} \Delta \mathbb{E}_t \pi'_{t+1}.$$

Inserting the adaptive expectation formation scheme (3.30) gives

$$\pi'_t = \frac{\mu'_{M,t} - \alpha O_{IV} \mathbb{E}_{t-1} \pi'_t}{1 - \alpha O_{IV}}.$$

Finally, re-inserting this into the expectation formation scheme $\langle 3.30 \rangle$ leads to the actual change in inflation expectations as a function of the lagged expectation $\mathbb{E}_{t-1}\pi'_t$ and current policy $\mu'_{M,t}$:

$$\Delta \mathbb{E}_t \pi'_{t+1} = \frac{\varkappa (\mu'_{M,t} - \mathbb{E}_{t-1} \pi'_t)}{1 - \varkappa O_{IV}}. \quad \langle 3.31 \rangle$$

In steady state, it is zero because $\mu'_{M,SS} = \pi'_{SS} = \mathbb{E}_{SS} \pi'_{SS}$.

Dynamics Outside of Steady State Suppose that in t the consolidated government decides to increase money growth over and above the steady-state level $\mu'_{M,t-1}$ ($= \mathbb{E}_{t-1} \pi'_t$):

$$\mu'_{M,t} = \mu'_{M,t-1} + \Delta \mu'_{M,t} = \mathbb{E}_{t-1} \pi'_t + \Delta \mu'_{M,t} \quad \langle 3.32 \rangle$$

As a direct effect, seigniorage earnings are increased, yet at the same time, rising inflation expectations indirectly depress seigniorage via money demand $\langle 3.25 \rangle$. In order to balance these two effects and thus be able to keep the desired budget deficit constant in Equation $\langle 3.26 \rangle$, a given change in money demand would require a certain adjustment of inflation expectations:

$$\begin{aligned} \Delta \left(\frac{\ln d_t}{O_{IV}} \right) &= \frac{\partial (\ln \mu'_{M,t-1} / O_{IV})}{\partial \mu'_{M,t-1}} \Delta \mu'_{M,t} - \frac{\partial \mathbb{E}_t \pi'_{t+1}}{\partial \mu'_{M,t-1}} \Delta \mu'_{M,t} \stackrel{!}{=} 0 \\ \Leftrightarrow \Delta \mathbb{E}_t \pi'_{t+1} &= \frac{1}{O_{IV} \mu'_{M,t-1}} \Delta \mu'_{M,t} \end{aligned} \quad \langle 3.33 \rangle$$

(The partial derivatives feature $\mu'_{M,t-1}$ to indicate the ‘point of departure,’ while $\Delta \mu'_{M,t}$ denotes the deviation of money growth from this steady state at time t . A slightly different derivation that arrives at the same result is laid out in Appendix A.1.4.2.)

However, the actual change $\langle 3.31 \rangle$ (combined with Equation $\langle 3.32 \rangle$)

$$\Delta \mathbb{E}_t \pi'_{t+1} = \frac{\varkappa}{1 - \varkappa O_{IV}} \Delta \mu'_{M,t} \quad \langle 3.34 \rangle$$

typically deviates from the requirement $\langle 3.33 \rangle$, leading to the question whether it is greater or smaller. Comparing the coefficients of Equations $\langle 3.33 \rangle$ and $\langle 3.34 \rangle$ (indicating steady-state money growth by the SS subscript), the actual is greater than the required

change in inflation expectations if

$$\frac{1}{O_{IV}\mu'_{M,SS}} < \frac{\varkappa}{1 - \varkappa O_{IV}} \Leftrightarrow \mu'_{M,SS} > \frac{1 - \varkappa O_{IV}}{\varkappa O_{IV}}. \quad (3.35)$$

This means that the ‘point-of-departure’ steady state has to be above a certain threshold, which is in line with Bruno and Fischer (1990, p. 357); in Figure 3.3, points B and B’ are natural candidates (I skip the discussion of points A and A’ here; see *Elegance* below).

Further, in order for $\Delta E_t \pi'_{t+1}$ not to change signs (i.e., in order for the denominator to be positive) in Equation (3.34), it must hold that

$$1 - \varkappa O_{IV} > 0 \Leftrightarrow \varkappa < \frac{1}{O_{IV}}, \quad (3.36)$$

otherwise inflation expectations *decrease* when money growth is increased. If these two conditions are fulfilled, an increase in money growth over and above its steady-state level will increase inflation expectations ‘too much,’ depressing money demand below the level that fits a constant budget deficit in Equation (3.26), thus leading the consolidated government to increase money growth even further in a futile attempt to defend said budget deficit. Because this happens on an explosive path, the result is hyperinflation.

Specific Example It can only serve as an example, but using the seigniorage-maximizing money growth rate (3.28), condition (3.35) becomes

$$\frac{1}{O_{IV}} > \frac{1 - \varkappa O_{IV}}{\varkappa O_{IV}} \Leftrightarrow \varkappa > \frac{1}{1 + O_{IV}}.$$

In short: Given μ'_M^{\max} , if the speed of expectation adjustment satisfies $(1 + O_{IV})^{-1} < \varkappa < O_{IV}^{-1}$, a further increase in money growth leads to hyperinflation. (How such hyperinflations *end* is explained by Sargent 1982b in a study that also emphasizes fiscal influences and that foreshadows some of the results derived in the subsequent chapters.)

3.6.2. Caveats and Remarks

Elegance I must concede that the present subsection is not as elegant in constructing a hyperinflationary result as the main literature followed (Bruno and Fischer 1990, Walsh 2010, pp. 156-159), which is mostly due to the attempted translation into discrete time. In the continuous-time version of the model, an increase in money growth creates a direct feedback between the respective equivalents of the actual adjustment of inflation expectations (3.31) and the 45° line (3.27): With the respective adjustments, the change

in inflation expectations adheres to

$$\frac{\partial \mathbb{E}_t \pi}{\partial t} = \frac{\alpha (\mu_M - \mathbb{E} \pi)}{1 - \alpha O_{IV}},$$

which has inflation expectations increasing instantaneously for money growth above the 45° line *et vice versa*. Such a feedback can not be generated as easily with discrete time if one cares for the exact timing of expectation formation. Strictly speaking, Figure 3.3 suffers from an inconsistency because Equations (3.29) and (3.27) require different ordinates: $\mathbb{E}_t \pi'_{t+1}$ for the former and π'_t for the latter. All the same, against the background of the discussion around Equation (3.31) (constant inflation and inflation expectations in steady state) and knowing that the derivation is smoother with continuous time, this seems tolerable.

Asymmetry To touch on another issue, the continuous-time dynamics also rest on condition (3.36), but nothing further (Equation (3.35) for instance). If it is satisfied, points B and B' in Figure 3.3 are unstable equilibria, leading to hyperinflation if money growth is increased or to the attracting equilibria A and A', respectively, if it is decreased. The unique equilibrium C is stable for lower and unstable for higher money growth rates because the respective graph lies below the 45° degree line entirely. This is an interesting result because it allows for hyperinflation but not hyperdeflation.

On the Importance of Adaptive Expectations Adaptive expectations are not crucial, similar results can also be obtained using rational expectations (although the stability properties of the equilibria are 'switched,' cf. Evans and Yarrow 1981). The condition is that agents' money holdings adjust only slowly, which amounts to introducing a certain friction to bring about the desired result (cf. Bruno 1989, p. 285; this approach is reminiscent of the commonplace insertion of Calvo 1983 pricing into real business cycle models, see Chapter 7). However, the justification of this friction seems arguable today because it is as easy as never before for the general public to adjust money balances given modern information technology and endogenous money supply.

The Olivera/Tanzi Effect Another alternative to sluggish adjustment of money balances proposed by Kiguel (1989, pp. 155-156) would be the introduction of tax-collection lags mocking the 'Olivera/Tanzi effect' (1967/1977), which posits that taxes can be devalued in real terms by the time they are collected if the respective collection lag or the inflation rate (or both) are sufficiently large. In addition, if treasury expenditure tracks price developments more closely—which does not seem too unrealistic given that a certain share of public spending is purchased from private vendors—there is a structural influence on deficits which might lead the consolidated government to seigniorage generation in

order to raise revenue.

The Olivera/Tanzi effect is an example of the interactions between monetary and fiscal policy in itself. However, its importance should be little and diminishing in low-inflation countries in which the advantages of information technology also diffuse into the tax-collection process, which is why it is only mentioned here in the context of high (or hyper-)inflations.

Key Takeaways from Chapter 3

Monetary and fiscal policy are inextricably intertwined and the treasury and the central bank are often considered to form a consolidated government. Money and unexpected inflation allow the latter to appropriate real resources over and above the exaction of taxes. A dominant fiscal policymaker might try just that, forcing the monetary authority to play along and accept loss of control over inflation (unpleasant monetarist arithmetic). Slipping beyond the optimal point of an inflation-tax Laffer curve, overdoing such seigniorage generation can lead to high or even hyperinflation.

4. Different Policy Regimes in the Baseline Optimizing Model

In the constant-endowment model of Chapter 2 *without money*, I equip monetary and fiscal policy with simple rules, the parameters of which are critical for determinacy, stability, and thus viability of equilibrium. I characterize the two stable (out of a total of four) equilibria and cover terminology.

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Derivations for this chapter can be found in Appendix B.

4.1. Modifications to the Baseline Model

Excluding Money Deviating from the constant-endowment variant of the baseline optimizing model laid out in Chapter 2, the representative household maximizes the period utility function

$$u(c_t) \tag{4.1}$$

subject to the budget constraint

$$P_t c_t + Q_t B_t = P_t y + P_t (g_t + t_t^{TH}) + B_{t-1}. \tag{4.2}$$

As there is no money, the price level P_t is the amount of treasury bonds B_t required to buy one goods unit.

Consolidated-Government Budget Constraint As a consequence of the household's budget constraint (4.2) and goods-market clearing (2.27), policies must satisfy the flow budget equation of the consolidated government:

$$B_{t-1} = Q_t B_t + P_t s_t \tag{4.3}$$

(With the exclusion of money from the model, the central bank practically becomes an institution without a balance sheet, so Equation (4.3) could also be called the flow budget equation *of the treasury*.) The present-value budget equation reads

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{4.4}$$

and can be derived analogously to Section 2.3.2. With $M_t = 0$ (or rather: nonexistent) in $Z_t \forall t$, the general formulation of Equation (2.34)/(2.42) makes it the appropriate transversality condition also in the present variant of the model.

4.2. Equilibrium Determination with Active and Passive Policies

4.2.1. Policy Rules and Laws of Motion

4.2.1.1. Monetary Policy and the Nominal Interest Rate

Taylor Rule Pursuing its inflation target π^* , the central bank sets the short-term nominal interest rate i_{t+1} according to

$$i_{t+1} = i_{SS} + \gamma_{\pi}^C (\pi_t - \pi^*) + \varepsilon_t^i, \quad \langle 4.5 \rangle$$

where SS subscripts demark steady-state values, γ_{π}^C is the Taylor coefficient, and ε_t^i denotes an interest-rate shock (or, put differently, discretionary monetary policy).¹¹ Tracing back to Taylor (1993), the central bank is said to adhere to the Taylor principle if $\gamma_{\pi}^C > 1$ and violate it for $\gamma_{\pi}^C < 1$.

Law of Motion for Inflation Combining the Taylor rule $\langle 4.5 \rangle$ with the (constant-real-rate) Fisher equation $\langle 2.47 \rangle$ yields the inflation difference equation

$$\mathbb{E}_t \pi_{t+1} - \pi^* = \beta \gamma_{\pi}^C (\pi_t - \pi^*) + \beta \varepsilon_t^i \quad \langle 4.6 \rangle$$

(see Appendix B.1.1 for derivations).

4.2.1.2. Fiscal Policy and Surpluses

Fiscal Policy Rule The treasury decides upon net transfers to households t_t^{TH} and is assumed to leave expenditure constant for simplicity ($g_t = g \forall t$) so that policy can be conveniently described in terms of the budget surplus s_t by Equation $\langle 2.10 \rangle$. Since some of the subsequent arguments loosely follow Leeper and Leith (2017), I also adopt their fiscal policy specification (cf. their p. 11), in which the surplus is adjusted by the treasury to deviations of debt from steady-state (coefficient γ_b^T):

$$s_t = s_{SS} + \gamma_b^T \left(\frac{b_{t-1}}{1 + i_t} - \frac{b_{SS}}{1 + i_{SS}} \right) + \varepsilon_t^s \quad \langle 4.7 \rangle$$

¹¹ The notation of the nominal interest rate i_{t+1} emphasizes that, while being set in t , it is connected to the future period $t + 1$. This might beg the question why it is not the (expected) future inflation rate π_{t+1} that the central bank reacts to but the current realization π_t . The full answer is beyond the scope of this thesis, but a short version reads: because of issues with determinacy other than those which are discussed here. In what follows, we could not develop Equation $\langle 4.6 \rangle$ as it is; rather, it would read $\mathbb{E}_t \pi_{t+1} - \pi^* = (\beta^{-1} - \gamma_{\pi}^C)^{-1} \varepsilon_t^i$ (cf. King 2000, p. 80). References relating to ‘inflation forecast targeting’ are Woodford (1994), Svensson (1997), and Bernanke and Woodford (1997), among others.

It considers the market value of debt, which is why the bond price $Q_t = (1 + i_t)^{-1}$ (cf. Equation (2.8)) also enters the equation. Similar to monetary policy, fiscal policy can be discretionary, which is represented by the policy shock variable ε_t^s . (See Section 4.4 below for an alternative surplus rule that helps distinguish ‘Ricardian’ from ‘non-Ricardian’ policy.)

Law of Motion for Real Debt Similar to the procedure for monetary policy, the combination of surplus rule (4.7) with the respective equilibrium condition (i.e., the flow budget equation of the treasury (4.3), which also leads to the intertemporal budget equation (4.4)), produces a difference equation in real debt:

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1 + i_{t+2}} - \frac{b_{SS}}{1 + i_{SS}} \right) = (1 + r - \gamma_b^T) \left(\frac{b_t}{1 + i_{t+1}} - \frac{b_{SS}}{1 + i_{SS}} \right) - \mathbb{E}_t \varepsilon_{t+1}^s \quad (4.8)$$

(See Appendix B.1.1.2 for derivations.)

4.2.2. Active and Passive Policies

Active vs. Passive Policy The laws of motion (4.6) and (4.8) have the same form as those of Leeper (1991, p. 136), who is often credited with a popular classification of policies: If the policy is such that the coefficient of the respective difference equation is greater than unity in absolute value (leading to an unstable difference equation), it is said to be *active*. Conversely, if the coefficient is smaller than unity in absolute value, the policy is called *passive*. Note that this classification does not refer to the coefficients of the individual policy rules (γ_π^C in the Taylor rule (4.5) and γ_b^T in Equation (4.7)) but to those of the respective laws of motion which are combinations of a policy rule and an equilibrium condition. (See Figures 4.3a-4.3b on p. 79 for a graphical representation.)

Active/Passive vs. the Taylor Principle Monetary policy is called active if the inflation process (4.6) is unstable because the central bank sets $\gamma_\pi^C > \beta^{-1}$; it is called passive if we have $\gamma_\pi^C < \beta^{-1}$ and thus obtain a stable difference equation. Since $\beta < 1 \Leftrightarrow \beta^{-1} > 1$, active monetary policy satisfies the Taylor principle ($\gamma_\pi^C > 1$, cf. Section 4.2.1.1). However, the reverse is not necessarily true: In a small interval $1 < \gamma_\pi^C < \beta^{-1}$, the Taylor principle is satisfied, but monetary policy still has to be considered passive because the respective law of motion (4.6) converges to the steady state value π^* .

Autonomous Policy A policy is called autonomous if it does not adapt to variables under control of the respective other policymaker. For instance, autonomous monetary policy does not take any fiscal variable into consideration (cf. Woodford 2001, p. 672; non-autonomous monetary policy is not part of the current discussion).

4.2.3. Equilibrium Determination

Counting Variables and Equations The basics of equilibrium determination are as described in Section 2.4 (in particular, Tables 2.1b-2.1a). One obvious difference is that money supply M and the respective first-order condition $\langle 2.14 \rangle$ drop out for the time being. Hence, there remain two equations—the flow budget constraint $\langle 4.2 \rangle / \langle 4.3 \rangle$ (replacing Equation $\langle 2.3 \rangle$ of the baseline model *with* money) as well as the Fisher equation $\langle 2.7 \rangle / \langle 2.47 \rangle$ —to determine the amount of bonds B and the price level P .

Fallacious Independence & Equilibrium Selection Superficial inspection of Equations $\langle 4.6 \rangle$ and $\langle 4.8 \rangle$ might make it appear as if they were independent of each other. In this case, a unique equilibrium could be found by setting the policy coefficients γ_π^C, γ_b^T such that they produce two unstable forward-looking difference equations with a single stationary state for each: For instance, in the case of monetary policy, the central bank would choose an active strategy with $\gamma_\pi^C > \beta^{-1}$ and thus produce a phase line steeper than the 45° line in Figure 4.1; like this, all initial values $\pi_0 \neq \pi^*$ would lead farther and farther away from the unique equilibrium π^* , which is why such a destabilizing initial value would never be implemented in a forward-looking equation (cf. Woodford 2003b, p. 128).

Joint Equilibrium Determination However, the two paths are *not* independent because the price level P_t is part of π_{t+1} as well as b_t . Equilibrium determination still works similarly to the procedure described in *Fallacious Independence & Equilibrium Selection*, but has to be mindful of said interconnection: A unique stable equilibrium only occurs for certain combinations of policy parameters γ_π^C, γ_b^T . (As it stands, the previous paragraph possibly contains a second fallacy regarding the validity of explosive equilibria which is unveiled in Chapter 9.4.1.)

Policy Regimes Four elementary regimes arise out of the combination of two policy fields (monetary and fiscal) with two strategies (active or passive): First, Section 4.3.1 describes *monetary dominance*, which consists of active monetary ($\gamma_\pi^C > \beta^{-1}$) and passive fiscal ($\gamma_b^T > r$) policy. Second, Section 4.3.2 on *fiscal dominance* describes a reversal of roles, so monetary policy is passive ($\gamma_\pi^C < \beta^{-1}$) and fiscal policy active ($\gamma_b^T < r$). Third, both policies could try to behave actively, and fourth, both could content themselves with being passive; these two regimes are described in Section 4.3.3.

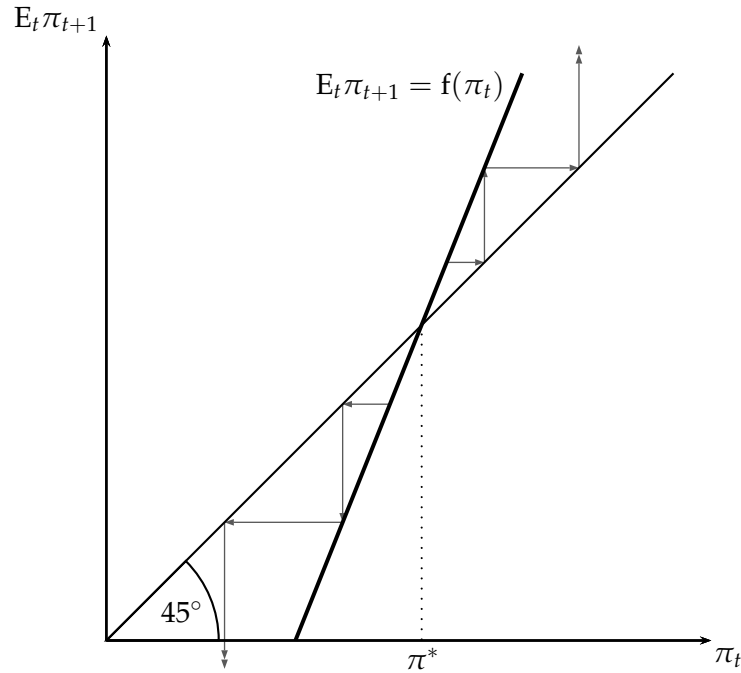


Figure 4.1: *Inflation Dynamics under Monetary Dominance.* \diamond Source: Own illustration based on Woodford (2001, p. 710). \diamond Explanations: See main text of Section 4.2.3 and, for ‘Monetary Dominance,’ Section 4.3.1.

4.3. Four Regimes

4.3.1. Monetary Dominance

4.3.1.1. Regime and Determinacy

Monetary Policy As already indicated, the inflation difference equation $\langle 4.6 \rangle$ has a unique solution

$$\pi_t = \pi^* - \frac{1}{\gamma_\pi^C} \mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{1}{\beta \gamma_\pi^C} \right)^j \varepsilon_{t+j}^i \quad \langle 4.9 \rangle$$

if

$$\left(\beta \gamma_\pi^C \right)^{-1} < 1 \quad \Leftrightarrow \quad \gamma_\pi^C > \beta^{-1}, \quad \langle 4.10 \rangle$$

i.e., if the Taylor coefficient is greater than the real interest rate, or put differently, if the central bank is the (only) active player. Formally, selection of this equilibrium in-

cludes the imposition of a “forward convergence condition” (Cho and Moreno 2011, p. 260; given by Equation $\langle B.1 \rangle$ in Appendix B.1.1.1) which is somewhat similar to the transversality condition in that it disallows explosive behavior of the inflation path—however, stark emphasis has to be put on the ‘*somewhat*’ since this is the crucial point in an argument about the validity of the different regimes’ equilibria below (again, cf. Chapter 9.4.1).

Return to Figure 4.1 for a graphical representation: Since the law of motion $\langle 4.6 \rangle$ is the combination of an equilibrium condition (the Fisher equation $\langle 2.5 \rangle$) and a policy rule (the Taylor rule $\langle 4.5 \rangle$), each point on the respective phase line $\mathbb{E}_t \pi_{t+1} = \mathbb{F}(\pi_t)$ is associated with equilibrium—even if this implies explosive paths for inflation and the price level, which occurs for all initial $\pi_0 \neq \pi^*$. Therefore, π^* is the unique *stable* equilibrium, but it only constitutes a *unique* equilibrium if we exclude explosive paths by adding an ‘*ad hoc* equilibrium condition’ such as the above forward convergence condition as a selection device. (Cf. Woodford 2001, pp. 709–711; Cochrane 2011, pp. 576–577; Canzoneri, Cumby, and Diba 2011, pp. 945–946. Besides Chapter 9.4.1, my Chapter 5.2 also picks up on this and makes related arguments.)

Fiscal Policy Following Leeper (1991) and Leeper and Leith (2017), fiscal policy is assumed to be passive here ($\gamma_b^T > r$). (Alternatively, monetary dominance could be characterized by the ‘less demanding’ requirement of Ricardian fiscal policy which would guarantee a stable real debt process for *all* price paths. It is introduced in Section 4.4 below.)

Equilibrium Determination To complete the analyses of Sections 2.4 and 4.2.3 for this specific case, note that Equation $\langle 4.9 \rangle$ returns a unique value for the current inflation rate. With P_{t-1} predetermined, this also pins down P_t . Finally, the flow budget constraint $\langle 4.2 \rangle / \langle 4.3 \rangle$ determines B_t as the residual.

4.3.1.2. Policy Shocks

Interest-Rate Shock Starting in steady state, consider a decrease in the path of ε^i , which is generally called discretionary expansionary monetary policy (increases would work *vice versa* in what follows). It lowers the interest rate in the Taylor rule $\langle 4.5 \rangle$ and increases inflation by Equation $\langle 4.9 \rangle$. The result is determined by three *ceteris paribus* effects:

1. An instantaneous increase in inflation raises the price level P_t .
2. With higher P_t , the value of outstanding debt on the left-hand side of the present-value budget equation $\langle 4.4 \rangle$ is diminished. This negative wealth effect to the representative household as holder of the bonds leads to a reduction in aggregate demand and, thus, downward pressure on the price level (cf. also Woodford 2001, p. 684).

3. There is no instantaneous reaction by the treasury as it reacts only to lagged real debt (cf. the fiscal policy rule $\langle 4.7 \rangle$), but of course, the present reduction in real debt entails lower surpluses in the future. Equivalent to higher net transfers, this induces the household to increase demand and thereby push up the price level. Within the present-value budget equation $\langle 4.4 \rangle$, lower expected surpluses on the right-hand side are compatible with a higher price level on the left-hand side.

Which effect dominates? With $\gamma_b^T > r$, it might seem as if the treasury even added to the pressure on prices because overly decreased surpluses mean higher expected transfer streams (or, discounted to the present: wealth) to the household, which then strongly increases demand and causes the price level to rise even above the initial plan of the central bank. As already mentioned above, however, a surplus-rule coefficient greater than the net real rate of interest ensures that real debt always returns to its steady-state value over time. Even if the present model assumes it is rolled over every period, which might ‘disguise’ it as a flow, debt is a stock. If it starts with an expected long-run value of x in a distant-future period $t + n$ before the policy shock is announced and the expectation remains that it is still x at $t + n$ after all shock-induced (flow) adjustments have occurred, nothing concerning treasury policy has changed from today’s perspective; real debt started in steady state before the inflationary policy shock and returns there given a sufficient amount of time. Depending on the coefficient γ_b^T , it might take longer or shorter to do so, but it does not seem expedient to argue in this direction in an infinite-horizon setup. So while there might be compositional effects on the path of future surpluses, the net effect of treasury policy on household wealth is nil—items 2 and 3 cancel out. The end result is item 1, an increase in inflation brought about by the central bank.

Inflationary policy in more than just the current period also affects the nominal interest rate. Plugging the equilibrium inflation rate $\langle 4.9 \rangle$ into the Fisher equation $\langle 2.47 \rangle$ gives the equilibrium nominal rate of interest

$$1 + i_{t+1} = (1 + i_{ss}) - \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{\beta \gamma_{\pi}^C} \right)^j \varepsilon_{t+j}^i \quad \langle 4.11 \rangle$$

which is not affected by a policy shock in just the current period ε_t^i . An increase in the nominal interest rate (for longer-lasting policy shocks) is equivalent to lower bond prices Q_t by Equation $\langle 2.8 \rangle$ and thus affects the treasury in its debt issuance. Higher interest payments to households could be seen as a wealth effect, but this is also offset by higher future surpluses (cf. Leeper and Leith 2017, p. 2318).

Surplus Shock Consider now a discretionary policy that decreases only the current surplus s_t by one unit (for simplicity): $\varepsilon_t^s = -1$ in policy rule $\langle 4.7 \rangle$. With B_{t-1} predeter-

mined in t and P_t in the hands of the central bank, the only way to satisfy its flow budget equation (4.3) is for the treasury to increase nominal debt B_t by $-\varepsilon_t^s P_t / Q_t = P_t / Q_t$ units. In the next period, $t + 1$, the surplus rule (4.7) reacts to the increase in (then-) lagged debt B_t . To make it short, there is again no ‘relevant’ change in the present-value budget equation (4.4), only a recomposition of the path of surpluses. (This is an example of Ricardian equivalence brought about by Ricardian fiscal policy, cf. Section 4.4 below.)

4.3.2. Fiscal Dominance

All derivations for this subsection can be found in Appendix B.1.1.2.

4.3.2.1. Regime and Determinacy

Fiscal Policy As a second case, consider now a passive fiscal policy rule. For simplicity, it does not react only weakly to changes in real debt but it does not respond at all ($\gamma_b^T = 0$; see Section 4.4 for this kind of policy):

$$s_t = s_{SS} + \varepsilon_t^s, \quad (4.12)$$

This leads to the following solution for the respective difference equation (4.8):

$$\frac{b_t}{1 + i_{t+1}} = \frac{b_{SS}}{1 + i_{SS}} + \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \varepsilon_{t+j}^s \quad (4.13)$$

Monetary Policy Like before, the ‘other’ policymaker is assumed to be passive. Therefore, the central bank reacts only weakly to inflation ($\gamma_\pi^C < \beta^{-1}$ in the Taylor rule (4.5)).

Equilibrium Determination In Section 4.3.1.1, the unique equilibrium is found by analyzing the law of motion related to the active policy and then proceeding to the passive policy. Unfortunately, this isolationist procedure can not simply be repeated under fiscal dominance. The reason is that the nominal interest rate—which is the central bank’s policy instrument and closely linked to the inflation rate—also appears in the respective equations, namely (4.8) and (4.13), which therefore do not lend themselves as easily to a graphical analysis as the law of motion for inflation (4.6) in Figure 4.1. (Note that this also holds for simpler surplus rules than Equation (4.7), with an example given by Equation (4.20) in Section 4.4. However, the interest-rate peg discussed in Section 5.1.5 resolves this problem and should make good on the omission at this point.) The way out is the verbal description in the following subsection.

4.3.2.2. Policy Shocks

Surplus Shock To illustrate the properties of the fiscal-dominance regime, I play through a one-off discretionary decrease in surpluses by the treasury. For simplicity, this policy shock is assumed to occur only in period t and to show zero persistence ($\varepsilon_t^s < 0$ and $\mathbb{E}_t \varepsilon_{t+j}^s = 0 \forall j \geq 1$).

Following Equation (4.3), a decrease in the current surplus s_t could be offset by taking on more debt on the part of the treasury at t . With $\gamma_b^T = 0$ in policy rule (4.12), the respective law of motion (4.8) shows that real debt explodes once it deviates from steady-state. How could Equation (4.13) be a solution if it potentially violates the transversality condition (not shown in the main text, see Appendix B.1.1.2)? What is more, how should this conceivable real debt increase in t be reconciled with the fact that ε_t^s does not even appear in Equation (4.13)? The answer lies in the consideration of what could change in response to the surplus policy shock. In the following, it is shown that prices and the reaction of monetary policy are what allows for a regime with passive fiscal policy to be viable.

Plugging the solution for real debt (4.13) into the flow budget equation (4.3) pins down current inflation

$$\Pi_t = b_{t-1} \left(\frac{s_{ss}}{1-\beta} + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varepsilon_{t+j}^s \right)^{-1}. \quad (4.14)$$

Since last period's price level P_{t-1} is predetermined, this determines the current price level P_t . In reaction to increasing inflation (recall that $\varepsilon_t^s < 0$), the central bank raises the contemporary nominal interest rate via the Taylor rule (4.5):

$$i_{t+1} - i_{ss} = \beta \gamma_{\pi}^C \left[\frac{b_{t-1}}{\frac{\beta}{1-\beta} s_{ss} + \beta \varepsilon_t^s} - (1 + i_{ss}) \right] + \varepsilon_t^i \quad (4.15)$$

With surplus shocks equal to zero from $t+1$ on, the market value of real debt in period t remains at its steady-state level (note that the sum index starts at $j = 1$ in Equation (4.13)). Consequently, by Equation (4.14), inflation is not driven by discretionary fiscal impulses anymore from $t+1$ on. Nonetheless, as the one-period-ahead version of Equation (4.15) shows, there still is an interest-rate 'afterglow' of the central bank's period- t reaction i_{t+1} to fiscally induced inflation:

$$\mathbb{E}_t i_{t+2} - i_{ss} = \beta \gamma_{\pi}^C (i_{t+1} - i_{ss}) + \mathbb{E}_t \varepsilon_{t+1}^i \quad (4.16)$$

The initial fiscal policy shock leads to a path for nominal interest rates that is potentially

explosive, depending on the coefficient of the interest-rate difference equation $\langle 4.16 \rangle$. It is a defining characteristic of the fiscal-dominance regime that it is smaller than unity (i.e., $\gamma_\pi^C < \beta^{-1}$); the solution of the inflation difference equation $\langle 4.6 \rangle$ is thus backward-looking here. The central bank acts passively, adjusting interest rates only moderately to inflation, and thereby contains the amount of interest the treasury has to pay on its debt.

From the perspective of the household sector, a one-off reduction in surpluses without any compensating adjustments in the future is an increase of net wealth that allows for higher consumption demand. To present the other alternative, namely the central bank reacting to higher inflation by increasing the nominal interest rate more than one-to-one, higher interest income would add further to wealth-driven demand and upward inflation pressure, taking the economy on an explosive path via Equation $\langle 4.16 \rangle$. If this happens, (*per se* harmless) fiscally induced inflation is made unstable by active monetary policy (cf. Leeper and Leith 2017, p. 2320). In the context of Figure 4.1, fiscal policy typically implements a $\pi_0 \neq \pi^*$ so that active monetary policy leads to explosive behavior (see Section 4.3.3.1) whereas passive monetary policy leads back to π^* (cf. Woodford 2001, pp. 711-712).

Interest-Rate Shock As before, consider a single-period shock without any persistence. Assuming an increase, we have $\varepsilon_t^i > 0$ and $\mathbb{E}_t \varepsilon_{t+j}^i = 0 \forall j \geq 1$. The easiest way is to resort to the flow budget equation of the consolidated government $\langle 4.3 \rangle$ again: With B_{t-1} predetermined and surpluses unresponsive to the interest-rate hike ($\gamma_b^T = 0$ in rule $\langle 4.7 \rangle$), the equation can only hold if nominal debt B_t increases accordingly—a lower bond price Q_t means the treasury has to issue more debt titles B_t to achieve the same nominal volume. Equation $\langle 4.14 \rangle$ implies that current inflation Π_t and thus also the current price level P_t are not affected by these events. Therefore, real debt b_t increases with its nominal counterpart. Since these changes are proportional, or in other words, the fractions $B_t / (1 + i_{t+1})$ and $b_t / (1 + i_{t+1}) = B_t / (P_t [1 + i_{t+1}])$ remain constant, the law of motion $\langle 4.8 \rangle$ does not trigger an explosion in real debt. In the next period, however, Equation $\langle 4.13 \rangle$ comes into effect and (since the term in parentheses on the right-hand side is unaffected) increases future inflation Π_{t+1} proportionally to then-lagged real debt b_t , which itself increases proportionally to the nominal interest rate in the period before. This induces the central bank to raise interest rates via the Taylor coefficient in rule $\langle 4.5 \rangle$, so the whole process starts over, but since monetary policy is passive, its amplitude decreases over time similar to the proceedings relating to Equation $\langle 4.16 \rangle$. All the central bank achieves is a transitory increase in inflation.

4.3.3. The Two Unfriendly Regimes: Explosions and Sunspot Equilibria

4.3.3.1. On the Explosiveness of Real Debt in Doubly Active Policy Regimes

Active Monetary and Fiscal Policy Combining active monetary policy ($\gamma_\pi^C > \beta^{-1}$) with active (potentially non-Ricardian: $0 \leq \gamma_b^T < r$, see Section 4.4) fiscal policy yields two unstable difference equations [\(4.6\)](#) and [\(4.8\)](#) in which both respective processes explode unless their steady-state values are hit exactly. To show the interactions between both laws of motion, either one can be taken as a starting point:

- Beginning on the fiscal side, the solution [\(4.13\)](#) to the real-debt law of motion [\(4.8\)](#) also implies a certain inflation rate [\(4.14\)](#). If this is different from the inflation target π^* , the inflation process explodes (cf. Woodford 2001, p. 711). This, in turn, feeds back on the composite term in Equation [\(4.8\)](#) by affecting real debt $b = B/P$ as well as the nominal interest rate i . Therefore, even if the fiscal side is in steady state initially, it is pulled out of it later. Being active, treasury policy does not counteract this development strongly enough, so that the real debt process also explodes.
- Alternatively, one can start on the monetary side. Given last period's price level P_{t-1} , the target inflation rate π^* implies a current price level P_t as well as the nominal interest rate $i_{t+1} = i_{ss}$ (since inflation will only deviate from steady state in the subsequent period if a shock occurs), both of which may let the composite term in Equation [\(4.8\)](#) deviate from steady state and thus explode.

Of course, it is not entirely inconceivable that there are mutually consistent steady states for inflation and real debt so that neither of the two explodes right away. However, reconciliation of the respective targets would require close coordination between monetary and fiscal policy and such a steady-state equilibrium could be described as fragile at best.

Real-Debt Explosions A slightly rearranged variant of the law of motion for the composite real-debt term [\(4.8\)](#) reads

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1 + i_{t+2}} \right) = (1 + r) \frac{b_t}{1 + i_{t+1}} - r \frac{b_{ss}}{1 + i_{ss}} = \frac{b_t}{1 + i_{t+1}} + r \left(\frac{b_t}{1 + i_{t+1}} - \frac{b_{ss}}{1 + i_{ss}} \right), \quad (4.17)$$

where the policy shock $\mathbb{E}_t \varepsilon_{t+1}^s$ is dropped because the focus lies on the dynamics (for which it can be a trigger, however). Further, treasury policy is assumed not to react to debt at all ($\gamma_b^T = 0$), which is the most extreme case imaginable at this point (later, Section 5.2.1.2 goes even farther by suggesting a negative policy parameter as a policy prescription). Note that a special case can emerge for $0 < \gamma_b^T \leq r$, see *Active Ricar-*

dian Policy in Section 4.4.2. Equation (4.17) confirms that the composite real-debt term explodes. Two subcases can be distinguished:

- Starting to the right of the steady state, $b_t/(1+i_{t+1}) > b_{ss}/(1+i_{ss})$, the right-hand side of Equation (4.17) shows that the term constantly increases and approaches infinity. Such a process can be triggered by too low inflation, which increases real debt b_t via its effect on the current price level P_t and further leads to a lower nominal interest rate i_{t+1} via the Taylor rule (4.5).
- Starting to the left, $b_t/(1+i_{t+1}) < b_{ss}/(1+i_{ss})$, the term in parentheses turns negative and makes $\mathbb{E}_t[b_{t+1}/(1+i_{t+2})]$ smaller than its lag $b_t/(1+i_{t+1})$. Analogously to the above case, the respective trigger would be too high inflation (and the corresponding price level as well as nominal interest rate, which are also too high to keep the composite real-debt term in steady state). After a certain time, $b/(1+i)$ turns negative itself, that is, the treasury becomes a net lender to the private sector. In principle, the process approaches minus infinity.

While it is obvious that the real-debt process explodes, the more important question is whether or not it does so ‘fast enough’ to violate the transversality condition (2.42), which would then rule out the respective equilibrium-candidate paths.

Growth Rate of Real Debt Recall that Equation (4.8) is derived from the consolidated-government budget equation (4.3) and the surplus rule (4.7). With $\gamma_b^T = \mathbb{E}_t \varepsilon^s = 0$, the latter two can be combined to

$$B_{t-1} = Q_t B_t + P_t s_{ss} \Leftrightarrow \frac{b_{t-1}}{\Pi_t} = \frac{1}{1+i_{t+1}} b_t + s_{ss} \Leftrightarrow b_t = (1+i_{t+1}) \left(\frac{b_{t-1}}{\Pi_t} - s_{ss} \right)$$

and used to find the steady-state value:

$$b_{ss} = (1+r) \Pi^* \left(\frac{b_{ss}}{\Pi^*} - s_{ss} \right) \Leftrightarrow b_{ss} = \frac{(1+i_{ss}) s_{ss}}{r} \Leftrightarrow \frac{b_{ss}}{1+i_{ss}} = \frac{s_{ss}}{r}$$

Plugging this into Equation (4.17) gives

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1+i_{t+2}} \right) = (1+r) \frac{b_t}{1+i_{t+1}} - s_{ss},$$

which implies that the market value of real debt grows at the gross rate

$$\frac{\mathbb{E}_t \left(\frac{b_{t+1}}{1+i_{t+2}} \right)}{\frac{b_t}{1+i_{t+1}}} = \frac{(1+r) \frac{b_t}{1+i_{t+1}} - s_{SS}}{\frac{b_t}{1+i_{t+1}}} = 1 + r - \frac{s_{SS}}{\frac{b_t}{1+i_{t+1}}}. \quad \langle 4.18 \rangle$$

Depending on the sign of b_t , it is smaller or greater than the real interest rate r .

Satisfaction of the Transversality Condition? In Equation $\langle 2.42 \rangle$, the discount factor $v_{t,t+J+1}$ can be split into two parts by use of Equations $\langle 2.16 \rangle$ - $\langle 2.20 \rangle$, which allows to write

$$\lim_{J \rightarrow \infty} \mathbb{E}_t \left(v_{t,t+J} \frac{\Pi_{t+J+1}}{1+i_{t+J+1}} \frac{Z_{t+J}}{P_{t+J+1}} \right) = \lim_{J \rightarrow \infty} \mathbb{E}_t \left[\left(\frac{1}{1+r} \right)^J \frac{b_{t+J}}{1+i_{t+J+1}} \right] = 0 \quad \langle 4.19 \rangle$$

(note that $Z_t = B_t$ here because there is no money in the current setup). The term in brackets on the right-hand side can have an unpleasant implication: The market value of real debt $b_t/(1+i_{t+1})$ is discounted by the gross real interest rate, which can be greater than the growth rate of real debt given by Equation $\langle 4.18 \rangle$. In particular, real-debt explosions to the right of the steady state (cf. the first bullet point on p. 73) unambiguously deduct a non-negative value from the gross real interest rate so that the net growth factor, i.e., the combination of the growth rate and the discount factor, is smaller than unity. Like this, the transversality condition $\langle 4.19 \rangle$ is always satisfied and can thus not be used to rule out the respective real-debt paths.

Violation of the Transversality Condition? Starting with the obvious case, real-debt explosions to the left of the steady state clearly violate the transversality condition $\langle 4.19 \rangle$ because the growth rate $\langle 4.18 \rangle$ exceeds the discount rate for negative real-debt values (even if only marginally). As stated in the second bullet point on p. 73, lending from the treasury to the representative household would become infinitely large. To be precise, this represents a violation of the no-Ponzi condition $\langle 2.30 \rangle$, but it is also covered by the transversality condition.

However, some authors see a violation of the latter *in any case*: Christiano and Fitzgerald (2000, p. 14) argue that the last term on the far-right-hand side of Equation $\langle 4.18 \rangle$ becomes smaller and smaller over time so that the growth rate of the composite real-debt term converges to the gross real interest rate $(1+r)$. If this is true, the present value of the expected market value of real debt is not discounted all the way to zero, so that the transversality condition is violated and the respective paths do not constitute equilibria.

Remaining Doubts Given an infinite amount of time, both the discounting of a finite market value of real debt to zero in the transversality condition (4.19) and the convergence of the respective growth rate to the real interest rate in Equation (4.18) are straightforward—individually. Considering both processes jointly is more complicated, however, because the argument is about relative speeds in infinity then: If the growth rate of real debt in Equation (4.18) converges ‘first,’ that is, before real debt is discounted all the way to zero, the transversality condition is violated because some finite amount of real debt is multiplied and divided by the same gross rate $(1 + r)$ in every period afterwards.

Putting it bluntly, it is not clear infinity works this way: Assuming such convergence (in the spirit of Christiano and Fitzgerald 2000) is literally premature because it implies that one process which requires an infinite amount of time is completed before another process that also requires an infinite amount of time. The counterargument would be that, *in every period*, the market value of real outstanding debt is reduced more strongly by discounting than it is increased by the growth rate of Equation (4.18); therefore, the present value approaches zero and the transversality condition is always satisfied if real debt explodes to the right of the steady state. One could consider this result especially unsatisfying because it represents an asymmetry in a model which is so simple that it ‘should’ be symmetric (by contrast, the zero-lower bound on net nominal interest rates is a ‘welcome’ example as it embodies a necessary friction to make an otherwise symmetric model asymmetric in a certain respect, cf. Section 5.2.1).

To be fair, one could probably come upon good arguments for both positions in a discussion that turns ever more mathematical. Exactly this is the only conclusion I can support personally at this point: The validity of equilibrium-candidate paths in the active-monetary-active-fiscal policy regime hinges on a mathematical subtlety. Nonetheless, since this question of validity is either a non-issue or (in the case of the aforementioned Christiano and Fitzgerald 2000) treated like only a minor complication in the literature, and because of the asymmetry mentioned above, I choose to lean towards the generally accepted classification of active-active policy configurations as regimes that do not lead to equilibria (instead of describing them as asymmetric regimes with no equilibrium for negative and infinitely many indeterminate equilibria for positive deviations from the real-debt steady state).

4.3.3.2. Sunspot Equilibria

When both policies are passive, matters are much simpler. Figure 4.2 illustrates the situation generally: For any initial value $\square_t \neq \square^*$ (substitute π or $b/(1 + i)$ for \square , respectively), the process reverts back to the steady state \square^* over time. Since this also means that the transversality condition (2.42)/(4.19) is never violated, all initial values and the ensuing paths are valid equilibria, which makes the model indeterminate.

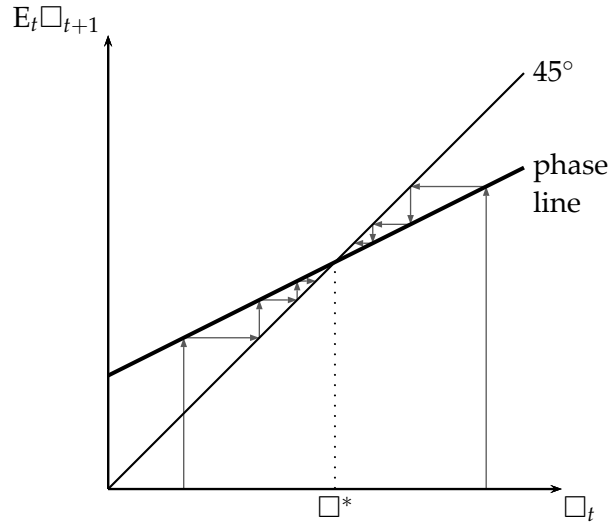


Figure 4.2: *Dynamics under Passive Policy.* \diamond Source: Own illustration. \diamond Explanations: Substitute π and $b/(1+i)$ for \square , respectively; see main text of Section 4.3.3.2.

In terms of Section 2.4, the \nearrow Infinite Recursion problem can not be solved fully because both monetary and fiscal policy fail to make the model determinate by ‘picking’ a unique equilibrium (cf. Canzoneri, Cumby, and Diba 2011, p. 948).

4.4. Ricardian Equivalence and Ricardian vs. Non-Ricardian Policy

4.4.1. Ricardian Equivalence

Alternative Surplus Rule Consider an alternative to surplus rule (4.7) here, namely

$$S_t = \gamma_B^T B_{t-1}, \quad 0 < \gamma_B^T \leq 1. \quad (4.20)$$

The treasury sets surpluses S_t so as to redeem a certain fraction γ_B^T of outstanding liabilities $Z_{t-1} = B_{t-1}$. (The shock term ε_t^s is omitted for simplicity. At least if modeled in the usual way, that is, either as white noise or a stable first-order autoregressive process driven by white noise, it would not add anything substantial to the argument to be made here.)

Effects on the Present-Value Budget Equation Combining the alternative surplus rule (4.20) with the flow budget equation of the consolidated government (4.3) (and Equat-

tion (2.18) for the stochastic discount factor v) yields

$$\mathbb{E}_t \left(v_{t,t+1} \frac{B_t}{P_{t+1}} \right) = \left(1 - \gamma_B^T \right) \frac{B_{t-1}}{P_t} \quad \langle 4.21 \rangle$$

and, by forward substitution,

$$\mathbb{E}_t \left(v_{t,t+J+1} \frac{B_{t+J}}{P_{t+J+1}} \right) = \left(1 - \gamma_B^T \right)^{J+1} \frac{B_{t-1}}{P_t} \quad \langle 4.22 \rangle$$

$$\Rightarrow \lim_{J \rightarrow \infty} \mathbb{E}_t \left(v_{t,t+J+1} \frac{B_{t+J}}{P_{t+J+1}} \right) = 0. \quad \langle 4.23 \rangle$$

Expression (4.23) is equivalent to the transversality condition (2.42). In other words, the latter is always satisfied for all paths of the endogenous variables, so its imposition as an equilibrium condition does not restrict the set of equilibria in this case.

Ricardian Equivalence Alternatively, plugging fiscal rule (4.20) into the present-value budget equation (4.4) (in the first line of Equation (4.24) below) and using Equation (4.22) (in the second line) then reveals that it is always identically satisfied:

$$\begin{aligned} \frac{B_{t-1}}{P_t} &= \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} s_{t+j} = \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \gamma_B^T \frac{B_{t+j-1}}{P_{t+j}} \\ &= \gamma_B^T \frac{B_{t-1}}{P_t} \sum_{j=0}^J \left(1 - \gamma_B^T \right)^j = \gamma_B^T \frac{B_{t-1}}{P_t} \frac{1}{1 - (1 - \gamma_B^T)} = \frac{B_{t-1}}{P_t} \quad \langle 4.24 \rangle \end{aligned}$$

Put differently, given the surplus rule (4.20), Ricardian equivalence obtains: The treasury ensures that the present value of future budget surpluses is equal to the real value of its currently outstanding liabilities. Even price level paths with unbounded growth or strong deflation do not impede this mechanism. (Cf. Woodford 1995, p. 29, for instance.)

The Bohn-Woodford Criterion The result that Ricardian equivalence holds for $\gamma_B^T > 0$ is shown by Bohn (1998, Technical Appendix A1). It leads to the following distinction used in this thesis: Fiscal policy is considered Ricardian for $\gamma_B^T > 0$ and non-Ricardian for $\gamma_B^T = 0$. To avoid confusion: Equation (4.20) is a simplified version of a broader class of policy rules that allows for other (unspecified) influences on surpluses apart from outstanding debt (cf. Bohn 1998, p. 951), so $\gamma_B^T = 0$ does not necessarily imply $S_t = 0 \forall t$ for *all* non-Ricardian rules. (To add confusion: Section 5.2.1.2 describes a policy proposal that actually sets the respective parameter below zero.)

The distinction between Ricardian and non-Ricardian fiscal policy by reference to the transversality condition (that is, its unconditional satisfaction via surpluses) is mostly credited to Woodford (1995; 2001). Canzoneri, Cumby, and Diba (2001b) derive basically the same criterion in a model that scales variables on nominal GDP in order to facilitate empirical considerations. In the same spirit, they add that, in an infinite-horizon setup, γ_B^T has to be “bounded away from zero infinitely often,” but not in every period. This increases the degree of plausibility associated with the Ricardian regime, but also introduces an issue which may complicate the interpretation of real-life events: “A stabilizing policy could be in effect every other year, or every third year, or every decade. Indeed, the fiscal retrenchment need not occur in the next 100 years, or in any finite dataset!” (Both quotes on their p. 1226.)

Finally, Cochrane (1999, p. 340) considers a real surplus rule $s_t(P_t) = S/P_t$ which is a function of fixed nominal surpluses (hence no time index) and the price level. He calls fiscal policy Ricardian if it moves in line with the price level, which may unnecessarily limit the focus on this single variable but, in his example, amounts to the same criterion as the previous approaches.

Implications for Determinacy The main finding is already indicated twice below Equations $\langle 4.23 \rangle$ and $\langle 4.24 \rangle$, but it is worth stating it clearly again: Ricardian fiscal policy does nothing to restrict a given set of possible equilibria but always satisfies the transversality condition $\langle 2.42 \rangle / \langle 4.23 \rangle$ (which is tantamount to the present-value budget equation $\langle 2.43 \rangle / \langle 4.4 \rangle$, cf. Section 2.3.2), irrespective of the path of other variables. Since this applies to prices in particular, it must be paired with active monetary policy as described in Section 4.3.1 to be able to achieve determinacy. (Cf. Woodford 2001, pp. 690-691. Cochrane 1999, p. 334 calls Ricardian policy the “fiscal analogue” to certain monetary regimes that lead to indeterminacy under the quantity theory, cf. Section 5.1.5 on interest-rate pegs; see also Section 9.3.1.)

4.4.2. Distinguishing Fiscal-Policy Classifications

Ricardian/Non-Ricardian vs. Active/Passive Policy The distinction between Ricardian and non-Ricardian policy is a ‘displacement to the left’ of the distinction between active and passive fiscal policy (cf. Section 4.2.2). While the latter revolves around whether the law of motion for surpluses $\langle 4.8 \rangle$ is converging or explosive, depending on whether $\gamma_b^T > r$ or $\gamma_b^T < r$, respectively, the former property is related to the transversality condition $\langle 2.42 \rangle / \langle 4.19 \rangle$ with Ricardian policy ($\gamma_b^T > 0$) leading to its satisfaction and non-Ricardian policy ($\gamma_b^T = 0$) leading to its violation. (Figure 4.3b summarizes the terminology graphically.)

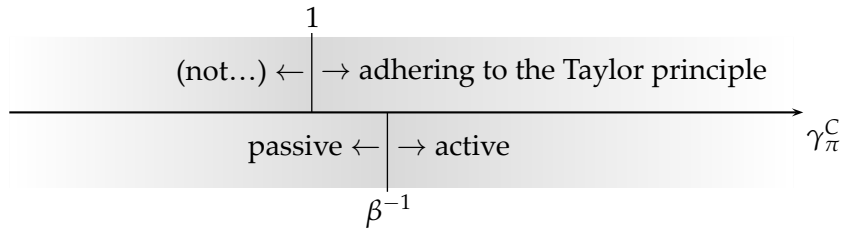


Figure 4.3a: *Terminology of Monetary Policy Rules.* ◇ Source: Own illustration.

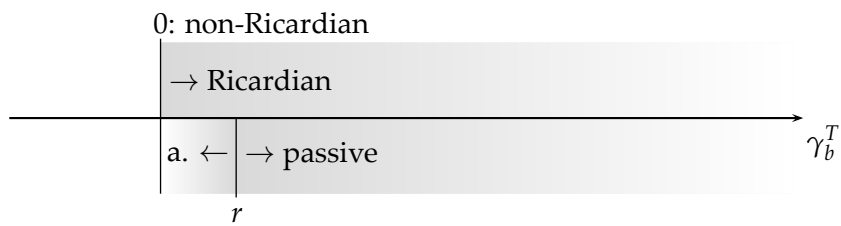


Figure 4.3b: *Terminology of Fiscal Policy Rules.* ◇ Source: Own illustration.

Active Ricardian Policy Inbetween, active Ricardian policy with $\gamma_B^T \in (0, r]$ means that the law of motion [\(4.8\)](#) can trigger explosions in real debt at a rate that is lower than the inverse stochastic discount factor. Therefore, these paths are not ruled out by the transversality condition [\(4.19\)](#). Such a passive-monetary/active-but-Ricardian-fiscal regime has a multitude of viable equilibria, that is, it is undetermined (cf. Cochrane 1999, p. 340; Canzoneri, Cumby, and Diba 2001b, p. 1226; Canzoneri, Cumby, and Diba 2011, p. 956). Active monetary policy seems to be the remedy here.

Which Approach Is Better? The two classifications—active/passive and Ricardian/non-Ricardian—appear against different backgrounds:

- Woodford (2001) presents a basic nonlinear DSGE model in which satisfaction or violation of the transversality condition puts fiscal policy in either the Ricardian or non-Ricardian corner. The advantage of this approach is its generality and that it naturally corresponds to said optimality condition, but as shown just above, active-Ricardian fiscal policy can lead to problems with determinacy.
- By contrast, Leeper (1991) introduces the active/passive classification in the context of a linearized model. He explicitly acknowledges that he therefore “cannot directly check that the transversality condition is satisfied” (p. 135). Indeed, the resulting assertions are limited to the vicinity around steady state; very large fluctuations are not covered by his analysis. Then again, using stability of the re-

spective difference equations avoids the kind of indeterminacy discussed above. Therefore, Leeper's requirements are both weaker and stronger than Woodford's, in different respects (cf. Woodford 1995, fn. 30, p. 27; Canzoneri, Cumby, and Diba 2011, p. 956).

It could probably be guessed that there is no clear answer to the introducing question; the suitability of approaches depends on the purpose followed. In any case, numerical analyses (which are mostly based on linearized models) will typically rely on the active/passive classification; the simulation of a sticky-price model in Chapter 7.3 of this study is one example. Further, Chapter 5.2.2 similarly presents arguments that distinguish between local and global analysis.

4.5. Final Notes on the Baseline Model with Policy

Saddle-Path(-Like) Stability Moving the derivation of solution $\langle 4.13 \rangle$ to Appendix B.1.1.2 obscures that, in switching from monetary to fiscal dominance, real debt turns from a backward-looking into a forward-looking variable. By contrast, inflation is forward-looking under monetary and backward-looking under fiscal dominance (cf. Cochrane 1999, p. 365).¹² Although Equations $\langle 4.6 \rangle$ and $\langle 4.8 \rangle$ do not form a linear difference model compatible to Blanchard and Kahn (1980), the deliberations above show that the present model displays saddle-path stability: If one policy maker behaves actively—that is, sets the coefficient of the respective difference equation above unity—the other must behave passively in order for the economy to reach a non-explosive solution; in other words, if one of the equations has a forward-looking (active) solution, the other one must be backward-looking (passive).

New Debt Sales Among other things, Sections 4.3.1 and 4.3.2 explain what happens in both regimes if the treasury engineers a surplus policy shock. In addition, Cochrane (1999, p. 347) also considers issuance of additional debt in period t : At first, nothing actually happens because B_t does not appear in Equation $\langle 4.4 \rangle$. In $t + 1$, however, either one of the remaining variables has to adjust. Under monetary dominance, the present value of surpluses increases accordingly so the price level is unaffected. By contrast, under fiscal dominance, surpluses do not change (that is, at all or strongly enough) so that the price level jumps to the value which makes Equation $\langle 4.4 \rangle$ hold.

¹² Use of these terms as well as notation is not uniform in the literature. Compare Azariadis (1993, Ch. 2) and Gandolfo (2010, Ch. 3), for instance: Both consider initial- as well as terminal-value problems, but apply different labels to the respective solutions. In Azariadis (1993, pp. 20-22), the “forward-looking solution” means starting from an initial value and going forward in time until the present date, yielding a solution that depends on past state variables. By contrast, Gandolfo (2010, p. 28) links the “forward solution” to a terminal-value problem and thus a sum term containing future realizations. I follow the latter classification.

Key Takeaways from Chapter 4

A policy is said to be active (passive) when the respective law of motion resulting from the policy rule and an equilibrium condition is unstable (stable). Four regimes arise: Under the conventional regime of monetary dominance, an active central bank sets the nominal anchor while a passive treasury establishes Ricardian equivalence. Under fiscal dominance, active fiscal policy determines the price level via wealth effects in the present-value budget equation of the consolidated government (passive monetary policy does not provoke explosive behavior). Doubly active regimes are considered explosive non-equilibria; doubly passive regimes lead to multiple-solution indeterminacy.

5. Extensions: What Happens When We Add...

Money is brought back into the model developed in Chapters 2 and 4. First, it is used as the central bank's policy instrument in three 'case studies' to further illustrate the functioning of fiscal price determination; later, it is endogenized in order to show how interest-rate pegs can overcome their reputation of leading to indeterminacy. In the second half of the chapter, I examine the effects of putting bounds on certain endogenous variables, namely, a zero lower bound on the nominal interest rate and a 'debt' limit on consolidated-government liabilities.

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Derivations for this chapter can be found in Appendix C.

5.1. Money

5.1.1. Setup

The basic model specifications are those of Chapter 2. Policy design in Section 5.1 draws especially on Woodford (1995).

Policy Monetary policy consists of setting an exogenous money supply M . This can be virtually any sequence that implies nonnegative nominal interest rates (cf. Buiter 2002, p. 466, for instance) or slightly more refined processes with constant or zero money growth (cf. Carlstrom and Fuerst 2000, pp. 25 or 27, respectively). Money-supply rules complicate the distinction between ‘active’ and ‘passive,’ which pertains to interest-rate policy; however, Sims (1999b, p. 419) thinks of exogenous money supply as active monetary policy as well (considering the implications for the now-endogenous nominal interest rate described in the case studies below, this seems fitting).

Fiscal policy sets net transfers to the household t^{TH} and keeps expenditure g constant. Because of Equation (2.10), it thus exogenously decides on budget surpluses s .

Counting Variables and Equations The basics of equilibrium determination are laid out in Section 2.4 (especially Tables 2.1b–2.1a). In the current policy arrangement, there remain three variables—the nominal interest rate i , the amount of bonds B , and the price level P —to be determined by three open equations: the budget constraint (2.3), the first-order condition with respect to money (2.14) (or one of the implicit money-demand functions (2.22) and (2.25) derived from it), and the Fisher equation (2.5)/(2.7). Of course, this does not entirely solve the issue of *Infinite Recursion* (p. 36) in the present setup so the model is still indeterminate at this point (cf. also Woodford 1995, pp. 4, 14).

5.1.2. Fiscal Price Determination

Assumptions Recall that utility (2.1) is additively separable in its arguments c_t and m_t and that the properties (2.2) apply to each of them individually. In addition, the marginal utility of real money is assumed to never exceed even the smallest-possible marginal utility of consumption (brought about by the highest-possible endowment y^{\max}) here:

$$u_m(m_t) < u_c(y^{\max}) \quad \forall m > 0, t \quad (5.1)$$

Equilibrium Determination Using money demand (2.22) and noting that $\partial u(\cdot)/\partial m_t = P_t \partial u(\cdot)/\partial M_t$, the present-value budget equation (2.44) can be rearranged to

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{u_M(M_{t+j})}{u_c(c_{t+j})} M_{t+j} + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \quad (5.2)$$

Since all other variables are already established (c_{t+j} is constant, M_{t+j} as well as $s_{t+j} = -[g_{t+j} + t_{t+j}^{TH}]$ are policy choices, and Z_{t-1} is predetermined), it pins down the current price level P_t , which then enables money demand (Equation (2.14)/(2.22)/(2.25)) to determine the nominal interest rate i_{t+1} . As a side note, Woodford (2001, p. 683) keeps

outstanding liabilities Z_{t-1} from turning negative by assuming the surplus process s_t to ensure that the right-hand side of Equation (5.2) remains positive.

Given the nominal and (constant) real interest rate, the Fisher equation (2.47) implies values that are consistent with equilibrium for the gross inflation rate $\mathbb{E}_t \Pi_{t+1}$ and thus the expected price level $\mathbb{E}_t P_{t+1}$. The last variable on the list (cf. *Counting Variables and Equations* in Section 5.1.1) is the amount of bonds B_t . While B is the pivotal element in moving from a flow ((2.3)) to the present-value budget equation ((2.44)), the latter only features the predetermined realization B_{t-1} so that the current realization B_t can be extracted as the residual from the former. Given that the sequences of M and s over the entire future time horizon are known, the problem of *Infinite Recursion* is solved.

5.1.3. Three Cases from Woodford (1995)

5.1.3.1. Introduction

A Special Rule for Transfers As a special case, Woodford (1995, p. 15) introduces a tax-collection rule which, in my notation, becomes the transfer rule

$$T_t^{TH} = P_t \tau_t^{TH} + \dot{M}_{t+1} M_t. \quad (5.3)$$

Transfers T_t^{TH} are now influenced by both the treasury and the central bank. τ_t^{TH} captures the treasury's commonplace decision about how much to tax or pay as transfers (on net). The second term on the right-hand side, by contrast, is somewhat unconventional: The treasury extends a 'seigniorage rebate,' compensating the interest savings that are generated by issuing money instead of debt, which clearly depends on decisions made by the central bank.¹³ The purpose of this kind of rule is to highlight and, at the same time, neutralize the effects of money-supply changes on treasury policy. It might be argued that it does the exact opposite, namely, force the treasury to adjust to decisions of the central bank when it would not have to do so with a more common type of rule; however, the understanding here is that the net-tax position of the treasury should be rid of windfalls (or, in the case of decreasing money supply, 'scourges') that it is not in control of.

As a consequence, the transfer rule (5.3) simplifies the present-value budget equation

¹³ The rate τ^{TH} is introduced only now because a more conventional rule that does not depend on the central bank's decision about money supply M_t would simply read $t_t^{TH} = \tau_t^{TH}$, which seems superfluous. Since endowment income is fixed in this model, τ^{TH} would be non-distortionary even if it were modeled as an income-tax rate, cf. Woodford (1995, p. 15).

⟨2.44⟩ to

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = -\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (g_{t+j} + \tau_{t+j}^{TH}) \quad \langle 5.4 \rangle$$

and the law of motion for outstanding consolidated-government liabilities ⟨2.37⟩ (sans the ‘dis’ superscripts) to

$$Z_t = (1 + i_{t+1}) (Z_{t-1} + G_t + P_t \tau_t^{TH}). \quad \langle 5.5 \rangle$$

Equilibrium Determination Again The present-value budget equation ⟨5.4⟩ reflects the central proposition of Woodford (1995, p. 17) that, with treasury policy following rule ⟨5.3⟩, the current price level P_t solely depends on fiscal variables whereas the path of money supply M_t is irrelevant. The law of motion ⟨5.5⟩ can then be used to track the evolution of outstanding liabilities.

In what follows, three case studies demonstrate in more detail how the equilibrium price level is influenced by fiscal policy (given the more specific assumptions taken from Woodford 1995).

5.1.3.2. Changes in Money Supply

Policy In period t , the economy starts in stationary state with constant policy variables and prices (see Appendix C.1.1). Let a dot above a variable ($\dot{\square}$) denote its new value after a hypothetical deviation from an initial state in the same period; put differently, ‘dot-variables’ denote states with the policy move under consideration while variables without dots denote the initial situation without any policy change. Hence, a permanent increase in money supply by the arbitrary fraction \dot{o} of the original amount M_{t+j} means that $\dot{M}_{t+j} \equiv (1 + \dot{o})M_{t+j} \forall j$.

Current Price Level Because the right-hand side of Equation ⟨5.4⟩ makes no reference to money balances at all and outstanding liabilities Z_{t-1} on the left-hand side only contain lagged money M_{t-1} , the contemporary price level P_t is unaffected by an increase in current money supply M_t .

Nominal Interest Rate In this case of simple money-supply increases without movement in the concurrent price level, it is obvious from Equation ⟨2.24⟩ that the nominal interest rate goes down as real money increases. The exact deviation depends on marginal utilities and could therefore only be quantified given a specific utility function (in a similar argument, the importance of the interest elasticity of money demand is also demonstrated by Carlstrom and Fuerst 2000, pp. 25-28).

Future Price Levels A misleading first intuition, possibly brought about by definition $\langle 2.28 \rangle$, could be that raising the money supply increases consolidated-government nominal liabilities in the next period—but this is wrong.

In general, a money-supply increase has two effects. Rearrange Equation $\langle 2.37 \rangle$ (in combination with definition $\langle 2.28 \rangle$) for clarity:

$$B_t + M_t = (1 + i_{t+1}) (Z_{t-1} - \dot{I}_{t+1} M_t - S_t)$$

First, holding all other policy variables constant, it decreases treasury debt. Second, at least at positive nominal interest rates, it generates seigniorage. *In the present case*, that is, with transfer rule $\langle 5.3 \rangle$ leading to Equation $\langle 5.5 \rangle$, the second effect is shut off; concurrent nominal liabilities Z_t remain constant and merely shift from debt to money. However, as already described, the \nearrow *Nominal Interest Rate* declines. Inserting all of this into the law of motion $\langle 5.5 \rangle$ shows that nominal liabilities actually *decrease* to $\dot{Z}_t < Z_{SS}$. Naturally, with the right-hand side still unchanged in the present-value budget equation $\langle 5.4 \rangle$, the future price level declines in reaction to a previous increase in money supply, to $\dot{P}_{t+1} < P_{SS}$. The resulting real money balances are even higher and thus decrease the nominal interest rate further which, in turn, decreases outstanding liabilities in period $t + 2$, and so on. The outcome is a deflationary process in which real money grows without bound and the nominal interest rate converges to zero.

Note that this resembles a ‘doubly active’ regime (cf. Chapter 4.3.3.1). Apart from the initial increase, money supply remains constant. What happens to transfers, and thus surpluses, is ambiguous because of the seigniorage rebate: Real money approaches infinity while $\lim_{i \rightarrow 0} \dot{I} = 0$ (cf. its definition $\langle 2.23 \rangle$). In the best case, the rebate vanishes over time, leaving only the constant τ^{TH} in transfer rule $\langle 5.3 \rangle$; in the worst case, it also grows without bound, *decreasing* surpluses by Equation $\langle 2.10 \rangle$. Hence, since neither fiscal nor monetary policy react appropriately to the disturbance and the ensuing events, an explosive path seems like the natural outcome.

5.1.3.3. Helicopter Money

Policy Consider now a scenario in which the central bank increases money supply by \dot{M}_t as above and the treasury raises *only the concurrent* transfer rate by the same amount:

$$\dot{\tau}_{t+j}^{TH} = \begin{cases} \tau_{SS}^{TH} + \dot{o} \frac{M_{t+j}}{\dot{P}_{t+j}} & j = 0 \\ \tau_{SS}^{TH} & \forall j \geq 1 \end{cases} \quad \langle 5.6 \rangle$$

Restating the first line as $\dot{P}_t \dot{\tau}_t^{TH} = \dot{P}_t \tau_{SS}^{TH} + \dot{o} M_t$ indicates what the consolidated government actually does: print money and give it to the household.

Current Price Level This time, the contemporary price level *is* affected: The difference between two instances of the present-value budget equation (5.4)—before and after the policy innovation—is given by

$$\frac{B_{t-1} + M_{t-1}}{\dot{P}_t} - \frac{B_{t-1} + M_{t-1}}{P_t} = -\left(g_t + \dot{\tau}_t^{TH}\right) + \left(g_t + \tau_{SS}^{TH}\right) = -\dot{o} \frac{M_t}{\dot{P}_t},$$

where the unaffected future terms on the right-hand side are already canceled out. Since the comparison is between initial stationary-state money balances $M_t = M_{SS}$ before the policy move and a deviation \dot{M}_t thereafter, the ‘before’ amounts do not change over time, allowing to write $M_t = M_{SS} = M_{t-1}$ and thus express the deviating price level \dot{P} in terms of the original one:

$$\dot{P}_t = \left(1 + \dot{o} \frac{M_{t-1}}{B_{t-1} + M_{t-1}}\right) P_t \quad (5.7)$$

With a positive amount of outstanding bonds, the net growth rate of the price level equals the net growth rate of money times the money share in total liabilities (both growth rates refer to the alternative states within period t); put differently, the price level does not rise as much as nominal money supply. Without bonds, prices increase one-for-one with money.

Nominal Interest Rate In case outstanding consolidated-government liabilities are purely monetary, money and prices increase proportionally according to Equation (5.7) ($B_{t-1} = 0$; this is the case examined by Woodford 1995, p. 20) so that the interest rate stays the same. By contrast, if there is a positive amount of outstanding bonds, the price level does not increase as much as money supply, raising real money balances and hence decreasing the nominal interest rate via (2.24).

Future Price Levels without Initially Outstanding Bonds In the simple case without bonds outstanding initially, use two instances of the law of motion (5.5) to get

$$\dot{Z}_t = (1 + \dot{o}) Z_{t-1}, \quad (5.8)$$

which means that outstanding liabilities after the money-supply increase are $(1 + \dot{o})$ times higher than they would be without it (see Appendix C.1.2 for the derivation).

Since there is no rebate in period $t + 1$, the right-hand side of the present-value budget equation (5.4) is back to the level of the initial time- t stationary state, implying that the future price level stays elevated: $\dot{P}_{t+1} = \dot{P}_t = (1 + \dot{o}) P_{SS}$.

Again, real money stays constant, so the nominal interest rate does as well. The right-

hand side of the law of motion $\langle 5.5 \rangle$ is equal in $t + 1$ and the original state of t in real terms. Since the latter is stationary (cf. Appendix C.1.1), the situation in $t + 1$ must also be. Helicopter money with no initially outstanding treasury debt keeps the economy in stationary state, albeit with nominal variables elevated by the factor $(1 + \dot{o})$.

Future Price Levels with Initially Outstanding Bonds Unfortunately, the more realistic case with outstanding bonds does not yield similarly clear-cut results. This is because the nominal interest rate decreases by an unknown amount (cf. \nearrow *Nominal Interest Rate* above and in Section 5.1.3.2) while the transfer rate increases ($\dot{\tau}_t^{TH} > \tau_{SS}^{TH}$). The question is whether the interest-rate effect outweighs that of the transfer rate in the law of motion $\langle 5.5 \rangle$:

- If so, future outstanding liabilities decrease to $\dot{Z}_t < Z_{SS}$. Since there is no rebate on the transfer rate anymore in period $t + 1$, the right-hand side of the present-value budget equation $\langle 5.4 \rangle$ returns to its original (higher) level, lowering the price level in $t + 1$ markedly: $\dot{P}_{t+1} < P_{SS} < \dot{P}_t$. Like this, real money is increased once again, further depressing the nominal interest rate, outstanding liabilities, and the price level. The economy is trapped in a deflationary spiral similar to that in Section 5.1.3.2.
- Otherwise, if the increase of the transfer rate has a stronger effect than the decrease in the nominal interest rate, outstanding liabilities grow by Equation $\langle 5.5 \rangle$ and, consequently, the price level rises by Equation $\langle 5.4 \rangle$. In $t + 1$, the move to \dot{Z}_{t+1} via the law of motion $\langle 5.5 \rangle$ depends on the transfer rate (which returns to $\dot{\tau}_{t+1}^{TH} = \tau_{SS}^{TH}$), the actual result for outstanding liabilities \dot{Z}_t , and the nominal interest rate i_{t+2} . The latter two require that either a specific utility function be known or that one distinguishes even more subcases based on the size of $\dot{M}_{t+1}/\dot{P}_{t+1}$ relative to M_{SS}/P_{SS} and \dot{M}_t/\dot{P}_t . Because it is conceivable that there are ‘intermediate’ results which do not necessarily continue a trend commenced in the respective previous period (as in the bullet point above), this process is possibly endless.

To put it bluntly, distinguishing analytically all possible subcases of such a specific case study is a tedious exercise that does not seem fruitful enough to warrant further effort at this point (which is probably why Woodford 1995 also abstains from it).

5.1.3.4. Helicopter Debt

Policy In the final case, the consolidated government keeps money supply constant forever ($M_t = M_{SS} \forall t$) and increases transfers to the household *once* (in period t). Since the fraction \dot{o} can take any arbitrary value, we can still use Equation $\langle 5.6 \rangle$ to describe transfer policy even though it is not logically bound to money anymore.

Current Price Level As before, the increased transfer raises the price level, $\dot{P}_t > P_t$ (cf. *Current Price Level* and especially Equation (5.7) in Section 5.1.3.3).

Nominal Interest Rate Constant nominal money supply and an increased price level \dot{P}_t imply lower real money balances and, therefore, a higher nominal interest rate by Equation (2.24).

Future Price Levels Increased transfers and a higher nominal interest rate both work to increase the amount of outstanding nominal consolidated-government liabilities \dot{Z}_t in Equation (5.5), so the present-value budget equation (5.4) implies that $\dot{P}_{t+1} > \dot{P}_t > P_{SS}$. This depresses real money and increases the nominal interest rate further. Fortunately, the counteractive change of transfers back to their initial level τ_{SS}^{TH} does not complicate the analysis from $t + 1$ on again (like it does in Section 5.1.3.3) because the stationary state can act as a reference point: Comparing the iteration of Equation (5.5) between $t - 1$ and t (which is stationary) to that between $t + 1$ and $t + 2$ (with a higher interest rate) reveals that the latter leads to an increase in outstanding liabilities, indicating that the end result is an inflationary spiral (cf. Woodford 1995, p. 21).

5.1.4. Generalization with Money

The distinction between Ricardian and non-Ricardian fiscal policy refers to satisfaction or violation of the transversality condition. However, the latter only includes treasury debt B in Section 4.4 as money is nonexistent in that variant of the model. Therefore, the distinction is extended here to include money: Ricardian fiscal policy satisfies the transversality condition *as implied* by forward iteration of the respective flow budget condition (cf. Section 2.3.1), non-Ricardian fiscal policy does not. Formally, this amounts to whether the reaction coefficient γ_Z^T in the respective surplus rule

$$S_t = \gamma_Z^T Z_{t-1} - \dot{I}_{t+1} M_t \quad (5.9)$$

is zero or not. (The second term on the right-hand side is the seigniorage rebate introduced in Section 5.1.3.1. Appendix C.2 shows that rule (5.9) is Ricardian and therefore lets the present-value budget equation (2.43) hold identically.)

Reassessing his 1995 paper, which uses the narrower definition of Ricardian policy pertaining only to treasury debt (cf. his pp. 26-27), Woodford (2001, fn. 26, p. 690) argues that the broader definition including money is “conceptually preferable” because, given such a Ricardian policy, the *entire* transversality condition is rendered irrelevant for equilibrium determination.

Adding money to debt is also supported by Cochrane 1999, p. 330, but for a different reason: Using the example of a commodity standard, he argues that the value of

both types of consolidated-government liabilities diminishes if its backing is insufficient. Woodford (1995, p. 12) shares this view—“even in the case of inconvertible fiat money” (a notion he inherits from Sargent 1982b).

5.1.5. Interest-Rate Pegs

Background The traditional study of Sargent and Wallace (1975) holds that pegged interest rates lead to indeterminacy. This section largely follows Woodford (1995; 2001) in explaining how fiscal policy can lead to determinacy of the model when monetary policy is passive like that. It does so also graphically and thus rectifies an omission from Section 4.3.2.1.

Policy Consider again the baseline optimizing model of Chapter 2. In contrast to Section 5.1, in which money supply M was the instrument of choice, the central bank now controls the nominal interest rate i . In particular, it keeps it pegged at a constant rate, which is why time indices for all affected variables are suppressed in this subsection. This pertains not only to i and \dot{I} , but also to liquidity preference $L(c, i) = L$ as a measure of real money demand (cf. Section 2.4), if one assumes endowments y as well as treasury expenditure g to be constant.

Indeterminacy? Obviously, this pins down the right-hand side of the money-demand equation (2.25). The two variables on the left-hand side, M and P , however, are undetermined individually: While real money M/P may satisfy the equation, this can happen with infinitely many different pairings of M and P (cf., among others, Cochrane 1999, p. 350; Buiter 2002, p. 471).¹⁴

Fiscal Price Determination It will be guessed that the proposed solution is to resort to the present-value budget equation (2.44) again. Substituting in money demand (2.25), it is expressed in terms of liquidity preference L instead of real money m :

$$\frac{Z_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j (\dot{I}L + s_{t+j}) \quad (5.10)$$

¹⁴ While he admits that it never really gained traction, McCallum (1986, p. 137; 2003b, pp. 1156-1157) points to an interesting distinction concerning indeterminacy: The one discussed here is about price-level indeterminacy and the inability of the model to determine any nominal variable. This problem arises if the agents it portrays are only interested in real variables and policy does not provide a nominal anchor. By contrast, the issue discussed in Chapter 4 (in particular Section 4.2.3) is a *real* indeterminacy in the sense that the model can not uniquely pin down a single equilibrium path from a multitude of possible candidates. As already explained, this is connected to the existence of expectations about future endogenous variables.

Since all variables on the right-hand side are determined, and so are outstanding nominal liabilities of the consolidated government Z_{t-1} , Equation (5.10) pins down the price level P_t . Money demand (in the form of either Equation (2.22) or (2.25)) then gives the required amount of money that the central bank has to supply in order to implement the desired nominal interest rate. The present-value budget equation (2.44) / (5.10) is also the reason why the central bank can not simply supply an arbitrary amount of money M in order to determine P via money demand (2.22) / (2.25) directly: the two equations would conflict unless, by chance, it chose the ‘correct’ amount of money consistent with equilibrium.

As in the case studies of Section 5.1.3, the law of motion (2.37) can be used to infer the future values of outstanding consolidated-government liabilities Z ; in combination with liquidity preference (2.25), it reads

$$Z_t = (1 + i) (Z_{t-1} - i P_t L - S_t) . \quad (5.11)$$

Given the processes of the policy variables i and S , Equations (5.10) and (5.11) thus determine the development of the price level P and outstanding liabilities Z .

Link to Section 2.4 At the most basic level, Equations (2.3), (2.7), and (2.14) determine the three remaining variables B , M , P . As already mentioned in the *Sequential vs. Simultaneous Equilibrium Determination* in Section 2.4, this happens simultaneously. Directly linking the determination of individual variables to specific equations is difficult in this case because if one connects the household flow budget constraint (the origin of the present-value budget equation (2.44) / (5.2)) with the price level P_t and money demand (2.25) (stemming from Equation (2.14)) with money supply M_t , the Fisher equation (2.7) would have to determine B_t , which is of course nonsensical because it does not feature the amount of bonds. Therefore, it must actually be the Fisher equation—which is part of the construction of the present-value budget equation (see p. 32)—that determines the price level while the amount of bonds is the residual in the household’s flow budget constraint. Recall that the ‘ $\mathbb{E}_t P_{t+1}$ problem’ (cf. Sections 2.4, 5.1.1) is resolved by construction of the present-value budget equation, which incorporates the entire sequence of future policy.

Graphical Stability Analysis Determination of the price level P_t can also be analyzed graphically. To do so, consider a slightly rearranged version of the consolidated-government budget equation (2.40) which also underlies the present-value budget equation (2.44) / (5.2) / (5.10):

$$\mathbb{E}_t \frac{Z_t}{P_{t+1}} = \beta^{-1} \frac{Z_{t-1}}{P_t} - \beta^{-1} (iL + s_t)$$

It is shown in Figure 5.1. Finding the steady state follows the same logic as in Figure

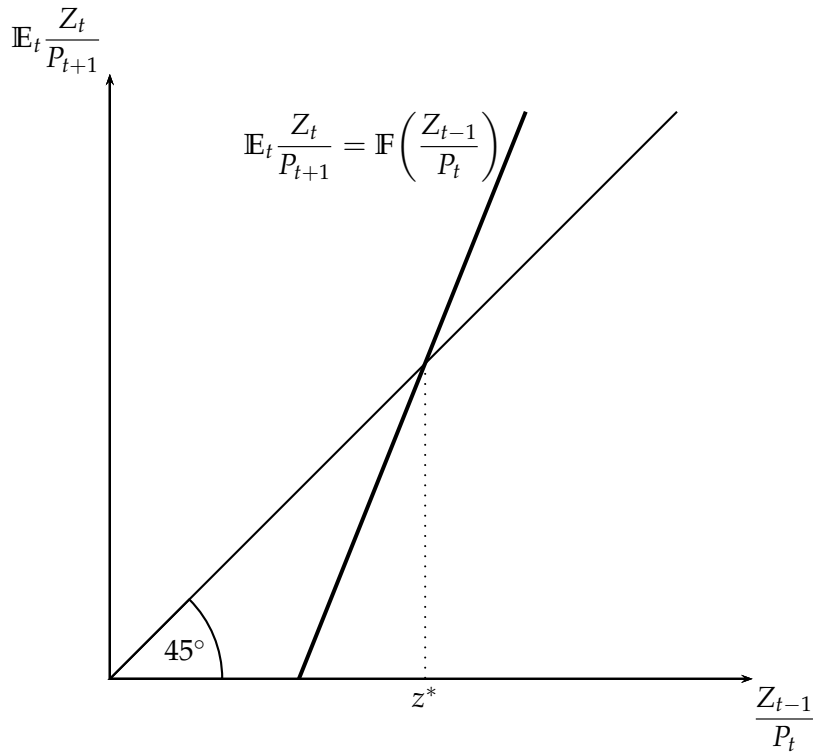


Figure 5.1: *Dynamics of Real Outstanding Consolidated-Government Liabilities Under an Interest-Rate Peg.* ♦ Source: Own illustration.

4.1: Since $\beta^{-1} > 1$, the phase line is steeper than the 45° line, so for any initial value $Z_1/P_0 \neq z^*$ set in $t = 1$ the process would explode and thus violate the transversality condition (2.42). With all variables on the right-hand side given exogenously and Z_{t-1} predetermined in period t , it is the price level P_t that jumps to an appropriate value, just as in the present-value budget equation (5.10). The exact value of the steady state z^* can be found by setting both Z/P terms to z^* and solving for it:

$$z^* = \frac{1}{1 - \beta} (\dot{I}L + s_t) \quad (5.12)$$

While the graphical approach might be more instructive than looking at the present-value budget equation, one has to be wary of its drawbacks. A hint at the difficulties associated with the graphical analysis of compound variables such as the real market value of treasury debt $b/(1+i)$ is already given in Section 4.3.2.1. With the nominal interest rate i pegged in the present setup, ‘day-to-day coordination’ with the central bank is not necessary anymore. However, surpluses s can still vary, and this has profound consequences: For one, the graphical method assumes that one can not only infer

$\mathbb{E}_t \square_{t+1}$ from \square_t via a given phase line but that one can also move $\mathbb{E}_t \square_{t+1}$ from the ordinate to the abscissa via the 45° line in order to subsequently infer $\mathbb{E}_t \square_{t+2}$, $\mathbb{E}_t \square_{t+3}$, and so on. Arriving at a stable solution—that is, a steady or stationary state—thus requires the corresponding phase line to not change (noting that the steady state z^* can also be found by assuming a constant s_t in Equation (5.10) underlines this)—but this is exactly what happens when the treasury *does* change surpluses. Furthermore, changes that are strong enough (*ceteris paribus*) to turn the intercept positive also imply that the treasury suddenly becomes a creditor to the household because the steady state z^* is moved into negative territory in Figure 5.1.

Why Peg the Interest Rate? Even if this regime is stable and determinate, it begs the question why the monetary authority should choose to peg the interest rate at a certain value. Woodford (1995, p. 1000; 2001, pp. 681–685, 689) suggests that interest-rate pegs are especially appealing in times of great and mostly unexpected fiscal strain, such as wartime—“when government purchases vary for reasons largely independent of the state of the economy or the government’s budget” (Woodford 2001, p. 681) or, put differently, when they are truly exogenous. In particular, he argues that the situation in the United States during and after world war II can be analyzed using the model at hand (see also Chapter 8.3).

5.2. Boundaries for Endogenous Variables

5.2.1. The Zero Lower Bound and the Perils of Taylor Rules

5.2.1.1. Falling Into Liquidity Traps

A Lower Bound for the Nominal Interest Rate In Section 4, especially within the regime of monetary dominance in Section 4.3.2, it is assumed that the central bank actively sets the nominal interest rate according to a linear Taylor rule with $\gamma_\pi^C > \beta^{-1}$. In this, it is not bounded from above or below. Of course, this is not very realistic because the existence of money as a store of value puts a lower bound on the nominal interest rate that is equal (or close) to zero. In order to take account of this limitation, the original Taylor rule (4.5) is replaced by

$$i_{t+1} = \max \left[i^* + \gamma_\pi^C (\pi_t - \pi^*) + \varepsilon_{t+1}^i, 0 \right] \quad (5.13)$$

in this section.

Implications for the Inflation Process Equation (5.13) can still be combined with the Fisher equation (2.7) to form a law of motion for inflation similar to Equation (4.6);

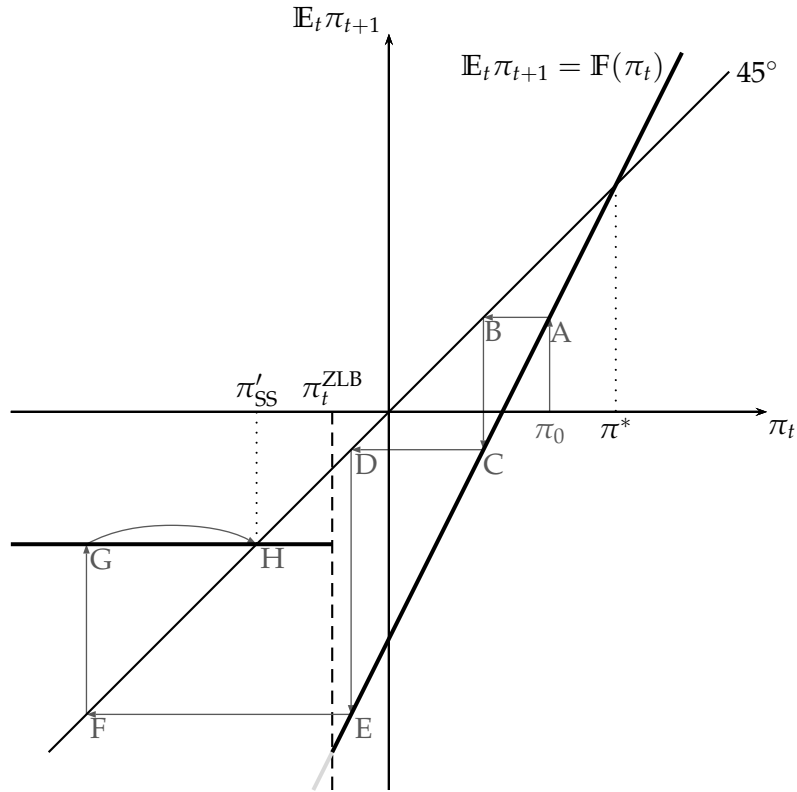


Figure 5.2: Inflation Dynamics in Consideration of the Zero Lower Bound on Nominal Interest Rates. ♦ Source: Own illustration.

however, this functions properly only within a certain domain, namely, approaching a critical value

$$\pi^{\text{ZLB}} = \frac{\gamma_{\pi}^{\text{C}} - (1+r)}{\gamma_{\pi}^{\text{C}}} \pi^* - \frac{r}{\gamma_{\pi}^{\text{C}}} \quad \langle 5.14 \rangle$$

from above (the monetary policy shock term ε_t^i is discarded here; see Appendix C.3.1 for derivations). π^{ZLB} is the lowest inflation rate to which the central bank can react ‘normally,’ that is, linearly with $\gamma_{\pi}^{\text{C}} > \beta^{-1}$ in the present specification. In the domain $[\pi^{\text{ZLB}}, \infty)$, the inflation process is described by Equation $\langle 4.6 \rangle$ algebraically and by the upward-sloping part of the phase line $E_t \pi_{t+1} = F(\pi_t)$ in Figure 5.2 graphically. (For the sake of completeness: π^* corresponds to π^* in Figure 4.1.)

The Liquidity Trap If current inflation π_t falls below the critical value π^{ZLB} , the central bank is powerless; the best it can do is leave the nominal interest rate at zero. Regarding the law of motion, the Fisher equation $\langle 2.7 \rangle$ with $i_{t+1} = 0$ implies that $E_t \pi_{t+1} = -r/(1 +$

r). Assuming the situation is as depicted in Figure 5.2, it actually gives rise to a second steady state

$$\pi'_{SS} = -\frac{r}{1+r} \quad \langle 5.15 \rangle$$

(Appendix C.3.2 describes a slightly more complicated scenario). The rationale is as follows: Starting from an arbitrary $\pi_0 < \pi^*$, movement until point E is straightforward and similar to what would happen in Figure 4.1 (keeping track of time, point A corresponds to $\mathbb{E}_t\pi_{t+1}$, point C to $\mathbb{E}_t\pi_{t+2}$, and point E to $\mathbb{E}_t\pi_{t+3}$). As before, the expectation one period further ahead ($\mathbb{E}_t\pi_{t+4}$) can be found by moving the ‘current’ value $\mathbb{E}_t\pi_{t+3}$ from the ordinate to the abscissa via the 45° degree line (point F). But since, at this point, $\mathbb{E}_t\pi_{t+3} < \pi^{ZLB}$, the market expects $\mathbb{E}_t i_{t+3} = 0$, which then leads to $\mathbb{E}_t\pi_{t+4} = \pi'_{SS}$ as implied by the Fisher equation. In point G, the inflation process has reached the horizontal part of the phase line. Finally, moving from the ordinate to the abscissa via the 45° line again, we arrive at point H, which confirms that π'_{SS} is a steady state.

Benhabib, Schmitt-Grohé, and Uribe (2002, pp. 538, 545) call the “unintended” steady state π'_{SS} a liquidity trap because the central bank loses its power to stabilize the economy and achieve its targets. Similarly, to the extent that $\pi'_{SS} < 0$, it would also be conceivable to speak of a deflationary trap; however, as the latter is often associated with accelerating—that is, *non-steady*—dynamics (up to a collapse of the real economy), whereas endowment-economy models such as the present one do not even allow for output gaps, ‘liquidity trap’ might be the more appropriate term here.

Continuous vs. Discontinuous Taylor Rules The discontinuity introduced by the zero lower bound on nominal interest rates complicates algebraic treatments profoundly. While manageable, even the graphical analysis using phase diagrams is somewhat tedious, as the explanations revolving around Figure 5.2 (and Figure C.1 in Appendix C.3.2) demonstrate.

Therefore, one might be inclined to devise a *continuous* policy rule that respects the zero lower bound. One way to achieve this is to employ an exponential function; note that such a policy rule can be active or passive depending on the level of inflation as its main argument (cf. Bullard 2010, p. 341). Still, as Figure 5.3 illustrates, active monetary policy around the target steady state π^* comes with a second, unwelcome steady state π'_{SS} (cf. Woodford 2003b, Ch. 2.4; Cochrane 2011, pp. 576–577).

As a side note, Benhabib, Schmitt-Grohé, and Uribe (2001, p. 42) imply that continuity of the Taylor rule is less important than monotonicity. At the same time, of course, they also note that non-monotonous rules are of little relevance since there is certainly no sensible case in which the central bank has an incentive to define an interval in which it (continuously or discontinuously) increases the nominal interest rate if the inflation

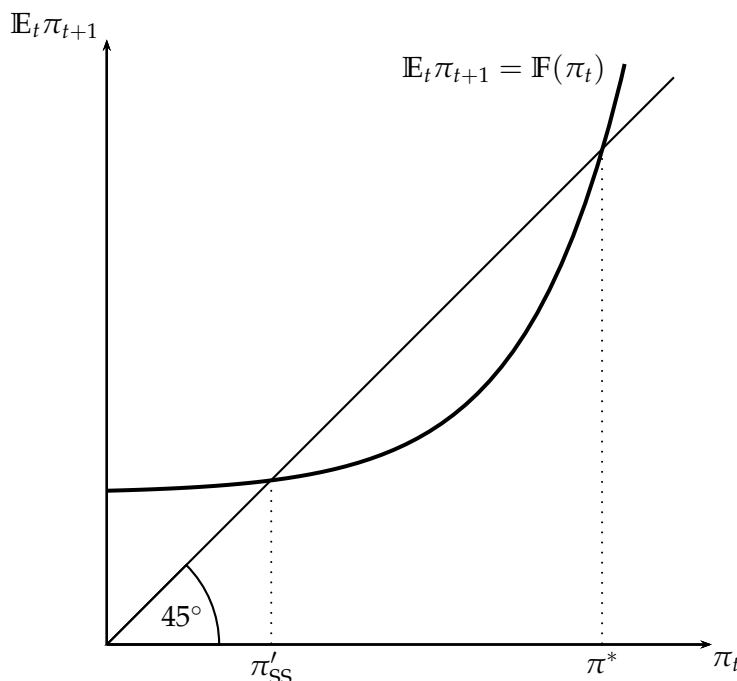


Figure 5.3: *Inflation Dynamics in Consideration of the Zero Lower Bound on Nominal Interest Rates (Continuous Nonlinear Taylor Rule).* \diamond Source: Own illustration based on Woodford (2003b, p. 126) and Cochrane (2011, p. 577).

rate declines.

The Perils of Taylor Rules The main results presented here emerge not only in simple setups such as the present one but also in production economies with price rigidities (cf. Benhabib, Schmitt-Grohé, and Uribe 2001). However, Benhabib, Schmitt-Grohé, and Uribe (2002, fn. 7, p. 545) note that the low steady state π'_{SS} might actually be preferred to the inflation target π^* in endowment economies because the associated higher level of real balances raises utility (while consumption remains constant; sticky-price models, by contrast, allow for the ‘intended’ steady state π^* to be welfare-superior).

Finally, it is important to emphasize the self-fulfilling nature of these liquidity traps: In a very reduced deterministic setup, it *might* be arguable that $\pi_0 \not\leq \pi^*$ so that the economy never slides off into the unintended steady state π'_{SS} (except for some blatant policy mistake). In stochastic setups, however, we can not rule out *a priori* that inflation is never pushed down too much by a shock. This enables self-fulfilling prophecies in that the mere expectation of $\pi_0 < \pi^*$ suffices to implement the liquidity trap (cf. Benhabib, Schmitt-Grohé, and Uribe 2002, pp. 546-547). Therefore, naïve reliance on Taylor rules as a panacea to stabilize the economy is perilous.

5.2.1.2. Avoiding Liquidity Traps

Following Benhabib, Schmitt-Grohé, and Uribe (2002), one way to avoid deflationary traps is through the appropriate design of fiscal policy. In this subsection, I briefly discuss three approaches.

Inflation-Sensitive Surpluses For simplicity, maintain the assumption that treasury expenditure remains constant so that the real interest rate does as well (see Section 2.4). The treasury sets t_t^{TH} such that, by Equation (2.10),

$$S_t = \dot{\gamma}_Z^T Z_{t-1} - \dot{I}_{t+1} M_t, \quad (5.16)$$

where

$$\dot{\gamma}_Z^T \equiv \dot{\gamma}_Z^T(\pi).$$

In words, surpluses S_t are raised to redeem an inflation-dependent fraction $\dot{\gamma}_Z^T$ of outstanding consolidated-government liabilities Z_{t-1} ; at this, analogously to rule (5.3) in Section 5.1.3.1, the treasury transfers $\dot{I}_{t+1} M_t$ (the interest savings it generates by issuing money instead of debt) back to the household. An important additional assumption about the fraction $\dot{\gamma}_Z^T$ is that

$$\begin{aligned} 0 < \dot{\gamma}_Z^T(\pi^*) &\leq 1 \\ -1 < \dot{\gamma}_Z^T(\pi'_{SS}) &< 0 \end{aligned} \quad (5.17)$$

which makes fiscal policy Ricardian for $\pi_t = \pi^*$ and non-Ricardian for $\pi_t = \pi'_{SS}$. To avoid problems with discontinuities or non-monotonicity, Benhabib, Schmitt-Grohé, and Uribe (2002, p. 547) assume that the coefficient is strictly increasing in inflation ($\partial \dot{\gamma}_Z^T / \partial \pi > 0$).

The fact that $\dot{\gamma}_Z^T(\pi'_{SS}) < 0$ is not entirely compatible with the original definition of non-Ricardian policy in Section 4.4 and Figure 4.3b is not problematic for it merely ‘overstates the case.’ The usual definition of non-Ricardian policy would imply $\dot{\gamma}_Z^T(\pi'_{SS}) = 0$ (cf. also Section 5.1.4) because it was probably not conceivable to its originators why the consolidated government should not only ignore outstanding liabilities but actively make them explode in a ‘super-non-Ricardian’ fashion.

Plugging the surplus rule (5.16) into the consolidated government’s flow budget equation (2.40) and iterating forward (cf. Equation (4.23) on p. 76) produces two different outcomes depending on the inflation rate: For π^* , fiscal policy is Ricardian so that the

transversality condition (2.42) is satisfied. By contrast, for π'_{ss} , the treasury effectuates an explosive real-debt path, and since the latter is ruled out by the transversality condition, expectations will never coordinate on the unintended steady state π'_{ss} . In short, the possibility of self-fulfilling liquidity traps is eliminated.

Liability Growth Targeting A variation on the approach laid out above is to target the growth rate of outstanding consolidated-government liabilities

$$\mu_Z \equiv \frac{Z_t - Z_{t-1}}{Z_{t-1}}, \quad (5.18)$$

which implies the surplus rule

$$S_t = \frac{i_{t+1} - \mu_Z}{1 + i_{t+1}} Z_{t-1} - \dot{I}_{t+1} M_t \quad (5.19)$$

(see Appendix C.3.3 for derivations). In principle, nominal liabilities Z_{t-1} depend on choices made by both the treasury and the central bank, but since the latter is assumed to follow the Taylor rule (5.13) and to disregard any further concerns, the growth target (5.18) must be achieved by a surplus rule that, again, takes interest savings on money balances into account. At this, it is important to have

$$i'_{ss} < \mu_Z < i^*, \quad (5.20)$$

that is, the growth rate of outstanding liabilities μ_Z must be greater than the interest rate associated with the liquidity trap i'_{ss} and smaller than the nominal interest rate i^* in the target steady state π^* .

Evidently, the effects of Equations (5.19)-(5.20) are very similar to those of \nearrow *Inflation-Sensitive Surpluses* (Equations (5.16)-(5.17)): Fiscal policy becomes ('super-')non-Ricardian for the unintended steady state π'_{ss} with i'_{ss} , hence ruling it out and making the target steady state π^* with i^* (for which surpluses behave in a Ricardian fashion) the unique equilibrium (cf. Benhabib, Schmitt-Grohé, and Uribe 2002, p. 549). This result is confirmed by Woodford (2003b, Ch. 2.4.2), who also shows that whether monetary policy uses money balances or the nominal interest rate as its instrument is almost irrelevant (there are minor differences, cf. his p. 133) while fiscal policy is the universal solution to avoid self-fulfilling deflationary traps.

Balanced Budgets Finally, another proposal is to require that the secondary deficit (the primary deficit plus interest due on outstanding debt) be zero at all times. In the context of the consolidated-government budget equation (2.38), which is more explicit about

interest on outstanding debt as it uses coupon instead of discount bonds, this policy prescription can be rearranged to the surplus rule

$$S_t = i_t B_{t-1} = \dot{I}_t Z_{t-1} - \dot{I}_t M_{t-1}, \quad \langle 5.21 \rangle$$

where the second equality also makes use of definition $\langle 2.29 \rangle$. According to the definitions in Sections 4.4 and 5.1.4, this type of fiscal policy is Ricardian as long as i_t and thus \dot{I}_t are not equal to zero indefinitely. Consider the respective transversality condition $\langle 2.34 \rangle$ (combined with Equation $\langle 2.20 \rangle$) to see this:

$$\lim_{J \rightarrow \infty} \mathbb{E}_t Z_{t+J} \prod_{j=0}^J \frac{1}{1 + i_{t+1+j}} = 0$$

If the economy is on a path towards the unintended steady state π'_{SS} , nominal interest rates will go to zero in finite time and remain there indefinitely. Therefore, outstanding liabilities in the ‘final period’ Z_{t+J} are not discounted strongly enough to yield a present value of zero; once again, the transversality condition is violated so that the respective path can not constitute an equilibrium. A balanced-budget rule like $\langle 5.21 \rangle$ averts the liquidity trap *without deficits* if—and only if—the central bank indeed sets $i'_{SS} = 0$ for π'_{SS} . Therefore, the balanced-budget proposal is a two-edged sword: It might be more appealing in that it does not rely on deficit creation, but it requires strong commitment on the part of the central bank to lower interest rates all the way to zero (cf. Benhabib, Schmitt-Grohé, and Uribe 2002, pp. 549-550).

5.2.1.3. Liquidity Traps in Money-Supply Regimes

Woodford (2003b, Ch. 2.4.2) extends the analysis of Section 5.2.1.1 to regimes in which the central bank uses money supply instead of the nominal interest rate as its instrument. Assume for simplicity that it supplies nominal money balances which grow at the constant rate

$$\mu_{M,t} \equiv \frac{M_t}{M_{t-1}} = \mu_M \quad \forall t. \quad \langle 5.22 \rangle$$

Phase Diagram As before, one can derive a phase diagram in order to visualize the development of real money balances. In order to do so, combine the Fisher equation $\langle 2.47 \rangle$ with the implicit money demand function $\langle 2.22 \rangle$ to get:

$$\mathbb{E}_t m_{t+1} = \frac{\mu_M}{\beta} \left[1 - \frac{u_m(m_t)}{u_c(c_t)} \right] m_t \quad \langle 5.23 \rangle$$

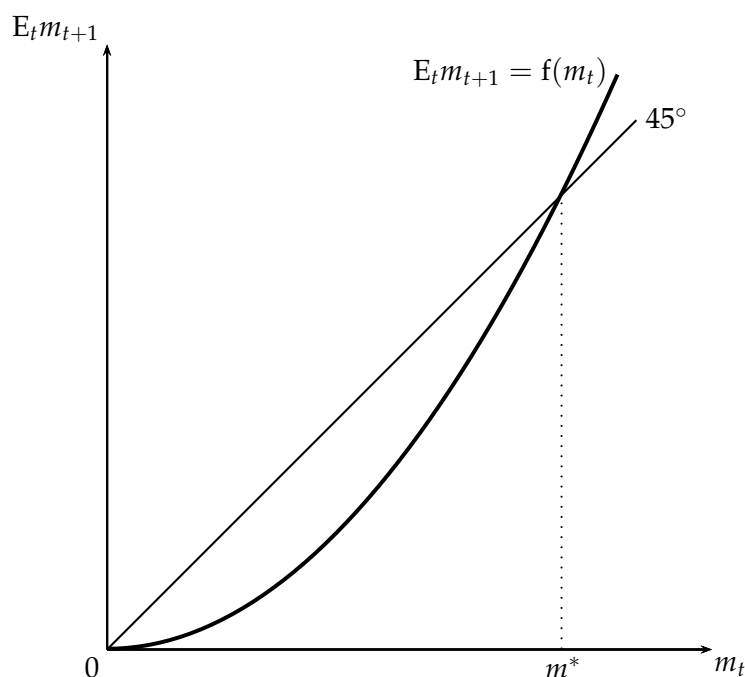


Figure 5.4: *Dynamics of Real Money Balances under a Money-Supply Policy.*
 ◇ Source: Own illustration based on Woodford (2003b, p. 130).

(See Appendix C.3.4 for derivations. This is similar to the derivations of the equations depicted in Figures 4.1 and 5.3, for which the Fisher equation is combined with the respective policy—i.e., Taylor—rule.)

What can we infer about the shape of the respective phase line? From the properties of the utility function (2.2) and (5.1), it can be inferred that the term in brackets lies in the interval $(0, 1]$. Since $u_m(m_t)$ is decreasing in m_t , the combination of m_t with the term in brackets is convex in m_t —at least at first: Assuming there is a satiation point in the demand for real balances beyond which $u_m(\cdot) = 0$, m_{t+1} becomes linear in m_t afterwards so that real balances eventually grow at the rate μ_M/β . Further, as m_t becomes very small, $u_m(m_t)$ increases, but never exceeds $u_c(c_t)$; therefore, m_{t+1} takes a very low value not quite equal to zero for small m_t . Relaxing assumption (5.1) by making it a ‘smaller than or equal to’ relationship would extend the above interval to $[0, 1]$ and thus allow for $m_{t+1} = 0$. To sum up, the function $\mathbb{E}_t m_{t+1}$ starts at zero, is convex with a slope below μ_M/β initially and becomes linear with slope μ_M/β beyond the satiation point. Whether this graph intersects the 45° line only in the origin or also in a second point therefore depends on the slope μ_M/β .

Deflation with Constant Nominal Money Growth Figure 5.4 sketches the law of motion (5.23) for $\mu_M/\beta > 1$. A starting value for real balances to the right of the intended

steady state ($m_0 > m^*$) implies that they grow without bound forever. This steady increase can not be driven by nominal money growth μ_M only since the growth rate of real money μ_M/β is greater than that; the economy hence experiences deflation at the rate π'_{SS} (setting the term in brackets in Equation [\(5.23\)](#) to unity and using the money-growth target [\(5.22\)](#) also leads to Equation [\(5.15\)](#)). Whether this can constitute an equilibrium depends on fiscal policy:

- Analyses in which money supply is the monetary instrument often assume that treasury debt is zero at all times, which renders fiscal policy non-Ricardian and would lead to a violation of the transversality condition [\(2.42\)](#) (cf. Woodford 2003b, pp. 131-132). Therefore, the deflationary trap could be excluded as an equilibrium.
- By contrast, if fiscal policy is Ricardian, treasury debt could be adjusted so as to guarantee satisfaction of the transversality condition; namely, the treasury would become a creditor to the household sector because ever-increasing real money balances require it to take a negative real debt position so as to make total consolidated-government liabilities equal to zero. (Cf. also Benhabib, Schmitt-Grohé, and Uribe 2002, pp. 551-552, for a rather sceptical view on such deflations.)

5.2.2. Debt Limits

Setup & Terminology Another bound that may constrain policy comes in the form of a limit on the creation of consolidated-government liabilities. Using discount bonds, it could be written as

$$0 \leq \frac{M_t + Q_t B_t}{P_t} \leq \bar{z} \quad \forall t, \quad (5.24)$$

where $\bar{z} > 0$ denotes the liability limit. In the literature, several expressions are often used synonymously. Therefore, one might also encounter the terms ‘debt limit’ or ‘borrowing limit’ for \bar{z} (cf. Woodford 2001, p. 696). In richer models, \bar{z} could be defined as a function of other variables; for instance, setting $\bar{z}_t = 0.6y_t$ would capture one of the Maastricht criteria for Euro zone members and candidates—roughly, since the Maastricht treaty only makes reference to public debt B but not money M . In the present constant-endowment setup, choosing the corresponding constant value for \bar{z} is equivalent. (Cf. Woodford 2001, p. 716; Benhabib, Schmitt-Grohé, and Uribe 2002, p. 553.) The zero lower bound on the left-hand side is added for simplicity. As shown in Appendix C.4.1, fiscal policy is Ricardian because the liability limit [\(5.24\)](#) necessarily implies satisfaction of the transversality condition [\(2.42\)](#).

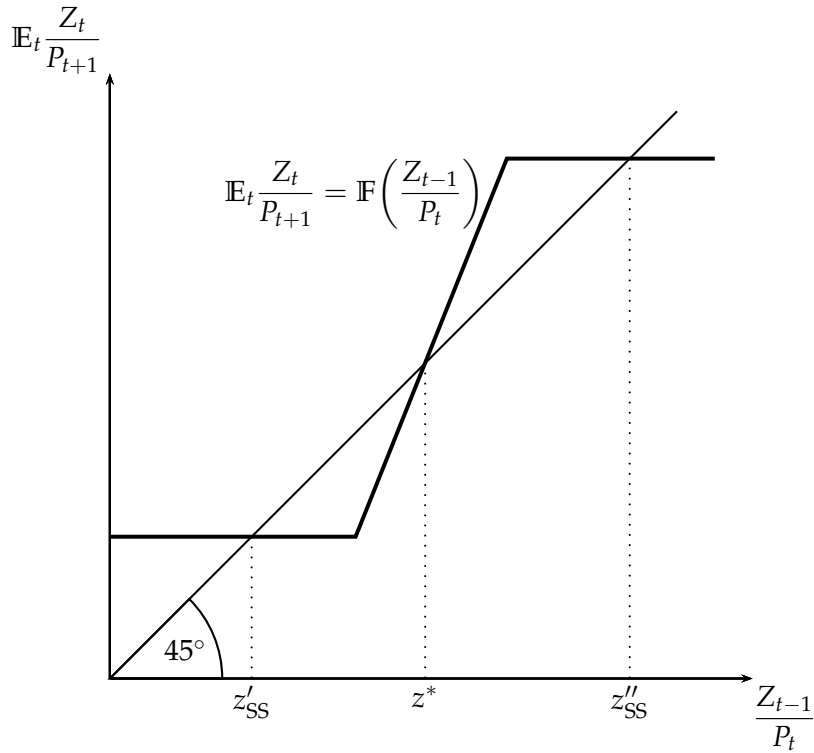


Figure 5.5: *Dynamics of Real Outstanding Consolidated-Government Liabilities Under a Liability Limit.* ♦ Source: Own illustration based on Woodford (2001, p. 700).

Steady States The consolidated-government budget equation (2.40) is repeated here as

$$\mathbb{E}_t \frac{Z_t}{P_{t+1}} = \beta^{-1} \frac{Z_{t-1}}{P_t} - \beta^{-1} (\dot{\mathbf{I}}_{t+1} m_t + s_t)$$

(under the assumption of constant endowments, cf. Section 2.4) and depicted by Figure 5.5. While the latter is very similar to Figure 5.1, the liability limit (5.24) implies that there are two additional steady states

$$\begin{aligned} z'_{SS} &= -\beta^{-1} \dot{\mathbf{I}} m \\ z''_{SS} &= \beta^{-1} (\bar{z} - \dot{\mathbf{I}} m) \end{aligned} \tag{5.25}$$

besides the original z^* given by Equation (5.12), where $z'_{SS} < 0 < z^* < z''_{SS}$ (cf. Appendix C.4.2).

Global Indeterminacy vs. Local Determinacy Any initial value $Z_1/P_0 \neq z^*$ eventually leads to one of the other two steady states, z'_{ss} or z''_{ss} , because of the ‘explosive’ behavior depicted in Figure 5.5. Therefore, the transversality condition (2.42) does not serve to rule out unstable equilibria anymore, or put differently, any trajectory starting between z'_{ss} and z''_{ss} constitutes an equilibrium, which makes the model (globally) indeterminate again.

Woodford (2001, pp. 698-699) calls this constellation “locally unique” in the sense that z^* is the only equilibrium in which the state variable Z/P remains in the *interior* of the effective bounds (5.25); accordingly, fiscal policy that would let Z/P run into these bounds is “locally non-Ricardian.” He refers to the analysis of Leeper (1991) and active fiscal policy (cf. Section 4.2.2) in particular, which is purely local in nature because it relies on linear approximations. Interestingly, Woodford seems to assume that the bounds (5.25) are “far from the equilibrium in question” so that local determinacy is sufficient for a usable model. To what extent this can be reconciled with a debt limit such as the one stipulated by the Maastricht treaty is questionable: On the one hand, the 60% debt-to-GDP ratio is a relevant constraint even for countries that do not seem to lie outside the ‘local domain’ in which linear approximations can be used in good faith (Germany probably comes to mind first, but the same could be argued for Eurozone countries which are affected much more strongly by the economic problems of the past decade, such as Italy, Portugal, or Spain, but without experiencing turbulent conditions). On the other hand, the ease with which the Maastricht treaty is violated as well as the lack of truly severe consequences from these violations points to the fact that the 60% debt-to-GDP criterion can not be the final frontier (or a true *limit* in the mathematical sense) that Woodford sees in z''_{ss} . But the more these bounds (5.25) are pushed outwards mentally, the more they effectively align with the transversality condition itself and the less they represent a distinct phenomenon. Therefore, as a matter of principle, local determinacy could be seen as one of the shakier concepts in the rigorous and concise world of modern DSGE modeling.

Key Takeaways from Chapter 5

Fiscal price determination also works with money and under money-supply policy. Interest-rate pegs do not necessarily lead to indeterminacy. The zero lower bound on nominal interest rates does, but again, fiscal policy can be a remedy. Debt limits can lead to indeterminacy as well, depending on whether the analysis is local (determinacy) or global (indeterminacy).

6. Monetary Unions

I extend the baseline model to a monetary union and study three cases relating to different fiscal-policy combinations of the two member countries.

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Derivations for this chapter can be found in Appendix D.

6.1. Adjustments to the Baseline Model

The basic adjustments to the single-country model are inspired by Woodford (1996). Bergin (2000) also studies a monetary union.

Countries and Agents The present economy is assumed to consist of two countries, Domestic (superscript D) and Foreign (F). The defining feature of a monetary union is that all member states use the same currency, issued by a common supranational central bank, while the respective treasuries retain the ability to make country-specific transfer and spending decisions.

Households & Endowments Each country is populated by a mass of households whose behavior is similar to that of the representative agent described at the beginning of Chapter 2.1 (\nearrow *Utility*). The endowments they receive are constant and equal in both countries ($y_t^D = y_t^F = y^D = y^F \forall t$).

Equality of Prices Without any trade restrictions on goods markets, the law of one price holds and both countries face a common price level $P_t^D = P_t^F = P_t \forall t$, implying that inflation is also similar at all times ($\Pi_t^D = \Pi_t^F = \Pi_t \forall t$).

Equality of Interest Rates The easiest case is that in which the central bank uses the nominal interest rate as its policy instrument and sets it equally for all member states. With the nominal interest rate as well as the inflation rate equal in both countries, so are real interest rates.

In a money supply regime, the argument, running from money supply to the nominal interest rate implied by money demand (2.22), would be a bit more involved. However, no-arbitrage on capital markets carries an implication for real interest rates: If Domestic and Foreign treasury debt is perfectly homogeneous except for the real return, households in both countries choose to hold only the return-dominant type. If both treasuries try to place debt on the market, they can only do so at equal real—and, because of equal inflation rates, nominal—interest rates. Alternatively, only the treasury offering the higher return will be able to sell bonds, but since it can sell to households in all countries and the respective Euler equations (2.21) of the latter are uniform, real interest rates (and nominal interest rates) have to be equal everywhere.

No Exchange Rates With only a single type of currency, a nominal exchange rate between Domestic and Foreign does not even exist. However, differing price levels in both countries would give rise to a real exchange rate in the sense of a relative price. While this is an often-used indicator in arguments about competitiveness in practice (cf. Sinn 2014a; Wyplosz 2013 for opposite positions on competitiveness as a trigger of the Eurozone crisis), the simplified setup presented here does not allow for any sensible price-level divergence—a real exchange rate different from unity—as this would require restrictions either on the flow of goods or the functioning of the price mechanism.

Household Budget Constraints The respective equivalents to the flow budget constraint (2.3) read

$$\begin{aligned} C_t^D + M_t^D + Q_t B_t^{DD} + Q_t B_t^{DF} &= Y_t^D + T_t^{DTH} + B_{t-1}^{DD} + B_{t-1}^{DF} + M_{t-1}^D \\ C_t^F + M_t^F + Q_t B_t^{FD} + Q_t B_t^{FF} &= Y_t^F + T_t^{FTH} + B_{t-1}^{FD} + B_{t-1}^{FF} + M_{t-1}^F. \end{aligned} \quad (6.1)$$

Double superscripts follow the pattern ‘holder, issuer’ (or ‘creditor, debtor’) so that, say, B_t^{DF} denotes debt issued by the foreign treasury at time t and held by the domestic household. This is necessary because each household can hold debt of each treasury, where the bond price Q_t is also equal across borders as it depends on the nominal interest rate i_{t+1} by Equation (2.8). T_t^{DTH} and T_t^{FTH} denote net transfers from the treasury to the household in Domestic and Foreign, respectively.

Market Clearing Within the monetary union, there is no equivalent to the goods-market-clearing condition (2.27) for any country individually as both populations are assumed to have the ability to trade freely on goods and capital markets. Rather, the union-wide goods market clears if

$$y_t^D + y_t^F = c_t^D + c_t^F + g_t^D + g_t^F \quad \langle 6.2 \rangle$$

While clearing of the treasury-debt markets is implicit in Chapter 2, it is crucial to make it explicit in the present analysis:

$$\begin{aligned} B_t^D &= B_t^{DD} + B_t^{FD} \\ B_t^F &= B_t^{DF} + B_t^{FF} \end{aligned} \quad \langle 6.3 \rangle$$

At this, B^D (B^F) denotes total outstanding debt of Domestic's (Foreign's) treasury and, as mentioned above, the terms on the right-hand sides denote individual debt holdings of the Domestic and Foreign households.

Similarly, money balances supplied by the central bank M have to equal exactly the sum of individual money demands (denoted M^D and M^F , respectively):

$$M_t = M_t^D + M_t^F \quad \langle 6.4 \rangle$$

6.2. Aggregation Problems

National Household Present-Value Budget Constraints As in the single-country case, the national flow budget constraints can be rearranged into present-value budget constraints for the respective households:

$$\begin{aligned} \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(c_{t+j}^D + \dot{I}_{t+j+1} m_{t+j}^D \right) &= \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(y_{t+j}^D + t_{t+j}^{DTH} \right) + \frac{Z_{t-1}^{DH}}{P_t} \\ \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(c_{t+j}^F + \dot{I}_{t+j+1} m_{t+j}^F \right) &= \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(y_{t+j}^F + t_{t+j}^{FTH} \right) + \frac{Z_{t-1}^{DF}}{P_t}, \end{aligned} \quad \langle 6.5 \rangle$$

where

$$\begin{aligned} Z_{t-1}^{DH} &\equiv B_{t-1}^{DD} + B_{t-1}^{DF} + M_{t-1}^D \\ Z_{t-1}^{FH} &\equiv B_{t-1}^{FD} + B_{t-1}^{FF} + M_{t-1}^F. \end{aligned} \quad \langle 6.6 \rangle$$

The difficulty—and the main difference with regard to the single-country equivalent described in Chapter 2.3.2—is that goods-market clearing can not be applied as easily: Equation (6.2) is an aggregate condition whereas the present-value budget constraints in Equation (6.5) only feature national variables.

Liabilities and Net Wealth A similar issue arises on capital markets: Since the distinction between Domestic and Foreign applies to both issuers and holders of treasury debt, it is no longer possible, on a national level, to simply identify outstanding consolidated-government liabilities with household net wealth carried over from the previous period, not least because the central bank is a supranational institution here. In what follows, let bond issues of the Domestic and Foreign treasuries be aggregated further into the union-wide supply of bonds

$$B_t^U \equiv B_t^D + B_t^F \quad (6.7)$$

and let

$$Z_t^U \equiv B_t^U + M_t \quad (6.8)$$

denote total outstanding liabilities of all public entities in the monetary union. Whether the latter definition is warranted or sensible from an institutional point of view is part of the investigation in this chapter (see *Risk Sharing and Realism* in Section 6.3.3 below, for example); in any case, is it useful with regard to notation.

Union Present-Value Budget Equation As already implied in the paragraph on *National Household Present-Value Budget Constraints* above, the goods-market-clearing condition (6.2) has to be combined with an aggregate household budget constraint, which is formed by summing up both Equations (6.1), which eventually yields

$$Z_{t-1}^U = \mathbb{E}_t V_{t,t+1} Z_t^U + S_t^D + S_t^F + \dot{I}_{t+1} M_t. \quad (6.9)$$

Equation (6.9) is of exactly the same form as the flow consolidated-government budget equation (2.37) / (2.39). Therefore, the analogue to the present-value budget equation (2.43) in the monetary union is given by

$$\frac{Z_{t-1}^U}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(s_{t+j}^D + s_{t+j}^F + \dot{I}_{t+j+1} m_{t+j} \right), \quad (6.10)$$

which corresponds to the transversality condition

$$\lim_{J \rightarrow \infty} \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}^U}{P_{t+J+1}} \right) = 0. \quad \langle 6.11 \rangle$$

National Consumption Levels Adding the assumption of constant and equal treasury expenditures ($g_t^D = g_t^F = g^D = g^F \forall t$) and applying goods-market clearing $\langle 6.2 \rangle$, we have

$$c_t^D + c_t^F = 2y - 2g, \quad \langle 6.12 \rangle$$

aggregate consumption is constant. While there is no union-wide version of the Euler equation $\langle 2.21 \rangle$, the argument about equal real interest rates (\nearrow Equality of Interest Rates, p. 106) in combination with the fact that household preferences are assumed to be homogeneous leads to the same result as in Chapter 2.4 (see \nearrow Constant Real Interest Rate, p. 34): constant real interest rates ($1 + r_t = \beta^{-1}$) and flat consumption profiles in both countries. Regarding the allocation of consumption, two cases can be distinguished:

- If lifetime household wealth, consisting of initial household wealth plus the present value of expected available income (the right-hand side of the present-value budget constraint $\langle 6.5 \rangle$), is the same in both countries, so are the respective consumption levels. Money demand $\langle 2.25 \rangle$ then implies that individual money demands are identical all the time as well. (Cf. Woodford 1996, pp. 27-28.) In this singular case, the monetary union can actually be viewed as the single economy of Chapter 2.4 ‘split in half,’ with no deviations in per-capita terms.
- If, by contrast, initial household wealth deviates—for instance, because one treasury decides upon a different path for transfers in a non-Ricardian fashion and thus (*ceteris paribus*) changes the present value of expected available income—national consumption levels are not the same. They remain constant and thus imply the same constant real interest rate, however. This scenario is explored in more detail in Section 6.3 below.

Treasury Budget Constraints Notwithstanding the above, national treasury budget equations do exist. After all, the treasuries constitute separate entities whose liabilities have to be described in terms of consistent time paths. Assuming equal remittances from the

central bank (see also ↗*Starting Point* on p. 111 below), we therefore have

$$\begin{aligned} B_{t-1}^D + T_t^{DTH} + G_t^D &= Q_t B_t^D + \frac{M_t - M_{t-1}}{2} \\ B_{t-1}^F + T_t^{FTH} + G_t^F &= Q_t B_t^F + \frac{M_t - M_{t-1}}{2}. \end{aligned} \quad \langle 6.13 \rangle$$

Given that the treasuries decide on the amounts of transfers as well as spending, that outstanding debt is predetermined, and that bond prices Q_t as a function of the nominal interest rate i_{t+1} (see Equation $\langle 2.8 \rangle$) are typically not in the hands of fiscal authorities either, Equations $\langle 6.13 \rangle$ determine the amount of debt each treasury has to incur in the current period.

If there are flow budget constraints, one can of course also iterate them forward infinitely often and receive the respective present-value budget constraints. The following equations are derived using the rearrangements of money-related terms described in Appendix D.1.2:

$$\begin{aligned} \frac{B_{t-1}^D}{P_t} &= -\frac{M_{t-1}}{2} + \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(s_{t+j}^D + \frac{\dot{I}_{t+j+1} m_{t+j}}{2} \right) \\ \frac{B_{t-1}^F}{P_t} &= -\frac{M_{t-1}}{2} + \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(s_{t+j}^F + \frac{\dot{I}_{t+j+1} m_{t+j}}{2} \right) \end{aligned} \quad \langle 6.14 \rangle$$

Summing up both equations $\langle 6.14 \rangle$ yields the union-wide present value budget equation $\langle 6.10 \rangle$. There is an important difference between individual Equations $\langle 6.14 \rangle$ and other present-value budget equations (including the union-wide variant $\langle 6.10 \rangle$), however: The latter are associated with transversality conditions that arise from the optimization problem (maximization on the part of households as well as borrowing limits, cf. Chapter 2) whereas there are no such individual equilibrium conditions for the former. Put differently, while transversality conditions have to be satisfied for individual households, the same is not true for individual treasuries. For them, it is conceivable that the pivotal elements of forward iteration (the terms which are equalized to zero in transversality conditions) deviate in opposite directions, i.e., infinity and minus infinity, but this is consistent with equilibrium as long as the transversality condition $\langle 6.11 \rangle$ is satisfied. (Cf. Bergin 2000, pp. 42-43, 49.)

Counting Variables and Equations In comparison to the baseline model of Chapter 2, the monetary-union variant features 11 additional variables as well as 14 additional equations (see Tables D.1a and D.1b in Appendix D.2).

In particular, six variables ($y, g, c, \lambda, s, t^{TH}$) are duplicated to differentiate between Domestic and Foreign and the money market is considered more explicitly, adding two

individual money demands (M^D, M^F). To enable a consistent description of household budget constraints, the bond market is described by six additional variables ($B^D, B^F, B^{DD}, B^{DF}, B^{FF}, B^{FD}$). Variables B and Z from Chapter 2 are replaced by B^U and Z^U , respectively.

Similarly, accounting for the two-country setup yields five duplicate equations (household budget constraint $\langle 2.3 \rangle / \langle 6.1 \rangle$, first-order conditions $\langle 2.12 \rangle / \langle 2.14 \rangle / \langle 2.15 \rangle$, definition of treasury surpluses $\langle 2.10 \rangle$) and adds six new equations (bond-market clearing $\langle 6.3 \rangle$, money-market clearing $\langle 6.4 \rangle$, treasury budget constraints $\langle 6.13 \rangle$, the definition of union-wide treasury debt $\langle 6.7 \rangle$). The definition of Z $\langle 2.29 \rangle / \langle 2.28 \rangle$ is replaced by the definition of Z^U $\langle 6.8 \rangle$.

Since there are also three additional exogenous or policy-determined variables (endowments y , treasury expenditure g and transfers t^{TH} are duplicated), the model remains closed.

6.3. Case Studies of Non-Ricardian Fiscal Policy in a Monetary Union

6.3.1. Introduction

Aggregate-Level Policy Regimes If the fiscal policies of both countries are studied on aggregate (in the sense of a centralized institution responsible for all member countries rather than just ‘summing up’ individual policy choices after the fact), the resulting interconnections with monetary policy are the same as in the single-country model of Sections 4-5. This directly hints at the object of interest in a monetary union: the coordination of fiscal policies which remain at the national level.

Starting Point Common ground for all further discussions in this section are the following assumptions:

- Treasury expenditure g is constant in both countries so as to avoid changes in the real interest rate (via goods-market clearing and consumption, which becomes especially hard to handle in the monetary union; see Section 2.4 for the basics).
- Surpluses are therefore set by adjusting transfers t^{TH} .
- The central bank has no authority or need for independent spending so that it can fully remit its seigniorage earnings to the two treasuries. It does so in equal parts.
- Finally, it is helpful to assume an initial steady state for the economy to start in. The peculiarities of the monetary union can then be pointed out by deviating from this initial situation. It exhibits a stable real level of outstanding union-wide liabilities brought about by identical fiscal policies of both countries, either because the latter are Ricardian or because they are non-Ricardian but happen to imply this

kind of stability irrespective of monetary policy (let the constant level of national surpluses needed to achieve this be denoted by s'). Hence, it corresponds to the first scenario for \nearrow National Consumption Levels in Section 6.2.

An Obvious Case Decentralized fiscal policies coincide with a hypothetical centralized one if both treasuries follow passive (Ricardian) surplus rules such as Equation (4.20). This constellation results in a regime of monetary dominance as described in Chapter 4.3.2 and does therefore not need to be discussed in greater detail anymore. (Cf. Woodford 1996, p. 30.)

6.3.2. One Non-Ricardian, One Ricardian

A National Ricardian Rule Sections 4.4 and 5.1.4 (including Appendix C.2) show that a Ricardian fiscal rule makes the respective present-value budget equation hold identically for all paths of all other variables (such as price levels or interest rates, for instance). In analogy to Equations (4.24) and (C.2) featured in these sections, the Ricardian surplus rule (5.9) can be adapted for use in Foreign,

$$\begin{aligned} S_t^F &= \gamma_Z^{FT} B_{t-1}^F - 0.5 (M_t - M_{t-1}) \\ \Leftrightarrow s_t^F &= \gamma_Z^{FT} \frac{B_{t-1}^F}{P_t} - 0.5 \left(m_t - \frac{m_{t-1}}{\Pi_t} \right), \end{aligned} \quad (6.15)$$

and combined with the union-wide present-value budget equation (6.10) to yield

$$\frac{B_{t-1}^D}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left[s_{t+j}^D + 0.5 \left(m_{t+j} - \frac{m_{t+j-1}}{\Pi_{t+j}} \right) \right]. \quad (6.16)$$

Irrelevance of Foreign's Policy for Prices Equation (6.16) provides a striking result: Given active fiscal policy in Domestic, the Foreign treasury setting surpluses in a Ricardian manner simply does not matter for price-level determination. In fact, it leads to exactly the same outcome as if Foreign were to adopt the policies of Domestic (multiplying Equation (6.16) by 2 makes it equivalent to Equation (6.10) with identical policies; cf. Woodford 1996, p. 32). The reason is that Foreign, by conducting Ricardian fiscal policy, takes prices as given instead of partaking in their determination, which is then completely up to Domestic.

Ricardian Equivalence Another 'irrelevance result' obtains for consumption. Consider the example of a transfer (T^{DTH}) increase in Domestic which is not offset by any means at any time because of the non-Ricardian nature of policy. The perceived gain in net

wealth leads Domestic households to increase demand for consumption, which drives up prices. As can be seen from the real version of rule (6.15), a higher price level leads to lower surpluses, that is, the Foreign treasury increases transfers to local households and thus compensates them for the initial price-driven real welfare loss. Therefore, Foreign households consume the same amount as before, and since the aggregate supply of goods does not change in the endowment economy, so do Domestic households.

6.3.3. One Non-Ricardian, One Super-Ricardian

A ‘Super-Ricardian’ Rule Now, replacing Equation (6.15), consider the case in which Foreign sets surpluses according to the ‘super-Ricardian’ rule

$$s_t^F = s_t' - (s_t^D - s_t'), \quad (6.17)$$

that is, such that they compensate deviations of Domestic surpluses from the hypothetical reference value s_t' . Determination of the latter is not the critical issue here; as indicated in *Starting Point* (p. 111) above, s' can be understood as the constant stream of national surpluses happening to be consistent with stability in all regimes—that is, also under active monetary policy, where it would have to support a current price level P_t consistent with the target inflation rate Π^* . Alternatively, it could also be the result of a Ricardian rule from which Domestic then decides to deviate.

Wealth Transfers Plugged into the union-wide present-value budget equation (6.10), rule (6.17) always maintains the initial situation and thus completely prevents any price-level deviation. However, such stabilization of prices possibly comes at a very high cost in terms of real goods: Because Foreign follows a policy that exactly balances the overall budget position, it effectively transfers wealth to Domestic every time and to the extent that the latter chooses to run budget deficits. Non-Ricardian fiscal policy by the Domestic treasury that hands out presents by decreasing (the present value of) taxes, which is tantamount to increasing (the present value of) transfers t^{DTH} , allows local households to consume more. At the same time, the Foreign treasury increases taxation of its constituency, lowering its wealth and thus consumption.

Behavior of Domestic’s treasury is not modeled more explicitly here, but it is obvious that such a “blank check” (Woodford 1996, p. 33) from another country is not an incentive to greater fiscal discipline. The likely end result is higher consumption in Domestic than in (or, put differently: paid for by lower consumption in) Foreign.

To add a technical note, the consumption Euler equation (2.21) shows that the real interest rate is constant if consumption is expected to be so, too, independent of its actual level. Therefore, different consumption levels are indeed consistent with a single union-wide real interest rate.

Risk Sharing and Realism The political viability of this scenario rests on a critical assumption: If households can perfectly insure themselves and thus ensure consumption, as assumed by Woodford (1996, p. 5), they do not really care about the distributional effects of the policies proposed in this subsection. Sims (1997, p. 14) and Bergin (2000, pp. 39, 44) criticize this as an oversimplification because it effectively rids households of their nationality. More importantly, with imperfect insurance, the realism of a model which has Foreign households enduring arbitrary wealth transfers to Domestic comes into question. While perfectly fine from a formal perspective, it does not seem politically feasible to maintain super-Ricardian policies to a greater extent. Case in point is the growing and oft-reported unwillingness of certain EMU members' constituencies to extend any kind of wealth transfers to fellow member countries like Greece.

6.3.4. Two Non-Ricardian Fiscal Policies

Aggregate Regime Finally, it is also conceivable that both countries decide to set surpluses without any concern to the level of their respective debt, not least because Foreign might recognize the futility of Ricardian policy if Domestic is non-Ricardian. In principle, the resulting constellation then resembles the description in Chapter 4.3.2; depending on monetary policy, the monetary union is governed by a doubly active regime or experiences fiscal dominance.

Wealth Transfers There is a non-negligible difference, however. While the situation is unambiguous with respect to the aggregate regime, a two-country setup introduces the possibility to redistribute wealth. If one of the countries increases transfers to households more than the respective other, its local households will gain more from higher real transfers than they lose from a rising price level (both decisions lead to *proportionate* upward pressure on the price level). Households of the more 'hesitant' (or less 'generous') country experience the opposite; their increased available income does not quite set off the loss from higher prices. Again, the behavior of treasuries is not modeled explicitly, but the incentive to try to outdo each other with tax presents is obvious. (Cf. Sims 1997, Section VI.)

6.4. Discussion of the Political Economy in a Monetary Union

Dilemma As the previous section makes clear, being part of a monetary union has severe consequences for a country if other members are not (or can not be effectively) prohibited from following non-Ricardian policies. In the presence of such 'partners,' policymakers of the country in question have to choose a position between two polar extremes, namely, price stability at the cost of (not even self-determined) wealth transfers and abandonment of the inflation target in an attempt to defend national wealth.

Dystopia Steadfast conclusions can only be drawn from setups that include the political process more explicitly, but indulging in a pessimistic worst-case scenario seems particularly easy in the monetary union. Consider the following an excursus in this direction.

A Critical View on the Institutional Setup in a Monetary Union A popular argument in favor of monetary unions holds that a supranational central bank is especially able to guarantee price stability because it is ‘elevated to another level’ compared to the rest of the economic and political system. In particular, fiscal policies remain national and numerous; furthermore, insofar as political unification lags behind, impeding supranational coordination, it is far more difficult for individual interested parties to attack central-bank independence (arguments in this direction are made by Fratianni and von Hagen 1993, for instance).

In tune with the tenor that fiscal policy matters for price-level and inflation determination, the present chapter paints a different picture: In a monetary union, fragmentation of fiscal policy increases the odds of failure from a normative point-of-view which considers price stability important.

Because of the way budget constraints and market-clearing conditions are aggregated, a single non-Ricardian fiscal policy suffices to turn an active-monetary-passive-fiscal into a doubly active regime (cf. Chapter 4.3.3.1) and thus refutes the ill-founded notion that active monetary policy ‘automatically’ achieves macroeconomic stability. While potentially stable, the second option—a passive-monetary-active-fiscal regime—is probably not in the interest of countries that are willing to follow Ricardian rules for the sake of price stability either (without going into further detail, the credibility of a commitment to price stability seems at least doubtful in case of fiscal dominance).

Attempted Freeriding This chapter assumes a monetary union consisting of only two equally large members. A larger number of countries and differences in size potentially introduce additional problems: It might especially be smaller nations in hopes of being able to freeride that turn out to be non-Ricardian, operating under the assumption that the wealth redistributions implied by their behavior are small enough to go unanswered by relatively larger countries (or the rest of the union *en bloc*, respectively). The worst thing that can happen from the perspective of such a freeriding state is that the others switch from being super-Ricardian to ‘just Ricardian’—at least at first instance.

Race to the Bottom Since behaving in a Ricardian manner has the same implications as exactly adopting the policies of the non-Ricardian country, initially Ricardian countries might simply do so (fiscal policy might feel much less constrained to politicians after all). At a minimum, Daniel (2001b, p. 304) argues that taking on foreign debt positions by adopting super-Ricardian policy runs counter to maximization of the domestic res-

idents' welfare; Sims (1997, p. 16) similarly asserts that, if consolidated governments "care about the welfare of their citizens, it will be difficult for them to maintain a commitment not to cut their primary surpluses." From there, it seems like only a question of time until union members engage in a 'race to the bottom' (or 'beggar-thy-neighbour competition') in which all countries try to gain from wealth redistributions by running ever more excessive budget surpluses and hoping that some of the inflationary costs this produces can be passed on to others (cf. Sims 1997, p. 16).

Enforcing Discipline The arguments above assume a very general kind of discretion on the part of national treasuries. This is not to be understood in the sense of the time-inconsistency literature à la Kydland and Prescott (1977) and Barro and Gordon (1983) in which a policymaker puts temporary before long-term gains but remains strictly within the limits of its own domain otherwise. Rather, the present situation is even worse because each treasury has the incentive to try and beggar its respective neighbors. (Cf. Sims 1997, p. 16.) While some countries, like Foreign in the examples above, might choose to follow a Ricardian rule, they can always opt out, become non-Ricardian, and try to receive wealth transfers as well.

What would be necessary is a way to prohibit this behavior. However, Sims (1997, p. 5) argues that controls and penalties, as envisioned in the Maastricht treaty, rather have a "perverse effect" in that they increase the incentive to exit the union and immediately devalue the currency (this would require that outstanding liabilities be denominated in national currency afterwards in order to be advantageous for the country in question). In particular, this relates to the excessive deficit procedure; for a short critical overview, see European Central Bank (2011, Box 2.1).

6.5. Selected References on Fiscal Price Determination in the Open Economy

Repetition: Monetary Union Bergin (2000) obtains the same results as in Sections 6.3.2-6.3.3 and presents the descriptions of consumption and treasury debt in more formal detail. His paper is also based on Woodford (1996, cited as Woodford 1998) but criticizes the assumption of perfect insurability of consumption (see *↗Risk Sharing and Realism* on p. 114 above). So does Sims (1997) who, furthermore, seems sympathetic to imposing *national* transversality conditions for reasons of plausibility (cf. p. 15).

Models with Exchange Rates Dupor (2000, especially pp. 623-625) shows that the determinacy under interest-rate pegs derived for the closed economy (see Chapter 5.1.5) does not persevere in an open-economy setup, the reason being aggregation problems similar to those discussed in Section 6.2. The price level, exchange rate, and real allocations

are all indeterminate under interest-rate pegs with more than one currency.

Daniel (2001b) contains the model of Dupor (2000) as a special case (to be fair, her more general case is even worse in that the “dimension of the indeterminacy” is two instead of one, see her p. 304). She resolves the indeterminacy problem by introducing “no-surplus” policy which places a zero upper bound on the present value of surpluses (note that positive surpluses in individual periods are fine as long as they are offset some other time, cf. p. 305).

Canzoneri, Cumby, and Diba (2001a) examine the consequences of monetary-fiscal coordination for managed floats, exchange-rate pegs, and monetary unions. Daniel (2001a) develops a “fiscal theory of currency crises.”

Key Takeaways from Chapter 6

Multi-country models face an additional problem in the form of aggregation. In a monetary union with a single interest rate, three regimes can be distinguished: If one country has non-Ricardian fiscal policy, the Ricardian policy of the other country ‘does not matter;’ fiscal policy is active overall. If the other country turns ‘super-Ricardian,’ fiscal policy becomes Ricardian overall; however, this situation is characterized by wealth transfers that are highly unrealistic from a political-economy perspective. If both fiscal policies are non-Ricardian, the monetary union might end up in a spiral of ever increasing nominal debt levels and inflation.

7. Monetary and Fiscal Price Determination with Nominal Rigidities

The baseline model is transformed into a ‘New-Keynesian’ variant by adding monopolistic competition and sticky prices. It is linearized and simulated in order to graphically study its behavior after technology and policy shocks.

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Derivations for this chapter can be found in Appendix F.

7.1. Adjustments to the Baseline Model¹⁵

7.1.1. Household Optimization

7.1.1.1. Preferences

Utility Instead of ‘the’ representative household invoked in Chapter 2, consider a unit-mass continuum of identical households that share the per-period utility function

$$u(c_t, m_t, \ell_t), \quad \langle 7.1 \rangle$$

where utility from consumption c_t and real-money holdings m_t have properties $\langle 2.2 \rangle$. By contrast, working ℓ_t hours yields convex disutility:

$$\begin{aligned} u'(\ell_t) &< 0 \\ u''(\ell_t) &< 0 \end{aligned} \quad \langle 7.2 \rangle$$

For later purposes (such as linearizing the model, cf. Section 7.2), it is necessary to assume a specific utility function. It is given by

$$u(c_t, m_t, \ell_t) \equiv \frac{c_t^{1-\rho}}{1-\rho} + \zeta_m \frac{m_t^{1-\nu}}{1-\nu} - \zeta_\ell \frac{\ell_t^{1+\eta}}{1+\eta} \quad \langle 7.3 \rangle$$

here and satisfies properties $\langle 2.2 \rangle$ and $\langle 7.2 \rangle$.

Variety of Consumption Goods In the present variant of the model, c_t does not denote a single multi-purpose consumption good anymore but instead the consumption bundle

$$c_t \equiv \left\{ \int_0^1 [c_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \quad \langle 7.4 \rangle$$

¹⁵ The model presented here is ‘plain vanilla’ for the most part. While readers familiar with monetary macroeconomics probably know it well from standard references like Woodford (2003b) and Galí (2008; 2015) among others, the discussion of fiscal price determination within a ‘New-Keynesian’ setting would be incomplete without a brief description of the underlying assumptions.

which consists of a variety of goods indexed over the interval $j \in [0, 1]$. Demands for individual goods j are derived in a separate optimization problem (see Appendix F.1.1) and given by

$$c_t(j) = \left[\frac{P_t}{P_t(j)} \right]^\theta c_t, \quad \langle 7.5 \rangle$$

where

$$P_t \equiv \left\{ \int_0^1 [P_t(j)]^{1-\theta} dj \right\}^{\frac{1}{1-\theta}} \quad \langle 7.6 \rangle$$

denotes the aggregate price level.

Goods usually are imperfect substitutes. From Equation $\langle 7.5 \rangle$, it can be derived that the elasticity of substitution between goods is equal to θ , which is a measure of the (uniform) degree of firms' market power. As θ increases, goods become closer substitutes; $\theta \rightarrow \infty$ then describes the limiting case of perfect competition in which firms have no market power (cf. Walsh 2010, p. 332).

7.1.1.2. Labor and Profit Income

In New-Keynesian models, households no longer receive exogenous (and possibly constant) endowments. Instead, they earn wages from supplying labor and profits from owning firms. This has several implications that are described in what follows.

Budget Constraint In comparison to budget constraint $\langle 2.3 \rangle$, the nominal endowment Y_t is replaced by labor income $W_t \ell_t$ and profit income Ψ_t :

$$\begin{aligned} C_t + M_t + Q_t B_t &= W_t \ell_t + T_t^{TH} + \Psi_t + B_{t-1} + M_{t-1} \\ \Leftrightarrow c_t + m_t + Q_t b_t &= w_t \ell_t + t_t^{TH} + \psi_t + \frac{b_{t-1} + m_{t-1}}{\Pi_t} \end{aligned} \quad \langle 7.7 \rangle$$

It is possible to substitute $C_t \equiv P_t c_t = \int_0^1 P_t(j) c_t(j) dj$ because of Equations $\langle 7.5 \rangle$ - $\langle 7.6 \rangle$, but since it does not add anything important, I opt for the simpler notation.

The subtleties of firm-household interconnections form an extensive literature in their own right and are left out here to save on space and time.¹⁶ Rather, I follow Galí (2015,

¹⁶ Just to give an impression: Blanchard and Kiyotaki (1987) introduce monopolistically competitive goods *and* labor markets, Ball and Romer (1989; 1991) abolish labor markets altogether by introducing 'yeoman farmers' (or worker-producers), and Ball and Romer (1990) consider both options. Woodford

p. ix) and McCandless (2008, p. 261) and assume that labor markets are perfectly competitive so that W_t and w_t denote the universally valid nominal and real wages per unit of labor, respectively. Similarly to Woodford (2003b, p. 146), I further assume that all households hold equal shares of all firms so that they earn identical incomes from nominal profits Ψ_t (real profits ψ_t).

Labor Supply With hours worked ℓ_t entering both the utility function and the budget constraint, the household faces an additional decision problem and the respective first-order condition:

$$u_\ell(c_t, m_t, \ell_t) + \lambda_t w_t \stackrel{!}{=} 0 \quad \Leftrightarrow \quad -u_\ell(c_t, m_t, \ell_t) = w_t u_c(c_t, m_t, \ell_t) \quad \langle 7.8 \rangle$$

(This can be derived from building a Lagrangian such as $\langle 2.11 \rangle$ with the adjustments discussed in the present chapter. The rearrangement uses Equation $\langle 2.12 \rangle$.) In optimum, marginal disutility from work must be equal to the marginal utility from consumption the household can afford given the real wage w_t . Given the specific utility function $\langle 7.3 \rangle$, the first-order condition $\langle 7.8 \rangle$ can be understood as a labor-supply function reading

$$\ell_t^\eta = \frac{w_t}{\xi_\ell c_t^\rho}. \quad \langle 7.9 \rangle$$

7.1.2. Firms in the Face of Price Rigidities

7.1.2.1. Preliminaries: Monopolistically Competitive Firms

Technology There is a continuum of firms indexed by $j \in [0, 1]$ producing differentiated goods with identical production functions

$$y_t(j) = a_t [\ell_t(j)]^\zeta, \quad \langle 7.10 \rangle$$

which exhibit constant returns to scale if the exponent of labor input $\ell_t(j)$ equals unity ($\zeta = 1$). The log level of economy-wide technology a_t is assumed to follow an autoregressive process

$$\ln a_t = \gamma_a \ln a_{t-1} + \varepsilon_t^a, \quad \langle 7.11 \rangle$$

(2003b, pp. 144-145) considers the implications (and equivalence) of assuming households that simultaneously supply all types j of labor as well as households that only supply one type and are equally numbered.

where ε_t^a is a white-noise shock and $\gamma_a \in [0, 1)$ so that technology always reverts to its steady-state level $\ln a_t = 0 \Leftrightarrow a_t = 1$ over time (cf. Woodford 2003b, pp. 148, 152; McCandless 2008, pp. 176-177; Galí 2015, p. 22, for instance).¹⁷ The model continues to abstract from capital for simplicity.

Nominal Rigidity and Aggregate Price Dynamics Following Calvo (1983), only a random share $(1 - \alpha)$ of firms, where $\alpha \in (0, 1)$, can readjust prices optimally in a given period while the remaining firms follow a simple rule. The optimal price in period t is denoted P_t^o .

This has implications for the behavior of the aggregate price level over time. If non-reoptimizing firms simply keep their prices constant, Equation (7.6) implies that

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1 - \alpha) (P_t^o)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

describes aggregate price-level dynamics. This simple rule can have some counterintuitive implications (cf. McCandless 2008, p. 265), but alternative rules are conceivable. To give an example, Benigno and Nisticò (2015, p. 13) have non-reoptimizing firms increase their prices at the target inflation rate Π^* , which makes

$$P_t = \left[\alpha (\Pi^* P_{t-1})^{1-\theta} + (1 - \alpha) (P_t^o)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (7.12)$$

the respective law of motion for the aggregate price level.¹⁸

7.1.2.2. The Firm's Profit-Maximization Problem and the Optimal Price

Profit Maximization Firms are assumed to maximize their market value, that is, the present value of expected profits. Firm j 's profits $\Psi_t(j)$ are given by total revenues less

¹⁷ Similarly to the design of labor and goods markets (cf. Footnote 16), other specifications are conceivable. For instance, Ireland (2004, p. 925) uses a logarithmic random walk with positive drift whereas Smets and Wouters (2003, p. 1132) use an AR(1) process for the level of the technology shock.

¹⁸ There is an exhaustive literature on the implications of 'Calvo pricing' and, in particular, the effects of different simple rules for non-reoptimizing firms. To give some examples: One of the first applications is by Yun (1996), who assumes that non-optimal prices remain constant. Erceg, Henderson, and Levin (2000) and Christiano, Eichenbaum, and Evans (2005) let non-optimal prices be indexed to steady-state or lagged inflation, respectively. Casares (2002) makes a threefold distinction in which non-reoptimizing firms are randomly assigned to keep prices constant or index them to target inflation.

In general, positive steady-state inflation has nontrivial effects, but full indexation heals this complication (cf. Ascari 2004, pp. 664-665; Ascari and Ropele 2009; Cogley and Sbordone 2008). It hence allows me to consider non-zero inflation targets (see Section 7.3.3.5).

cost of production:

$$\Psi_t(j) \equiv P_t(j) y_t(j) - \text{Cost}_t(j) \quad \langle 7.13 \rangle$$

$\text{Cost}_t(j)$ as a short form of $\text{Cost}_t[y_t(j)]$ denotes the total nominal cost of producing $y_t(j)$. (Since costs in absolute level do not appear very often in the description of the model, I allow myself this mannerism, including the denial to waste a symbol on them.) Aggregate profits Ψ are simply the integral over all firms (see also Appendix F.1.2.2):

$$\Psi_t \equiv \int_0^1 \Psi_t(j) dj = P_t y_t - W_t \ell_t \quad \langle 7.14 \rangle$$

As described in Appendix F.1.3, this leads to the optimality condition

$$\frac{P_t^o(j)}{P_t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} v_{t,T} m_{CT}(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} v_{t,T} y_T \frac{P_t}{P_T} (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}, \quad \langle 7.15 \rangle$$

where

$$\varrho \equiv \frac{\theta}{\theta - 1} \quad \langle 7.16 \rangle$$

denotes the desired mark-up in absence of the Calvo friction ($\alpha = 0$), which is then driven solely by the degree of market power of the monopolistically competitive firms. Further, nominal marginal costs are given by the product of the nominal wage and the marginal product of labor:

$$MC_t(j) \equiv \frac{W_t}{\zeta_{a_t} [\ell_t(j)]^{\zeta-1}} \quad \langle 7.17 \rangle$$

(A similar expression pertaining to the aggregate economy—eliminating the j indices from Equation $\langle 7.17 \rangle$ —is derived at a later stage, see Appendix F.3.1.5.)

7.1.3. Policies of the Consolidated Government

7.1.3.1. Central Bank

In its setting of the nominal interest rate, the monetary authority follows

$$1 + i_{t+1} = (1 + i_{SS}) \left(\frac{\Pi_t}{\Pi^*} \right)^{\gamma_\pi^C} \left(\frac{y_t}{y_t^{\text{nat}}} \right)^{\gamma_y^C} (1 + \varepsilon_t^i), \quad \langle 7.18 \rangle$$

where y^{nat} denotes the ‘natural’ aggregate output level that would obtain in the hypothetical case of perfect price flexibility (that is, with an ineffective Calvo rigidity, $\alpha = 0$; see Appendix F.1.4 and especially *Natural vs. Steady-State Output* on 246). The policy shock ε_t^i is autoregressive of first order with $\gamma_\varepsilon^C \in [0, 1)$ and ε_t^i white noise:

$$\varepsilon_t^i = \gamma_\varepsilon^C \varepsilon_{t-1}^i + \varepsilon_t^i \quad \langle 7.19 \rangle$$

7.1.3.2. Treasury

Treasury Expenditure on Differentiated Goods Introducing goods varieties also affects public spending. It is assumed that the treasury targets an aggregate bundle of goods g that is formed similarly to the consumption bundle c in Equation (7.4):

$$g_t \equiv \left\{ \int_0^1 [g_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \quad \langle 7.20 \rangle$$

Like households, it tries to achieve any given bundle at a minimal cost, which leads to analogous good-specific public demand functions

$$g_t(j) = \left[\frac{P_t}{P_t(j)} \right]^\theta g_t. \quad \langle 7.21 \rangle$$

As explained in Appendix F.1.2.1, treasury expenditure can be expressed in terms of consumption via the ‘public spending ratio’ Γ :

$$\Gamma_t \equiv \frac{g_t(j)}{c_t(j)} \quad \forall j \quad \Rightarrow \quad g_t = \Gamma_t c_t \quad \langle 7.22 \rangle$$

Surplus Rule The actual policy decision is not about the composition of expenditure, but about its level. To be precise, the treasury again follows a rule for budget surpluses,

namely

$$\begin{aligned} S_t &= P_t s^{\text{fix}} + \gamma_y^T (Y_t - Y_t^{\text{nat}}) + \gamma_z^T Z_{t-1} - \dot{I}_{t+1} M_t + P_t \varepsilon_t^s \\ \Leftrightarrow s_t &= s^{\text{fix}} + \gamma_y^T (y_t - y_t^{\text{nat}}) + \gamma_z^T \frac{Z_{t-1}}{\Pi_t} - \gamma_m^T \dot{I}_{t+1} m_t + \varepsilon_t^s, \end{aligned} \quad \langle 7.23 \rangle$$

where s^{fix} denotes the fixed component of the surplus rule and γ_m^T is a binary operator that allows to include the seigniorage rebate (cf. Section 5.1.3.1, $\gamma_m^T = 1$) or not ($\gamma_m^T = 0$).¹⁹ Again, the shock term ε_t^s is autoregressive of order one with $\gamma_\varepsilon^T \in [0, 1)$ and ε^s white noise:

$$\varepsilon_t^s = \gamma_\varepsilon^T \varepsilon_{t-1}^s + \epsilon_t^s \quad \langle 7.24 \rangle$$

Expenditure Rule The combination of surplus rule $\langle 7.23 \rangle$ and definition $\langle 2.10 \rangle$ leaves open the division between treasury expenditure g and transfers to households t^{TH} . Letting the latter be determined by Equation $\langle 2.10 \rangle$, two rules are considered here for public expenditure g :²⁰

1. With the first rule, the treasury defines some constant level of public expenditure g^{fix} from which g_t only deviates in case (and in the amount of) a surplus shock

¹⁹ The reason for defining an additive rather than multiplicative rule for the treasury is a technical one. In general, additive connections introduce ratios of steady-state values into the linearized equations. This can be avoided by defining multiplicative rules, as is done in Taylor rule $\langle 7.18 \rangle$, for example. There are several arguments against doing the same for surplus rules: For one, the budget constraint $\langle 7.7 \rangle$ is linear itself which automatically introduces a steady-state ratio into its linearized form $\langle 7.39 \rangle$. Further, granting the seigniorage rebate $\dot{I}m$ (see Section 5.1.3.1) is typically an additive operation. Therefore, the gains of avoiding steady-state ratios are minor because some of these ratios simply can not be avoided. The most important argument, however, is that a multiplicative rule including a term like $(Z_{t-1}/Z_{ss})^{\gamma_z^T}$ (or an equivalent expression in real terms) does not allow for the *The Bohn-Woodford Criterion* (p. 77) to be checked as easily: The additive case rests on exponentiating the expression $(1 - \gamma_z^T) \in [0, 1)$ infinitely and thereby demonstrating that the transversality condition is satisfied (see Sections 4.4, 5.1.4, and Appendix C.2). By contrast, with a non-linear rule, this argument can not be made anymore in the original model (while the linearized model is a bit, yet not drastically, easier to handle). The potential ‘middle ground’ of defining a rule with a multiplicatively inserted term like $(\gamma_z^T Z_{t-1})$, in turn, has the drawback of losing the policy coefficient γ_z^T in its linearized form.

²⁰ There certainly are more rules that could be analyzed. One is a ‘split rule’ which assigns treasury expenditure g and transfers to households t^{TH} specific weights (γ_g^T) in achieving a given surplus: $g_t = -\gamma_g^T s_t$ and $t_t^{TH} = -(1 - \gamma_g^T) s_t$. The reason for not considering this setup in more detail is that it can lead to negative public spending ($g < 0$), an unusual outcome which would require some imagination to yield a reasonable interpretation (if any). There are two obvious workarounds: First, placing limits on the parameters and policies such that negative treasury expenditure would never occur; however, this reduces the generality of the model. Second, simply adding a requirement that public spending never become negative by imposing $g \geq 0$. This resembles the zero lower bound on the nominal interest rate, and while there are solution approaches to this problem, their application to the ‘split rule’ would blow the present section out of proportion. (For such “occasionally binding constraints,” see Guerrieri and Iacoviello 2015 and Holden 2017.)

(‘shock-only rule’). Formally,

$$g_t = g^{\text{fix}} - \varepsilon_t^s. \quad \langle 7.25a \rangle$$

2. Alternatively, it could render the *public expenditure ratio* into a policy tool, setting a fixed proportion between private and public consumption. For simplicity, it is assumed that this relationship is constant ($\Gamma_t = \bar{\Gamma} \forall t$), so that

$$g_t = \bar{\Gamma} c_t. \quad \langle 7.25b \rangle$$

Formally, this is (very close to) a repetition of Equation $\langle 7.22 \rangle$, but it follows a different logic: The latter is part of a purely definitional process with the aim to simplify matters (but no active part in equilibrium determination) whereas Equation $\langle 7.25b \rangle$ is an integral part of closing the model if this type of treasury policy is chosen.

7.1.4. Equilibrium

Goods-Market Clearing In equilibrium, supply of each goods variety has to equal the sum of private and public demands,

$$y_t(j) = c_t(j) + g_t(j) = (1 + \Gamma_t) c_t(j) \quad \forall j, \quad \langle 7.26 \rangle$$

which, given an appropriate definition of aggregate output y_t , also implies aggregate goods-market clearing (cf. Equation $\langle 7.22 \rangle$ and Appendix F.1.2.1):²¹

$$y_t \equiv \left\{ \int_0^1 [y_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} = c_t + g_t = (1 + \Gamma_t) c_t, \quad \langle 7.27 \rangle$$

²¹ A further implication is that ‘total demand’ for a specific variety j follows the same pattern as private and public demands (cf. Equation $\langle F.5 \rangle$ in Appendix F.1.2.1):

$$y_t(j) = \left[\frac{P_t}{P_t(j)} \right]^\theta y_t$$

Labor-Market Clearing Similarly, the sum of individual labor demands $\ell_t(j)$ arising from the production functions $\langle 7.10 \rangle$ has to be equal to labor supply by households:

$$\int_0^1 \ell_t(j) dj = \ell_t \quad \langle 7.28 \rangle$$

Counting Variables and Equations Since the New-Keynesian model variant is different from the baseline version in several respects, it is helpful to again summarize all variables and describe how they are determined.

In comparison to the baseline model of Chapter 2 as summarized by Table 2.1a, 17 variables are added to Table 7.1a (ε^a , a , Ψ , $\Psi[j]$, MC , $MC[j]$, Γ , q , $c[j]$, $g[j]$, ℓ , $\ell[j]$, P^o , $P[j]$, W , y^{nat} , $y[j]$). Since the constant endowments are replaced by a production sector, the ‘quasi-exogenous’ variables c , g , r , and λ move into the main group of to-be-determined variables.

Table 7.1b shows that seven of these additional variables are mere definitions (a , Ψ , $\Psi[j]$, $MC[j]$, Γ , q , MC). The technology shock ε^a is white noise. Natural output y^{nat} is a hypothetical construct denoting a flexible-price benchmark value for output. The Calvo (1983) procedure determines whether a specific firm’s price $P(j)$ is optimal or carried over from the previous period (indexed to target inflation).

Therefore, seven of the additional variables still have to be determined ($c[j]$, $g[j]$, ℓ , $\ell[j]$, P^o , W , $y[j]$). This happens simultaneously, so a sequential description is only admissible insofar as one keeps in mind that each of the following ‘modules’ is subject to the *ceteris-paribus* assumption (however, the list gives a brief overview of the more detailed derivations in Appendix F.1):

1. Given j -specific prices, the Dixit and Stiglitz (1977) framework yields specific-good demands for household consumption $c(j)$ (Equation $\langle 7.5 \rangle$).
2. The same holds for the allocation of aggregate treasury expenditure on the different varieties $g(j)$, governed by Equation $\langle 7.21 \rangle$.
3. Specific-good market-clearing conditions $\langle 7.26 \rangle$ then determine total demand for each variety $y(j)$.
4. These ‘total demands,’ in turn, determine j -specific labor demands $\ell(j)$ via the production functions $\langle 7.10 \rangle$.
5. Labor-market clearing $\langle 7.28 \rangle$ then determines the necessary amount of aggregate labor ℓ .
6. Given aggregate consumption and labor, the first-order condition with respect to labor $\langle 7.8 \rangle / \langle 7.9 \rangle$ pins down the real and, in combination with the aggregate price

level, nominal wage. (Aggregate consumption and the price level are determined by the respective Dixit and Stiglitz bundlers along the way.)

7. Things come full circle when considering pricing decisions of firms: Optimal prices P^o follow first-order conditions $\langle 7.15 \rangle$ derived from profit-maximization, which itself depends on good-specific demands and wages, among others.

7.2. Linearization

7.2.1. Preparations

Definitions The New Keynesian model laid out in Section 7.1 is nonlinear and complex. Letting

$$\hat{\square}_t \equiv \ln \square_t - \ln \square_{ss} \quad \langle 7.29 \rangle$$

denote the logarithmic ('log') deviation of \square_t from its steady-state value \square_{ss} , it can be reduced to a linear model in fewer variables. Further, the resulting model is stated in percentage deviations, which often is more informative than levels.

There are some exceptions to the rule: First, it is common to cast the model in terms of the output gap between actual and natural (instead of steady-state) levels

$$\tilde{y}_t \equiv \ln y_t - \ln y_t^{\text{nat}} \quad \langle 7.30 \rangle$$

(see also \nearrow *Natural vs. Steady-State Output* on p. 246). Second, interest and inflation rates appear in levels ('without hats,' see also \nearrow *Gross and Net Rates* on p. 231). Finally, some of the level variables (treasury debt b , budget surpluses s , transfers to households t^{TH} , and consolidated-government liabilities z) can take negative values, which does not allow for log-linearization but requires 'just linearization.' In order to obtain percentage deviations of these variables as well, let

$$\check{\square}_t \equiv \frac{\square_t - \square_{ss}}{|\square_{ss}|} \quad \langle 7.31 \rangle$$

(see Appendix E.5 for the rationale behind and remaining limitations to Equation $\langle 7.31 \rangle$).

The necessary derivations are tedious and do not yield valuable insights into the interaction of monetary and fiscal policy, which is why they are relegated to Appendices E (a technical overview about linearization using Taylor-series approximation and logarithms) and F.3 (the actual log-linearization of the model).

no.	variable	determination	description
<i>'truly' exogenous</i>			
1	ε^a	exogenous	technology shock
<i>endogenous by definition</i>			
2	Π	$\langle 2.6 \rangle$	inflation rate
3	s	$\langle 2.10 \rangle$	budget surplus of the treasury
4	v	$\langle 2.16 \rangle$	real stochastic discount factor
5	V	$\langle 2.17 \rangle$	nominal stochastic discount factor
6	\dot{i}	$\langle 2.23 \rangle$	opportunity cost of holding money
7	Z	$\langle 2.28 \rangle / \langle 2.29 \rangle$	nominal wealth (household)/liabilities (cons. gov.)
8	P	$\langle 7.6 \rangle$	aggregate price level
9	a	$\langle 7.11 \rangle$	technology (AR[1] process)
10	Ψ	$\langle 7.13 \rangle$	aggregate profits
11	$\Psi(j)$	$\langle 7.14 \rangle$	firm j 's profits
12	$MC(j)$	$\langle 7.17 \rangle$	j -specific marginal costs
13	Γ	$\langle 7.22 \rangle$	ratio of treasury expenditure to consumption
14	ϱ	$\langle 7.16 \rangle$	desired markup
15	MC	$\langle F.31 \rangle$	aggregate marginal costs (approximation)
<i>to be determined by equations or set exogenously by policy</i>			
16	B		bonds
17	c		aggregate consumption
18	$c(j)$		j -specific consumption
19	g	policy	aggregate treasury expenditure
20	$g(j)$		j -specific treasury expenditure
21	i	policy	nominal interest rate
22	ℓ		aggregate labor supply
23	$\ell(j)$		j -specific labor demand
24	M		money supply
25	p^o		optimal price
26	$P(j)$		j -specific price
27	Q		bond price
28	r		real interest rate
29	t^{TH}	policy	net transfers from the treasury to the household
30	W		nominal wage
31	y		aggregate output
32	y^{nat}		natural (aggregate) output
33	$y(j)$		j -specific output
34	λ		Lagrange parameter for the budget constraint

Table 7.1a: Overview of Variables in the New-Keynesian Model. \diamond Explanations: See \nearrow Counting Variables and Equations on p. 128.

no.	Equation	description
<i>'normal' equations</i>		
1	$\langle 2.5 \rangle / \langle 2.7 \rangle$	Fisher equation
2	$\langle 2.8 \rangle$	no-arbitrage condition for bond prices
3	$\langle 2.12 \rangle$	first-order condition with respect to consumption
4	$\langle 2.14 \rangle$	first-order condition with respect to money
5	$\langle 2.15 \rangle$	first-order condition with respect to bonds
6	$\langle 7.5 \rangle$	j -specific goods demand
7	$\langle 7.7 \rangle$	flow budget constraint of the household
8	$\langle 7.8 \rangle / \langle 7.9 \rangle$	first-order condition with respect to labor
9	$\langle 7.10 \rangle$	j -specific production functions
10	$\langle 7.15 \rangle$	first-order condition with respect to the optimal price
11	$\langle 7.21 \rangle$	j -specific treasury expenditure
12	$\langle 7.26 \rangle$	j -specific goods-market clearing
13	$\langle 7.27 \rangle$	aggregate output/goods-market clearing
14	$\langle 7.28 \rangle$	labor-market clearing
<i>'mere definitions'</i>		
15	$\langle 2.6 \rangle$	inflation rate
16	$\langle 2.10 \rangle$	budget surplus of the treasury
17	$\langle 2.16 \rangle$	real stochastic discount factor
18	$\langle 2.17 \rangle$	nominal stochastic discount factor
19	$\langle 2.23 \rangle$	opportunity cost of holding money
20	$\langle 2.28 \rangle$	nominal wealth (household)/liabilities (cons. gov.)
21	$\langle 7.6 \rangle$	aggregate price level (Dixit and Stiglitz aggregator)
22	$\langle 7.11 \rangle$	AR(1) process for technology
23	$\langle 7.13 \rangle$	aggregate profits
24	$\langle 7.13 \rangle$	firm j 's profits
25	$\langle 7.17 \rangle$	j -specific marginal costs
26	$\langle 7.22 \rangle$	ratio of treasury expenditure to consumption
27	$\langle 7.16 \rangle$	desired markup
28	$\langle F.31 \rangle$	aggregate marginal costs (approximation)
<i>'everything else'</i>		
29	exogenous variable	
30-32	set by policy	
33	Calvo pricing	
34	'natural' (flexible-price) model	

Table 7.1b: *Equations Available for Equilibrium Determination in the New-Keynesian Model.* \diamond *Explanations:* See \nearrow *Counting Variables and Equations* on p. 128.

How Far Should the Model Be Reduced? The typical procedure in (relatively) simple models is to reduce the set of linearized equations to a minimum. A prime example is the basic New-Keynesian model as expounded, for instance, by Galí (2008; 2015), leading to a system of two difference equations (the “dynamic IS curve” and the “New-Keynesian Phillips Curve”) in two variables (the output gap \tilde{y} and, under the usual assumption of zero target inflation, the actual inflation rate π). With such a limited amount of variables and only a few accompanying parameters, many questions can actually be solved analytically; for example, the dynamic behavior of output and inflation can be described by (relatively) straightforward stability analysis based on eigenvalues.

As the amount of state variables and parameters of interest increases, however, the analytical approach quickly becomes intractable. The model developed here already suffers from this problem. *In principle*, it could be reduced to a system of ‘size three,’ adding consolidated-government liabilities \hat{z} and the respective law of motion to the picture. *Practically*, however, evaluating the model and working with it analytically seems impossible: Within the confines of conventional personal-computing equipment, it is possible to reduce the model to a 3×3 system in matrix notation using symbolic-math software, but calculating the determinant of the coefficient matrix is already close to the limit of usable computing power, and deriving and displaying symbolic forms of eigenvalues apparently are beyond it. Further, and probably more importantly, the resulting stability conditions would turn out way too complex to be of any practical use.

In the trade-off between richness and tractability, numerical simulations constitute a middle ground. Therefore, I reduce the model only somewhat further—making some of the respective expressions considerably longer in the process—in the rest of Section 7.2 and then move on to the simulation in Section 7.3. Of course, the latter is a numerical and, as such, quantitative procedure, but the resulting graphic impulse-response ‘functions’ should be understood as qualitative sketches rather than precise indications of real-world events.

7.2.2. The Linearized Model

7.2.2.1. The Reduced Model

Dynamic IS Curve The demand side of the economy is represented by

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \Xi_r (i_{t+1} - \mathbb{E}_t \pi_{t+1} - r_{SS}) + \Xi_a (\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t) + \Xi_g (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t), \quad \langle 7.32 \rangle$$

for which the following shorthands are defined:

$$\Xi_r \equiv \frac{1}{\rho} \frac{c_{SS}}{y_{SS}} = \frac{1}{\rho (1 + \Gamma_{SS})} \quad \langle 7.33 \rangle$$

$$\Xi_a \equiv \frac{1 + \eta}{\Theta \zeta} \quad \langle 7.34 \rangle$$

$$\Xi_g \equiv \frac{\rho}{\Theta} \frac{g_{ss}}{c_{ss}} - \frac{g_{ss}}{y_{ss}} = \frac{\rho \Gamma_{ss}}{\Theta} - \frac{\Gamma_{ss}}{1 + \Gamma_{ss}} \quad \langle 7.35 \rangle$$

$$\Theta \equiv \rho \frac{y_{ss}}{c_{ss}} - \frac{\zeta - 1 - \eta}{\zeta} = \rho (1 + \Gamma_{ss}) - \frac{\zeta - 1 - \eta}{\zeta} \quad \langle 7.36 \rangle$$

This is a rather general but otherwise standard version of the ‘dynamic IS’ equation (cf. Galí 2008, p. 49). The deviation of actual output y from its hypothetical flexible-price (‘natural’) level y^{nat} is driven by four determinants: First, its own expectation about the future. Second, the difference of the actual real interest rate from its steady state level, which reflects private-sector demand on goods markets. Third, the development of technology a . Fourth, the change in treasury expenditure g as a measure of public demand on goods-markets. Derivations can be found in Appendix F.3.2.

New-Keynesian Phillips Curve The supply side of the model is described by the New-Keynesian Phillips Curve

$$\pi_t - \pi^* = \beta (\mathbb{E}_t \pi_{t+1} - \pi^*) + \kappa \tilde{y}_t, \quad \langle 7.37 \rangle$$

where

$$\kappa \equiv \Theta \Theta'' = \left(\rho \frac{y_{ss}}{g_{ss}} - \frac{\zeta - 1 - \eta}{\zeta} \right) \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \frac{\zeta}{\zeta + \theta(1 - \zeta)} \quad \langle 7.38 \rangle$$

is a shorthand. It describes the supply-side relationship between marginal costs—for which the output gap \tilde{y} acts as a proxy—and inflation π . It is ‘expectation-augmented’ in that expected inflation affects the current inflation rate. See Appendix F.3.3 for derivations.

Consolidated-Government Liabilities Finally, deviations of real consolidated-government liabilities from their steady-state values follow

$$\check{z}_t = \beta^{-1} \check{z}_{t-1} + \frac{b_{ss}}{|z_{ss}|} (i_{t+1} - i_{ss}) - \beta^{-1} \frac{z_{ss}}{|z_{ss}|} (\pi_t - \pi^*) - (1 + i_{ss}) \frac{|s_{ss}|}{|z_{ss}|} \check{s}_t - \frac{i_{ss} m_{ss}}{|z_{ss}|} \hat{m}_t. \quad \langle 7.39 \rangle$$

They are influenced by their own lag, the stance of monetary policy in terms of the nominal interest rate, inflation, the fiscal-policy stance in terms of surpluses, and a measure of seigniorage. Derivations are given in Appendix F.3.4.

Degree of Reduction As indicated before (see *How Far Should the Model Be Reduced?* and the introduction to the present subsection), it is possible to reduce the model to three equations in three endogenous variables. Equations (7.32)-(7.39) do *not* represent the ultimate reduced model, however, as they still feature more variables. Further reduction would greatly complicate matters because replacing all ‘non-core’ variables, especially surpluses \hat{s} and real-money \hat{m} , is a multi-stage process leading to ever longer equations.

7.2.2.2. Closing the Model

Counting Variables and Equations The descriptions in Section 7.2.2.1 mention three equations in eight variables ($\hat{y}, i, \pi, \hat{a}, \hat{g}, \hat{z}, \hat{s}, \hat{m}$). Technology is given quasi-exogenously by Equation (7.11), which reads

$$\hat{a}_t = \gamma_a \hat{a}_{t-1} + \varepsilon_t^a \quad (7.40)$$

when linearized, still leaving four variables ‘too many.’ This issue is resolved by including the description of policy in what follows.

Policy Rules The respective policy rules for interest rates (7.18) and surpluses (7.23) in linearized form read

$$i_{t+1} = i_{SS} + \gamma_\pi^C (\pi_t - \pi^*) + \gamma_y^C \tilde{y}_t + \varepsilon_t^i \quad (7.41)$$

$$\begin{aligned} \check{s}_t = & \frac{1}{|s_{SS}|} \left(\gamma_y^T y_{SS} - \gamma_m^T \frac{m_{SS}}{1 + i_{SS}} \gamma_y^C \right) \tilde{y}_t - \frac{1}{|s_{SS}|} \left(\gamma_z^T \frac{z_{SS}}{\Pi^*} + \gamma_m^T \frac{m_{SS}}{1 + i_{SS}} \gamma_\pi^C \right) (\pi_t - \pi^*) \\ & + \gamma_z^T \frac{1}{\Pi^*} \frac{|z_{SS}|}{|s_{SS}|} \check{z}_{t-1} - \gamma_m^T \frac{m_{SS}}{|s_{SS}|} \hat{m}_t - \gamma_m^T \frac{1}{1 + i_{SS}} \frac{m_{SS}}{|s_{SS}|} \varepsilon_t^i + \frac{1}{|s_{SS}|} \varepsilon_t^s \end{aligned} \quad (7.42)$$

(see Appendix F.3.5 for derivations).

Treasury Expenditure and Transfers While the behavior of surpluses is explained by Equation (7.42), the underlying components, public expenditure \hat{g} and transfers to households \hat{f}^{TH} , are yet undetermined. As before (Section 7.1.3.2), transfers are determined as the residual in the log-linear counterpart of definition (2.10),

$$\check{s}_t = - \frac{|t_{SS}^{TH}|}{|s_{SS}|} \check{t}_t^{TH} - \frac{g_{SS}}{|s_{SS}|} \hat{g}_t, \quad (7.43)$$

while the deviation of treasury expenditure from steady state depends on the type of rule chosen for its level:

1. If treasury expenditure only reacts to surplus shocks (Equation (7.25a)), the linearized version of the rule becomes

$$\hat{g}_t = -\frac{1}{g_{ss}}\varepsilon_t^s. \quad (7.44a)$$

2. Alternatively, if the treasury simply sets expenditure in *constant* proportion to private consumption ($\Gamma_t = \bar{\Gamma} \forall t$ as a simplifying assumption about policy), Equation (7.25b) turns into

$$\hat{g}_t = \hat{c}_t. \quad (7.44b)$$

Further Reduction? As indicated before (p. 132), it is not very instructive to reduce the model further by plugging the policy rules into the equations they relate to (the dynamic IS curve (7.32) and the law of motion for consolidated-government liabilities (7.39), respectively) because the results would be long and unwieldy.

7.3. Simulation

7.3.1. Introduction and Procedure

Calibration The model utilizes two alternative baseline calibrations adopted from works which perform a similar exercise (simulating a linearized New-Keynesian DSGE model): While Kim (2003) is one of the standard references on the fiscal theory of the price level and directly compares the “passive monetary-active fiscal” to the “active monetary-passive fiscal” regime, Galí (2008) is an even more widely regarded reference on the ‘conventional’ New Keynesian model (but leaves out the ‘fiscalist’ regime). The respective sets of parameter values are given in Table 7.2. For the rationale behind certain parameter values, see also Appendix F.4.

Technical Specifications All simulations are executed using Matlab R2017b with the Dynare package version 4.5.4.²² The necessary files are available upon request.

Variable Names The plot titles of the impulse-response functions in this chapter do not use the same symbols for variables as the rest of the thesis. Instead, they consist of a simplified descriptor of the variable as listed in Table 7.3, followed by the shock depicted

²² For Matlab, visit <https://www.mathworks.com/products/matlab.html>. Dynare is available for free under <http://www.dynare.org>; see also Adjemian et al. (2017).

parameter	description	Kim (2003)	Galí (2008)	thesis
s^{fix}	constant part of the surplus rule			0.01
α	Calvo parameter	0.75	0.67	
β	discount factor	0.99 ^a	0.99 ^a	
γ_a	AR(1) coefficient of the technology shock	0.8	0.5	
γ_y^C	central-bank reaction parameter to the output gap	0	0	
γ_ε^C	AR(1) coefficient of the interest-rate shock	0.8	0.5	
γ_y^T	treasury reaction parameter to the output gap	0	0	
γ_ε^T	AR(1) coefficient of the surplus shock	0.8	0.5	
Γ	public spending relative to private consumption			0.67
ζ	scale parameter (production function)	0.67	1	
η	labor disutility coefficient	1	1	
θ	elasticity of substitution between goods	10	6	
ν	real-money utility coefficient	1 ^b	1 ^c	
ξ_ℓ	disutility weight of labor			1
ξ_m	utility weight of real money			0.001
π^*	inflation target	0 ^a	0 ^a	
ρ	consumption utility coefficient	5	1	

Table 7.2: *Baseline Calibrations for the New-Keynesian Model.* \diamond *Explanations:* These parameter values are used in the model simulations unless indicated otherwise. \diamond *Notes:* ^a Quarterly value. ^b Originally, $\nu = 5$ in Kim (2003), but since values above unity tend to provoke violations of the Blanchard and Kahn (1980) conditions, I set $\nu = 1$ here. ^c Implicit (cf. Bergholt 2012, pp. 8-10).

no.	simulation plots	normal use	no.	simulation plots	normal use
1	a	\hat{a}	10	pi	π
2	AR1i	ε^i	11	r	r
3	AR1s	ε^s	12	s	ξ
4	b	\check{b}	13	t	\check{t}^{TH}
5	c	\hat{c}	14	w	\hat{w}
6	g	\hat{g}	15	y	\hat{y}
7	i	i	16	ygap	\hat{y}
8	lbr	$\hat{\ell}$	17	ynat	\hat{y}^{nat}
9	m	\hat{m}	18	z	\hat{z}

Table 7.3: *Variable Names in the Simulation Plots and Under Normal Use.* \diamond *Explanations:* Plot titles feature the variable name followed by the depicted shock. For instance, **c_eps_a** depicts the log-deviation of consumption \hat{c} after a shock to technology a .

in the plot (`_eps_a`, `_eps_i`, or `_eps_s`). In order not to load the figure too much, not all impulse-response functions are shown.

Variables and Equations The full list of variables and equations used to simulate the linearized model is given by Tables F.1a and F.1b in Appendix F.5.

Impulse-Response Functions Horizontal axes denote quarters. Vertical axes give percentage deviations from steady state, except for interest rates and inflation, which are expressed as plain *yearly* rates. In the legends, “T” denotes the Taylor coefficient γ_π^C , “CGL” denotes the reaction coefficient of surpluses to consolidated-government liabilities γ_Z^T , “CByg” denotes the output-gap coefficient of the central bank γ_y^C , and “FPyg” denotes the output-gap coefficient of the treasury γ_y^T .

Policy Regimes Two basic regimes are considered here: The institutional setup commonly deemed conventional in which the central bank reacts strongly to inflation (with $\gamma_\pi^C = 1.5$ or $\gamma_\pi^C = 1.1$ in the Galí 2008 or Kim 2003 calibration, respectively) and surpluses adjust so as to contain consolidated-government liabilities (for this, $\gamma_Z^T = 0.1$ from Kim 2003 is also applied to the Galí 2008 calibration). Conversely, in the ‘fiscalist’ regime, neither policymaker reacts to ‘their’ variable of interest: $\gamma_\pi^C = \gamma_Z^T = 0$. Of course, there is a continuum of policy-parameter constellations inbetween (and, in the case of the conventional regime, beyond) these values, but the chosen constellations allow me to represent two ‘polar’ regimes and compare them to standard references in the literature.

7.3.2. Baseline Results

7.3.2.1. Technology Shocks

Active Monetary, Passive Fiscal Policy Figure 7.1a depicts the effects of a sudden and persistent deterioration in technology ($\varepsilon_t^a = -0.25$ with an autoregressive coefficient $\gamma_a = 0.5$). The solid lines representing the active-monetary/passive-fiscal regime tell a familiar story: Natural output declines more strongly than actual output so that the output gap (see Figure 7.1b) becomes positive,²³ which in turn puts upward pressure on inflation. The central bank reacts by strongly increasing the nominal interest rate

²³ This could be made more explicit formally. Since it is not the main concern, I only give the following hints: Natural output moves in line with technology by Equation (F.36) ($\partial \hat{y}_t^{\text{nat}} / \partial \hat{a}_t > 0$) whereas the output gap depends on the difference of expected future and current technology ($\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t$) in the dynamic IS equation (7.32) which, given the technology process (7.11), yields a negative coefficient ($[\gamma_a - 1] \hat{a}_t$, i.e., $\partial \hat{y}_t / \partial \hat{a}_t < 0$). Bringing in Equation (F.10), $\hat{y}_t = \tilde{y}_t + \hat{y}_t^{\text{nat}}$, then implies that the deviation of actual output from steady state must lie between the two. With $\hat{a}_t < 0$ here, it is greater than the (negative) deviation of natural output as the (positive) output gap is added to it; or *vice versa*, it is smaller than the (positive) output gap as the (negative) deviation of natural output is ‘added’ to it.

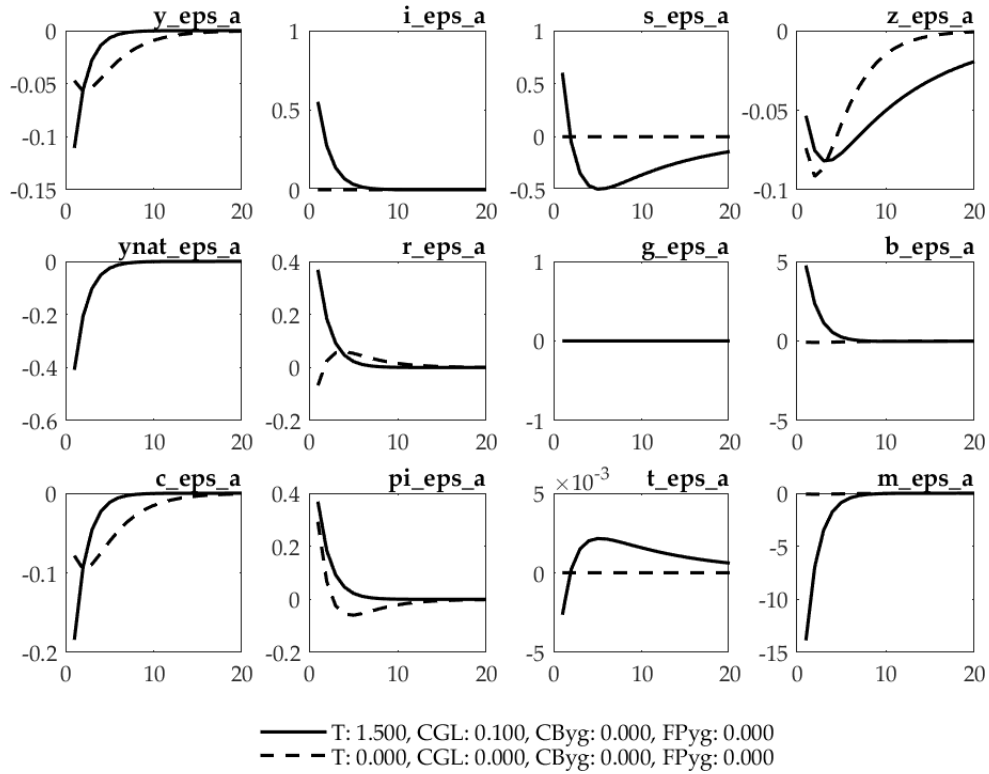


Figure 7.1a: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137.

and thus also the real interest rate. Given the decline in consumption as well as the increasing nominal interest rate, real money balances decline drastically.

The remaining adjustments are more complex (which is the downside of not dismantling interdependencies further, see also \nearrow *How Far Should the Model Be Reduced?*, p. 132). In the setup of Figure 7.1a, surpluses are driven by changes in consolidated-government liabilities as well as inflation (see the additional Figure 7.1b, which features the two non-zero summands on the right-hand side of Equation (7.42)). While the reaction to the former is as expected, one might wonder why higher inflation leads to an *increase* in surpluses instead of lowering them on account of real devaluation of the outstanding debt. The answer is that steady-state treasury debt and consolidated-government liabilities are actually *negative* in this ‘conventional’ regime (see Table F.2 in Appendix F.5); hence, the treasury reacts to the implicit transfer of wealth to the private sector by increasing surpluses in order to restore its long-run creditor position ($z_{ss} < 0$ in the linearized surplus rule (7.42); see also Section 7.3.3.1 below for a brief analysis of this phenomenon). As the influence of inflation wears off, consolidated-government liabilities become the main driving force behind surpluses over time. With treasury ex-

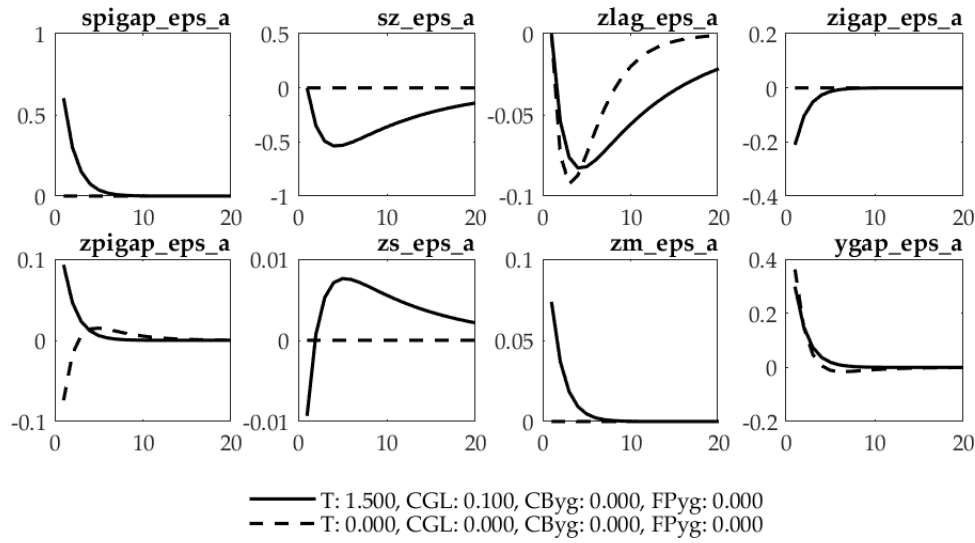


Figure 7.1b: *Simulation of the Sticky-Price Model: Technology Shock (Addendum)*. \diamond Source: Own illustration. \diamond Explanations: See Figure 7.1a. The variable names follow the logic of Table 7.3, where **sygap**, **spigap**, **sz**, and **sm** denote the individual reactions of surpluses \check{s} to the output gap \tilde{y} , the inflation gap $(\pi - \pi^*)$, consolidated-government liabilities \check{z} , and real money balances \hat{m} , respectively. Similarly, **zlag**, **zigap**, **zpigap**, **zs**, and **zm** denote the summands on the right-hand side of Equation (7.39).

penditure remaining constant, surpluses have to be brought about wholly by transfers to households.

In the explanation just above, it was taken for granted that surpluses move in line with outstanding liabilities (i.e., fall). But why do consolidated-government liabilities decrease in the first place? This can be explained along the summands on the right-hand side of the respective law of motion (7.39) (and with the help of Figure 7.1b):

- Recurring to the interdependencies mentioned above, surpluses have a stake in this, but it is rather small numerically.
- Noting that b_{ss} and z_{ss} are negative here, we have counteracting influences from the nominal-interest and inflation rate gaps, increasing and decreasing the net-creditor position of the treasury, respectively. In line with the Taylor principle, the former effect outweighs the latter so that public-sector liabilities move deeper into negative territory (claims on households grow larger).
- Real money declines strongly and, as indicated by the last summand, rids the treasury of seigniorage earnings.
- The misconception that these falling real money balances could abruptly lower z via their definition (2.28) / (F.42) is already debunked in Section 5.1.3.2 (see \nearrow Future

Price Levels, p. 87; and it would of course contradict this very train of thought). Rather, the first summand in Equation (7.39) embodies the logic behind consolidated-government liabilities: They are stocks that need to be rolled over. All the other summands could thus be understood as ‘adjustments’ to this necessity. (This is where the definition (2.28)/(F.42) has its place: With real money given and consolidated-government liabilities just determined, it is upon treasury debt to adjust.)

Considering the starting points of the impulse-response functions in Figures 7.1a-7.1b, the ‘adjustments’ point to consolidated-government liabilities below their steady-state value (coming out of steady state, the lag $\mathbf{zlag}/\tilde{z}_{t-1}$ is equal to zero), even with treasury debt increasing to a less negative value and thus generating less interest income for the fiscal authority.

Passive Monetary, Active Fiscal Policy The dashed lines in Figure 7.1a show some differences in response to the technology shock if the treasury is the active policymaker and monetary policy is passive: For one, the central bank does not raise the nominal interest rate in response to increasing inflation. By consequence, the real interest rate has a mildly attenuating influence on actual output and consumption (while natural output behaves just as above). Surpluses, like treasury expenditure and transfers to households, remain flat because $\gamma_Z^T = 0$. Consolidated-government liabilities, whose steady-state value is now positive (again, see Section 7.3.3.1), are devalued by inflation, and this effect is reinforced by the lag component $\mathbf{zlag}/\tilde{z}_{t-1}$ at first (see Figure 7.1b). Note that the movements in treasury debt as well as money coincide with those in consolidated-government liabilities, they are just hard to recognize because of scaling.

Evaluation Comparing the two regimes, it should be noted that output and consumption recover slower in the fiscalist than in the conventional regime. Overall, the results presented here are similar to those of Kim (2003, pp. 769-771). In particular, the model exhibits “inflation reversal” in the passive-monetary/active-fiscal regime: Because the central bank does not combat the arising inflation, an “inflation tax” accrues, and in the absence of any reaction on the part of the treasury, intertemporal budget balance (i.e., the present-value budget equation (2.45)) requires a negative inflation tax later on.

To add another reference, the impulse responses are also consistent with those in Galí (2008, p. 55) who considers a favorable technology shock, i.e., $\varepsilon^a > 0$, but since the models are ‘symmetric’ (because there is no zero lower bound on the nominal interest rate, for instance), the comparison is not complicated.

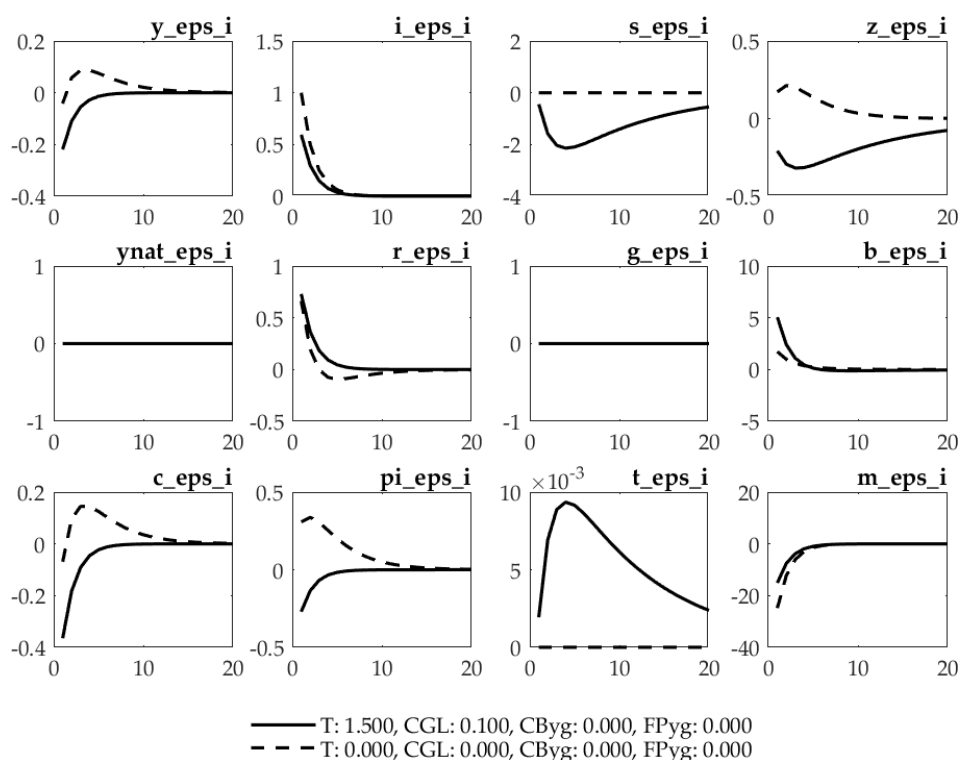


Figure 7.2a: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^i = 1$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137.

7.3.2.2. Interest-Rate Shocks

Active Monetary, Passive Fiscal Policy In Figure 7.2a, the central bank performs a discretionary increase of the nominal interest rate of 1% that is persistent with the autoregressive coefficient $\gamma_\varepsilon^C = 0.5$. Again, the events in the conventional case are familiar: The real interest rate increases and depresses actual output as well as consumption. Natural output, by contrast, remains unmoved so that the emerging negative output gap (depicted in the additional Figure 7.2b) puts downward pressure on prices. Following its policy rule, the central bank reacts to inflation below target by lowering interest rates, which is why the nominal rate never reaches the full value of the shock but starts at about 0.6%.

Similar to Section 7.3.2.1, surpluses only react to inflation and consolidated-government liabilities. In contrast to the technology shock, however, inflation also decreases so that surpluses are below their steady-state value during the whole adjustment process. Again, treasury expenditure has no part in achieving surpluses, this entirely falls on transfers to households.

Consolidated-government liabilities, which are negative in steady state, decline be-

cause higher rates award the treasury with more interest income (bonds are negative as well again) and lower inflation transfers wealth to creditors. The decreases in surpluses and money balances tend to push liabilities up, but are too weak numerically to counteract the aforementioned effects noticeably. As before, higher treasury debt substitutes for lower money balances.

Passive Monetary, Active Fiscal Policy With the Taylor coefficient set to zero, the interest-rate shock can develop fully. The usual intuition would be that it should also pull up the real interest rate which would in turn negatively affect the output gap. However, from natural output (which remains flat) and actual output (which increases), we can infer that this is not the case. Rather, the impulse-response functions resemble the events of the flexible-price model described in Chapter 4.3.2.2: The higher rate leads to higher interest income of the private sector, for one because it increases service on treasury debt, but also due to lower money demand and the associated substitution into bonds. With surpluses unchanged now and in the future, consolidated-government liabilities increase, which is tantamount to an increase in private-sector wealth. Richer households spend more on consumption and thus drive up actual output as well as the output gap. The resulting inflation is not answered by rate increases on the part of the central bank; therefore, the real interest rate is not too far away from that of the active-monetary/passive-fiscal regime. (It has to be noted, however, that consumption and output start out below steady state. As Figure 7.2b shows, the reason is that the effect of the increasing real interest rate outweighs that of the expectation term $E_t \tilde{y}_{t+1}$ in the first two periods.)

Evaluation The analysis of an interest-rate shock is more interesting than a technology shock since the former displays diverging behavior of some variables while adjustments only differ gradually in the latter. In particular, output, consumption, and inflation diverge after the interest-rate shock.

Again, the results are similar to those in Kim (2003, pp. 768-769), who points out that government liabilities increase for two reasons (substitution out of money and into bonds as well as the higher interest rate on these bonds) in the fiscalist regime and that output and consumption actually start below zero while inflation is elevated from the beginning. He also offers an alternative explanation for this phenomenon: Because the treasury does not adjust budget surpluses, increases in consolidated-government liabilities have to be financed via an inflation tax so that inflation rises immediately despite higher interest rates. Further references confirming the results obtained here are Galí (2008; for the conventional regime only) as well as Canzoneri, Cumby, and Diba (2011; for both regimes, but only four variables, namely consumption, output, the real interest rate, and inflation).

Some differences arise in comparison to Kim (2003), whose impulse-response analysis

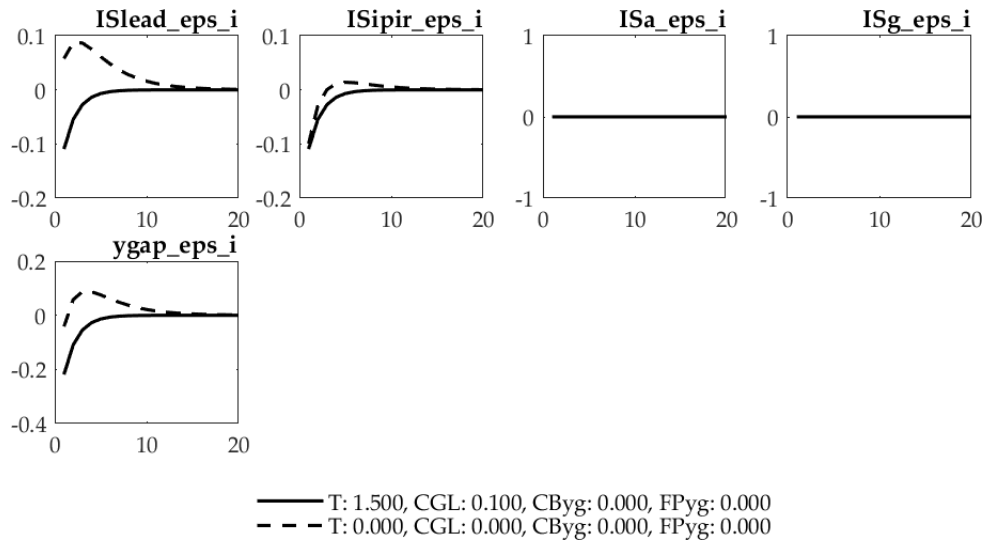


Figure 7.2b: *Simulation of the Sticky-Price Model: Interest-Rate Shock (Addendum)*. \diamond Source: Own illustration. \diamond Explanations: See Figures 7.2a and 7.1b. **ISlead**, **ISipir**, **ISa**, and **ISg** signify, respectively, the lead, rate-difference, technology, and public-spending summands on the right-hand side of the dynamic IS equation (7.32).

is more comprehensive than those of the other authors. For instance, the crudely defined growth rate of consolidated-government liabilities (following from Equation (7.31)),

$$\Delta \check{z}_t \equiv \check{z}_t - \check{z}_{t-1} = \frac{z_t - z_{t-1}}{|z_{ss}|},$$

deviates somewhat from the respective figure in Kim (2003). Further, transfers to households in this study and taxes (i.e., the opposite) in his both adjust from above steady state in the active-monetary/passive-fiscal regime, but this is likely due to differences in the details about fiscal policy (expenditure rule (7.44) and his equivalent).

7.3.2.3. Surplus Shocks

Active Monetary, Passive Fiscal Policy Figure 7.3a displays the dynamic adjustment to a discretionary reduction of surpluses by the treasury ($\varepsilon^s = -0.01$, $\gamma_\varepsilon^T = 0.8$). Following expenditure rule (7.25a)/(7.44a), public spending increases and leads to some crowding-out of private consumption, but it is rather mild since actual output increases as well. So does natural output, which is why the output gap opens up only a bit (see Figure 7.3b) and inflation is not very pronounced. Still, the central bank adheres to the Taylor principle and increases the nominal as well as the real interest rate accordingly.

Equation (7.42) lists and Figure 7.3b depicts the potential drivers of surpluses on the

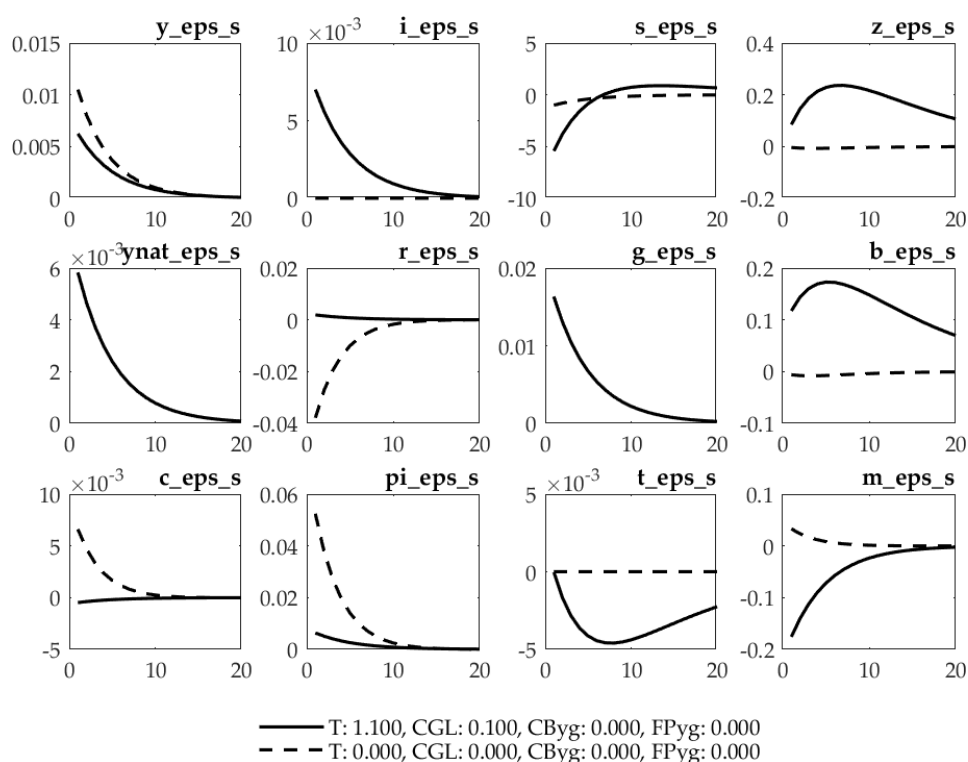


Figure 7.3a: *Simulation of the Sticky-Price Model: Surplus Shock.* ♦ Source: Own illustration. ♦ Explanations: $\epsilon^s = -0.01$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also ↗ *Impulse-Response Functions* on p. 137.

right-hand side. Inflation plays a neglectable role numerically. Much more, surpluses are driven strongly by the shock, at least in the beginning. As the latter dissipates slowly, the influence of consolidated-government liabilities increases so that the deviations of surpluses from steady state undergo a sign switch and turn positive moving from period 6 to 7, that is, after one-and-a-half years—the treasury makes good for the ‘profligacy’ of the early stages.

The effects on the remaining variables should be straightforward with the explanations given in the previous subsections: Transfers to households (taxes) are a residual and not very large quantitatively in comparison to surpluses and public spending. With the nominal interest rate going up slightly, real money balances are in less demand. The negative shock to surpluses increases consolidated-government liabilities initially, but as explained just above, this feeds back into surpluses which then pull liabilities back down towards steady state from period 7 on. In terms of model equations, treasury debt is a residual of consolidated-government liabilities and real money; with regard to a story behind the dynamics, the increase in it as well as the slight increase in interest *on* it are an additional burden on the treasury in the ‘redemption phase.’

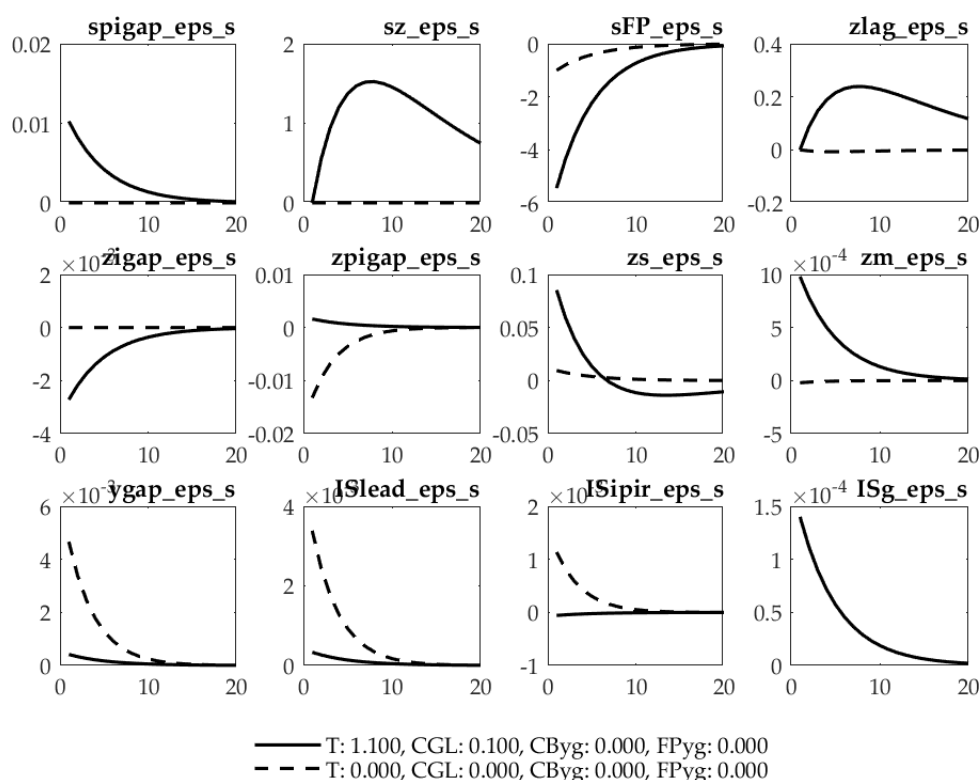


Figure 7.3b: *Simulation of the Sticky-Price Model: Surplus Shock (Addendum)*. \diamond Source: Own illustration. \diamond Explanations: See Figures 7.3a and 7.1b/7.2b.

Passive Monetary, Active Fiscal Policy When the treasury is the active policymaker, the only influence on surpluses is its decision to implement a shock (impulse responses that stay flat in both regimes are not shown in Figure 7.3b). Given that public spending moves inversely by Equation (7.25a)/(7.44a), transfers to households do not move. Knowing that the treasury will not balance its budget intertemporally by increasing surpluses in the future, households consume more. This drives up actual output over and above natural output so that the resulting output gap provokes inflation. With a passive monetary authority, however, the nominal interest rate does not react; rather, the depressed real interest rate reinforces the boom.

Consolidated-government liabilities change only subtly. The (mathematically) positive influence of surpluses is overcompensated by the devaluation through inflation. All in all, the maximal amplitude is -0.0069 in the fourth quarter, which is hardly noticeable in the graph. Real money increases moderately for a transactions-demand motive. Treasury debt is also rather passive.

Evaluation With $s_{SS} = -0.001837 \approx -0.002$ (see Table F.2 in Appendix F.5) in the active-monetary/passive-fiscal regime, a first-period shock of $\epsilon_t^s = -0.01$ amounts to

roughly five times the steady-state value of surpluses. In other words: It is huge. Yet, considering the responses of other variables, its effects are moderate, if not miniscule. By contrast, in the passive-monetary/active-fiscal regime, the same shock amounts to ‘only’ -100% given the steady state $s_{SS} = 0.01$, but the consequences are way more tangible with regard to central variables such as output, consumption, or inflation.

Further, the adjustment processes look very different in the two regimes: With monetary policy active and fiscal policy passive, much of the ‘left half’ of Figure 7.3a (consumption, the real interest rate, and inflation) shows only weak responses while the ‘right half’ is characterized by a long adjustment process far beyond the 20 periods depicted here—reversion to steady state actually occurs around period 40-50 for surpluses, transfers to households, treasury debt, and thus also consolidated-government liabilities. By comparison, the fiscalist regime offers an adjustment that could be called ‘quick and dirty’: 10 periods of increased consumption at the cost of high inflation, but hardly any change in the ‘right-half’ variables.

7.3.3. Analysis of Particular Items

7.3.3.1. A Peculiarity of Surpluses in Steady State

Steady-state consolidated-government liabilities z_{SS} are determined via Equation (F.19) in Appendix F.2.2:

$$z_{SS} = \frac{1}{\beta^{-1}(1 - \gamma_z^T) - 1} \left[(1 + i_{SS}) s^{\text{fix}} + (1 - \gamma_m^T) i_{SS} m_{SS} \right]$$

In principle, this looks intuitive: β^{-1} is related to the gross real interest rate (consider Equation (2.46), for instance, which states that $\beta^{-1} = 1 + r_t \forall t$ in the constant-endowment economy). It is adjusted by taking into account how strongly the treasury reacts to outstanding liabilities via the expression $(1 - \gamma_z^T)$. Next, it is turned into a net rate by subtracting unity. Finally, the term in brackets sums terms representing the ‘fundamental’ part of treasury surpluses s^{fix} and a potential seigniorage rebate (the expression $[1 - \gamma_m^T] i_{SS} m_{SS}$). In short, steady-state liabilities of the consolidated government are equal to a perpetuity of a certain amount of surpluses.

The interesting part, however, is that the fraction in front of the brackets can be positive or negative depending on the fiscal regime. Trying out different calibrations reveals that, with active monetary policy, the Blanchard and Kahn (1980) conditions are violated for the *system* if $\gamma_z^T < 1 - \beta$, while the other regime with passive monetary policy yields a stable solution.²⁴

²⁴ To shed light on this, consider a somewhat crude line of reasoning: Combining the budget equation (2.37) (or its close relative (F.18)) with surplus rule (7.23) yields a difference equation in consolidated-

The peculiarity is that, in the regime with active monetary and passive fiscal policy, a positive constant s^{fix} leads to *negative* consolidated-government liabilities z_{SS} and, by Equation (7.23), to negative steady-state surpluses s_{SS} . Of course, the latter two variables having the same sign is in line with the present-value budget equation (Equation (2.45), for example). Further, the ‘reversal’ between s^{fix} and s_{SS} does not occur with passive monetary and active fiscal policy ($\gamma_Z^T = 0$; see Table F.2 in Appendix F.5 for the steady-state values).

7.3.3.2. Different Expenditure Rules

General Remarks Switching the expenditure rule from Equation (7.44a) to (7.44b) only has unspectacular effects on the model outcomes. That there are any effects in the first place is due to goods-market clearing (Equation (F.28) in Appendix F.3.1.4) which puts treasury expenditure \hat{g} into a relationship with private consumption \hat{c} and actual output \hat{y} ; since transfers to households \hat{t}^{TH} pick up any slack by Equation (7.43), there would be no effect otherwise. The respective impulse responses are given in Appendix F.5 (Figures F.2b-F.4h).

Technology Shocks The reactions of actual output \hat{y} and natural output \hat{y}^{nat} are a bit stronger than under the initial rule (7.44a) used in the baseline results of Section 7.3.2. Since the output gap does not open up just as wide, inflation and thus also the nominal interest rate increase a little less.

Interest-Rate Shocks Results for interest-rate shocks are similar, but since the natural output \hat{y}^{nat} is unaffected under rule (7.44a), the negative influence under rule (7.44b) is more striking at first sight.

Surplus Shocks Fiscal-policy shocks to surpluses are the most interesting case because they highlight the main features of the two policy regimes. Under rule (7.44a) (Figure 7.3a), treasury expenditure tracks the surplus shock ϵ^s and thus also affects output (positively) and consumption (negatively) in the active-monetary/passive-fiscal regime. By contrast, rule (7.44b) (depicted in Figure 7.4) makes public spending move in line with private consumption in the same regime. However, nothing happens—and this is exactly what the theory holds: Households know that passive (Ricardian) fiscal policy

government liabilities. Considering it in isolation (the original sin in this argument) reveals that it is stable only if $\beta^{-1}(1 - \gamma_Z^T) < 1$, which makes the fraction in the above equation ((F.19)) negative. Of course, an isolated treatment of the equation is invalid because it is part of an equation system (which is why I abstain from stating this ‘charlatanry’ even more explicitly). Taking it as a mere guideline for the calibration of the linearized system (combining Equations (7.39) and (7.42) leads to the same isolated stability condition) and trying to simulate this in Dynare, by contrast, is not forbidden.

evens out surpluses over time, so there is no net wealth effect and therefore no reason to adjust consumption. Results are different in the fiscalist regime: Consumption is elevated under both rules (with only slight differences) because the surplus shock increases households' wealth perceptions and hence drives consumption, output, and inflation.

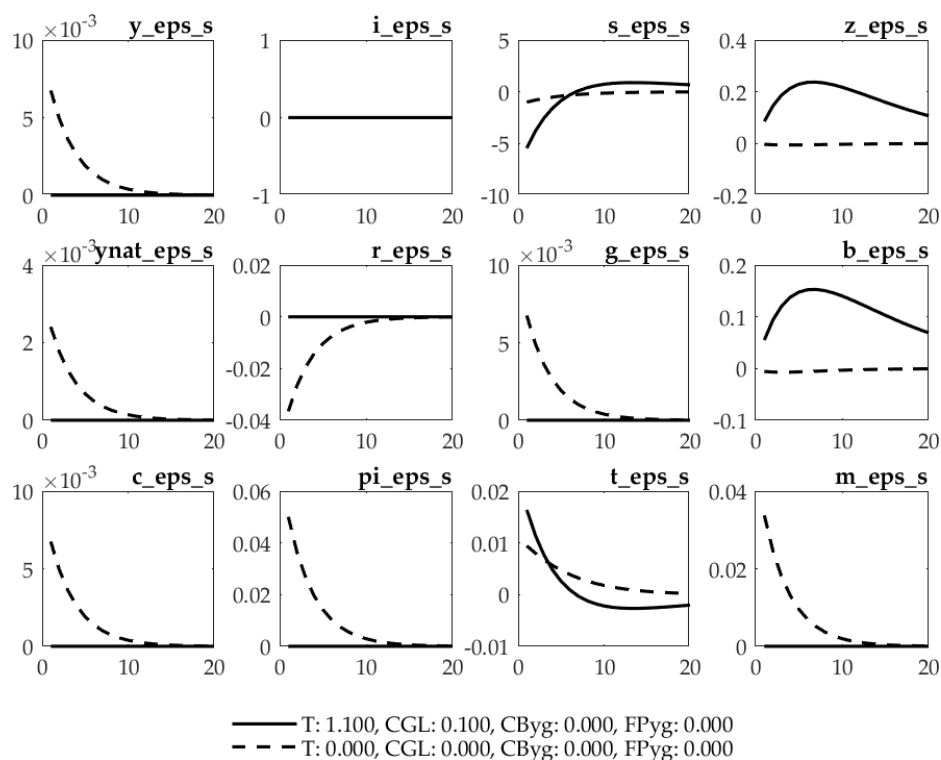


Figure 7.4: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also *Impulse-Response Functions* on p. 137.

7.3.3.3. The Seigniorage Rebate

There are some modest effects of granting the seigniorage rebate, but they are not strong enough to change the general shape of impulse-response functions, for instance. This might be considered a dispensable non-result, but at least the stylized fact that seigniorage does not play a substantial role empirically (see Section 3.4.2) is reflected in the model as well.

7.3.3.4. Admitting Reactions to the Output Gap

Another variation of the model is to let policy react to the output gap. In order to keep regimes clear-cut, this is only done in the active-monetary/passive-fiscal regime. The reaction parameter is set uniformly to 0.125, a value used by Galí (2015, p. 68) for the

Taylor rule and also transferred to the surplus rule here. While accounting for the output gap in policy rules has some effects, they are mostly not drastic (which is in line with a similar comparison within the model of Galí 2008; 2015; not shown). Therefore, and in particular because the plots are barely distinguishable from each other visually, only three examples are given in Figures F.5a-F.5c in Appendix F.5 while the remaining figures are left out.

Some remarks are in order, though. If policymakers take the output gap into account, respective changes in impulse responses are most pronounced for surpluses and consolidated-government liabilities. The reason for this is setting $\gamma_y^T = \gamma_y^C$, which implies a stronger relative effect of the output gap on surpluses than on interest rates (cf. the steady-state values in Table F.2). This can even lead to sign reversals, see the response of consolidated-government liabilities to an interest-rate shock in Figure F.5b. For the same reason, the responses to a surplus shock are so close together that they might appear as the same line in Figure F.5c.

7.3.3.5. Target Inflation

Varying the inflation target π^* affects the steady-state nominal interest rate. However, real money is the only variable that depends on the *net* rate (cf. Equation (F.41)); in every other instance, it appears as a gross rate. Therefore, only the impulse-response functions for real money and real debt (which is calculated as the residual from real money and consolidated-government liabilities by Equation (F.42)) change notably if the inflation target is adjusted. The respective figures are omitted for this reason.

Key Takeaways from Chapter 7

Adding the usual ‘New-Keynesian’ frictions complicates the baseline model to the point where it can not be properly handled analytically anymore. Simulations of the linearized model confirm the previous findings, however: Aside from the conventional regime, fiscal dominance constitutes another regime producing stable results.

8. Notes on the Monetary-Fiscal Model in Practice: Empirics and Applications

This chapter indicates some empirical issues related to the ‘detection’ of policy regimes by briefly covering simple as well as more advanced methodologies and referencing possible fiscal interpretations of historical real-world episodes.

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8.1. Observational Equivalence in Simple Tests of Policy Rules

Surpluses One might be inclined to test for regimes by running a regression of surpluses s_t on B_{t-1}/P_t and equate the resulting regression coefficients to γ_B^T in rule $\langle 4.20 \rangle$ ($S_t = \gamma_B^T B_{t-1}$ or $s_t = \gamma_B^T B_{t-1}/P_t$, respectively). Bohn (1998) follows this approach, for instance.

As Section 4.4.1 shows, if the coefficient associated with outstanding liabilities γ_B^T is positive, such a rule leads to a tautologic satisfaction of the present-value budget equation $\langle 4.4 \rangle$. Finding a positive regression coefficient could therefore be seen as evidence of a active-monetary/passive-fiscal regime.

By way of contrast, consider a non-Ricardian surplus rule ($\gamma_B^T = 0$) in the form of a plain AR(1) process with $\gamma_s^T \in (0, 1)$:

$$s_t = \gamma_s^T s_{t-1} + \epsilon_t^s$$

Plugged into the present-value budget equation (4.4), this yields

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\beta \gamma_s^T \right)^j s_t = \frac{1}{1 - \beta \gamma_s^T} s_t \quad \Leftrightarrow \quad s_t = \left(1 - \beta \gamma_s^T \right) \frac{B_{t-1}}{P_t}.$$

Bringing s_t to the left-hand side, which is done in the equation after the equivalence sign, demonstrates that the same regression as above (of surpluses on debt and the price level) yields a regression coefficient in the interval $(0, 1)$, which is likely to lead to the fallacious conclusion that fiscal policy is Ricardian and follows surplus rule (4.20) with $\gamma_B^T \in (0, 1)$. A low level of outstanding debt is understood to require little surpluses. In the true model, however, low surpluses (determined exogenously in a non-Ricardian fashion) lead to a low real valuation of debt. The regimes are *observationally equivalent*, that is, they can not be distinguished simply by ‘crunching the numbers.’ (Cf. Cochrane 1999, pp. 340-341. In making this argument, I use the setup of Chapter 4 because it is the most parsimonious; nothing substantial changes if we switch to a richer model including money and possibly more sophisticated policy rules.)

Leeper and Leith (2017, p. 2384) note that estimating just the policy rule (Equation (4.20) or the unnumbered equation above) suffers from *simultaneity bias* because the “bond valuation equation” (2.43) is missing (cf. Wooldridge 2013, Chapter 16.2, for a simple textbook treatment).

Interest Rates Similar objections can be made with respect to the Taylor rule. In the context of a simple model, Cochrane (2011, pp. 573-574) shows that regressions of the nominal interest rate i on the inflation rate π do not measure the Taylor coefficient γ_π^C but the serial correlation parameter of the policy disturbance, rendering empirical studies in search of estimates for actual Taylor coefficients (such as Lubik and Schorfheide 2004) useless. Again, the same equilibrium outcomes can be generated no matter whether $\gamma_\pi^C > 1$ or $\gamma_\pi^C < 1$. The reason is connected to the way the New-Keynesian model works (letting inflation jump to the only stable value for the sake of determinacy, see Sections 4.2.3 and 9.4.1).

8.2. Making Sense Of and With Vector Autoregressions

8.2.1. Introduction to a Tandem of Papers

The Tandem Canzoneri, Cumby, and Diba (2001b, pp. 1227-1231) try to tackle the question of regimes empirically by employing and interpreting a vector autoregression with

postwar U.S. data.²⁵ Cochrane (1999) does the same but arrives at different conclusions. Since both papers acknowledge and discuss the respective other, it is best to present them jointly.

Notation The notation in Canzoneri, Cumby, and Diba (2001b) is different from mine in two respects: First, they state outstanding liabilities of the consolidated government as a beginning-of-period variable. Second, nominal variables are scaled on nominal GDP P_t in order to make the time series stationary (similarly, Cochrane 1999, p. 361, scales on consumption c). Since I only repeat their arguments and do not carry out empirical investigations myself, I ignore the second adjustment. With respect to the first one, however, I define

$$Z'_t \equiv Z_{t-1} \quad \Leftrightarrow \quad z'_t \equiv \frac{Z'_t}{P_t}$$

for the present chapter because it makes the analysis of leads and lags in time series data more legible.

Theoretical Implications The underlying idea is that variables should behave differently *over time* in the two regimes: In the conventional case with active monetary and passive fiscal policy, an increase in the current surplus s_t reduces the amount of future outstanding consolidated-government liabilities z'_{t+1} .

Matters are a bit more complicated in the fiscalist regime with passive monetary and active fiscal policy. Repeating the corresponding present-value budget equation (2.43) in the current notation,

$$z'_t = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}) ,$$

helps illustrating three distinct cases:

1. If the innovation in the current surplus s_t is not correlated with future surpluses (and discount factors), there is no effect on current outstanding liabilities. This is an implication of the above budget equation (shifted forward one period) in which z'_{t+1} is only affected by surpluses from $t + 1$ onwards.

²⁵ Vector autoregressions have been proposed for use in macroeconomic contexts by Sims (1980). Prime examples of applications to monetary policy are given by Christiano, Eichenbaum, and Evans (1999) and Stock and Watson (2001). Textbook treatments include Hamilton (1994) and Becketti (2013).

2. Conversely, if there is a positive correlation between the innovation in surpluses and future surpluses, the increased right-hand side of present-value budget equation (2.43) requires a similar increase on the left-hand side, that is, in z'_{t+1} .
3. Finally, there could also be a negative correlation between the current innovation and future surpluses.

The conventional regime and the first two cases of the fiscalist regime yield clear perceived implications: If an increase in surpluses s_t is followed by a reduction in the real value of outstanding liabilities z'_{t+1} , the active-monetary/passive-fiscal regime is identified. If z'_{t+1} does not fall, this implies the passive-monetary/active-fiscal regime.

Unfortunately, there still is the third case of the fiscalist regime. Since the innovation is negatively correlated with future surpluses here, the right-hand side of the present-value budget equation decreases, so the left-hand side must follow suit. In other words: Real outstanding liabilities z'_{t+1} decrease in the subsequent period and thus lead to the same result in the fiscalist as in the conventional regime. This possibly prohibits identification of the policy regime based on the sequence of surpluses and real outstanding liabilities.

8.2.2. Empirical Findings

Vector Autoregression Canzoneri, Cumby, and Diba (2001b) run a vector autoregression in s_t and z'_t , examining reactions to exogenous shocks to the former. (I omit the technical details because this is supposed to be an overview rather than a replication.) Their Figure 3 is reprinted here as Figure 8.1; it shows that both orderings lead to similar results:²⁶ A shock to the surplus in one period leads to a negative response of liabilities in the subsequent period (the dashed standard error bounds imply that this finding is significant over the entire ten-year time horizon). As discussed at the end of the previous subsection, this could imply either the active-monetary/passive-fiscal regime or the third case of the passive-monetary/active-fiscal regime with a negative correlation among surpluses.

Additional Stylized Facts As a hint as to which regime is actually in play, Canzoneri, Cumby, and Diba (2001b, p. 1229) find significant positive autocorrelation in surpluses

²⁶ The order of variables is a choice in the design of a vector autoregression and determines their reactions to each other. If a variable is ordered first (or 'at the top'), it is not affected by concurrent shocks to other variables but might influence the latter. Conversely, variables ordered later (in descending order, towards the 'bottom') are influenced by prior variables but do not affect these prior variables concurrently. Inverting the argument, variables should be ordered in ascending order of responsiveness to changing economic conditions (sluggish ones first, highly reactive ones last). Since the ordering does not seem to matter in Canzoneri, Cumby, and Diba (2001b), I leave it at this verbal description. See the references cited in Footnote 25 if necessary.

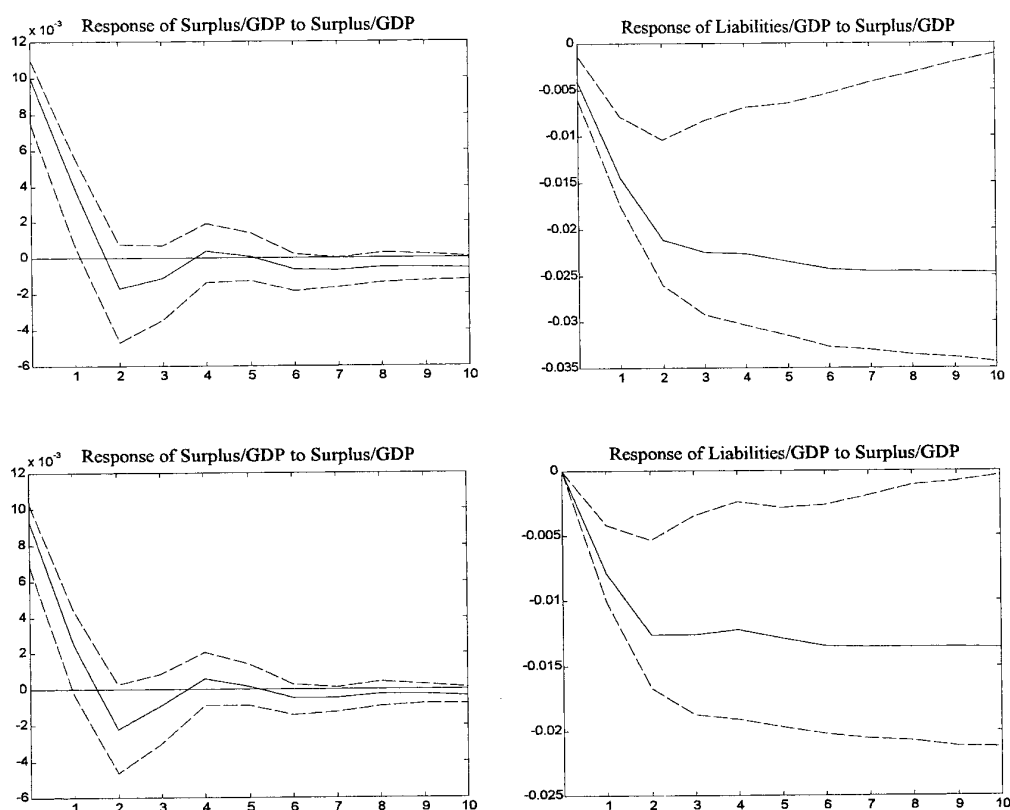


Figure 8.1: *Vector Autoregression in Surplus/GDP and Liabilities/GDP.* \diamond Source: Reprinted from Canzoneri, Cumby, and Diba (2001b, p. 1228). \diamond Explanations: In the top panels, the ordering places surplus/GDP before liabilities/GDP; in the bottom panels, the ordering is reversed (i.e., liabilities before surplus). See also \nearrow Vector Autoregression on p. 154.

over nine years while the impulse-response functions of the vector autoregression depicted in Figure 8.1 also show a positive reaction of surpluses in the first period after the shock but nothing (significant) beyond that. In any case, it seems safe to rule out a negative reaction of surpluses to a positive shock at least in the first ten years.

In addition, Cochrane (1999) presents several stylized facts about postwar U.S. data. The most important of them are summarized in Table 8.1.

8.2.3. Alternative Explanations

The Conventional Regime as an Easy Explanation Explaining the impulse responses in Figure 8.1 with the active-monetary/passive-fiscal regime is straightforward: An increase in the current surplus s_t decreases the real value of outstanding consolidated-government liabilities in the next period z'_{t+1} . The positive response of s_{t+1} reinforces this initial decrease. Canzoneri, Cumby, and Diba (2001b, p. 1229) argue that election and business cycles typically span several years so shock terms such as ε^s in Equation

no.	variables	stylized fact
1	ζ^i, ζ^m	Seigniorage is an insignificant fraction of government revenue.
2	s, y	There is a secular relation between surpluses and output; most variation in the surplus is the predictable result of output variation.
3	s, π	Surpluses and inflation are both procyclical; business-cycle movements in inflation are positively correlated with the surplus.
4	s, π	Longer-term variation in the surplus and inflation are negatively correlated.
5	s, z'	The surplus is very well negatively correlated with debt growth.
6	s, z'	The real value of debt moves much more slowly than the surplus.
7	s, z'	The <i>level</i> of debt is poorly correlated with the surplus.
8	z'	Real and nominal debt behave similarly over time.

Table 8.1: *Stylized Facts About Postwar U.S. Data.* \diamond Source: Cochrane (1999, pp. 361-363, 365, 368, 371). \diamond Explanations: Replace “debt” with “liabilities” to make the stylized facts more compatible with the rest of the thesis.

$\langle 4.20 \rangle$ can be quite persistent. However, the fact that redeeming liabilities now leads to a lower burden in the future (*ceteris paribus*) is not reflected in their analysis, at least not within the first ten years they illustrate.

Solution to a Fiscalist Puzzle Following Cochrane (1999, p. 367), an AR(1) process for surpluses can only explain a positive correlation between surpluses and debt (be reminded of Chapter 4.3.2 for the basic story and Section 8.1 for an actual AR[1] process in use). However, this behavior does not match the stylized facts derived by him earlier (p. 365-366) which hold that the *level* of real consolidated-government liabilities is hardly and its *growth rate* is negatively instead of positively correlated with surpluses.

In order to explain these observations with a fiscal theory, he turns to a more complicated story, arguing that this might be the more realistic case after all: In recessions, surpluses go down because taxes plummet and public spending increases. The treasury sells nominal debt to raise revenue, but “the only way extra nominal debt sales can raise revenue is if they come with a promise to raise surpluses in the future” (p. 367). Modeling surpluses as an AR(2) instead of an AR(1) process allows for the possibility of lower surpluses now and higher surpluses later, where the latter increase the real value of consolidated-government liabilities and thus implement the observed negative correlation between current surpluses and growth in liabilities.

In terms of the analysis of Canzoneri, Cumby, and Diba (2001b), using the passive-

monetary/active-fiscal regime to explain that z'_{t+1} falls after a positive shock to s_t is only possible if the negative correlation between current and future surpluses is strong enough to reduce their present value *in spite of positive* (or at least non-negative) *correlation during the first ten years*. Importantly, they add that a policy which generates such positive short-run and negative long-run correlations can not be related to any argument about redeeming outstanding liabilities at some rather distant ‘time of reckoning’ because it is a defining characteristic of active (non-Ricardian) fiscal policy that outstanding liabilities do not play any important role in policy considerations (such as rule (4.20), see also *The Bohn-Woodford Criterion*, p. 77).

Artificial Statistical Model To achieve such a kind of policy, Cochrane (1999, p. 368) splits the surplus into two parts: The short-term “business-cycle component” is driven by output fluctuation at constant net tax rates (think of ‘automatic stabilizers’) while the long-term component reflects tax and spending decisions that are deliberate policy choices rather than reactions to business-cycle fluctuations (perhaps because a new government comes into office which implements taxation and spending levels that, *ceteris paribus*, differ from those before). The short- and long-run components are negatively correlated. This two-tiered surplus is then combined with a present-value budget equation into a simple model which is calibrated to match the stylized facts (see Table 8.1) and results of a vector autoregression. Cochrane finds that his artificial model replicates the observations well. In particular, it has surpluses and the real value of consolidated-government liabilities, which equals the present value of the *entire* surplus stream in the present-value budget equation, moving into opposite directions (see also Canzoneri, Cumby, and Diba 2001b, p. 1230). The negative long-term correlation in the surplus sequence explains this property as well as the fact that the level of real liabilities does not display a tight relationship with the surplus (the level is ‘getting mixed signals’ in that the current surplus decreases while expected future realizations go up).

Inflation Smoothing Figure 8.2 is a copy of his Figure 12 and serves to illustrate his argument, summarized in what follows. The solid line depicts growth in real consolidated-government liabilities while the dashed and solid-scored lines represent actual and simulated nominal growth, respectively. Inflation is not drawn in this figure, but can be inferred as the difference between real and nominal growth (his Figure 11 plots actual and simulated inflation and is an input in the construction of his Figure 12/my Figure 8.2). His finding is that fluctuations in real *and* nominal liability growth are larger than fluctuations in inflation, where the synchronization between the former explains the latter: Growth in real liabilities is very volatile. The reason for inflation being relatively stable in comparison is that nominal liability growth is also very volatile—in this way, fiscal policy contributes to inflation stabilization. Consider two alternative policies for further illustration: If nominal liabilities perfectly tracked real liability growth, there would be

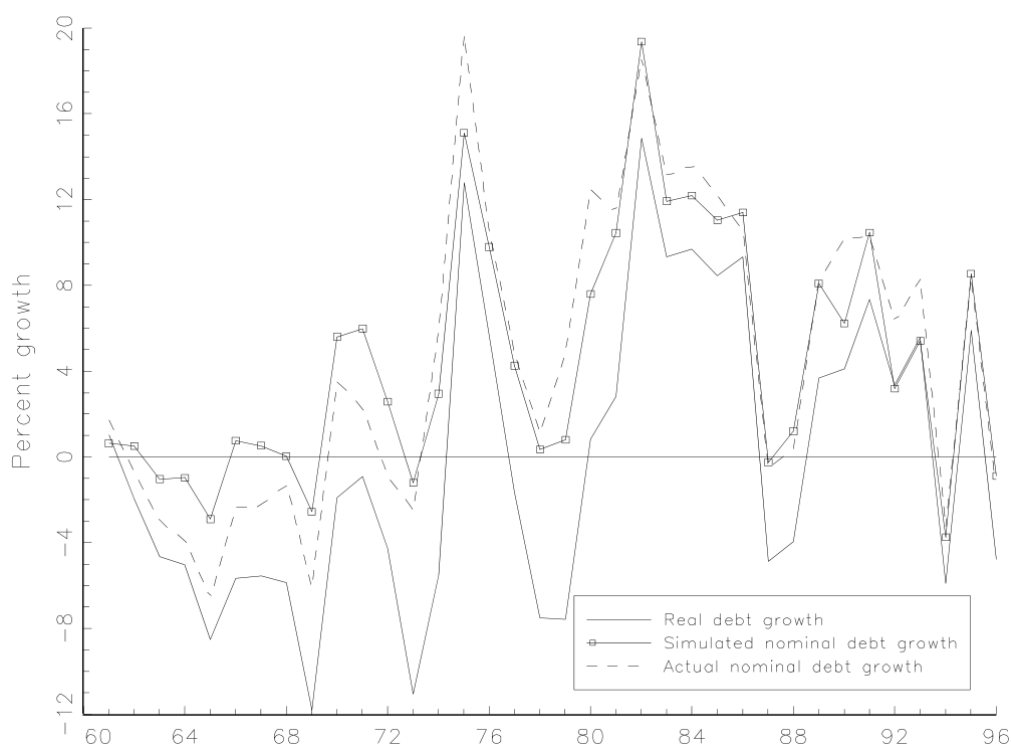


Figure 8.2: *Simulated and Actual Debt Growth in Cochrane (1999)*. ♦ Source: Reprinted from Cochrane (1998 [sic!], p. 47; this figure is more legible than the one on p. 373 of the 1999 paper but seems to be similar otherwise). ♦ Explanations: See ↗*Artificial Statistical Model* on p. 157.

no inflation; this would be an exaggeration of his view on actual policy. Conversely, if nominal liability growth adhered to a ‘ $k\%$ rule’ à la Friedman (1959) even in the face of fluctuating real liability growth, the inflation rate would be much more volatile. (Cf. Cochrane 1999, pp. 367, 372, 374.)

Mutual Objections In his ↗*Artificial Statistical Model*, Cochrane (1999, pp. 369-370) derives a very strong negative correlation between short- and long-run surplus innovations (-0.95) from the stylized relationship between innovations in the surplus and the real value of consolidated-government liabilities. Canzoneri, Cumby, and Diba (2001b, p. 1231) object that this high value can not be found in the available data and submit a much lower value of 0.06 instead. Hence, while acknowledging the identification problems associated with testing for policy regimes, their conclusion is that the simpler story with active monetary and passive fiscal policy is the more plausible explanation of U.S. data.

Then again, Cochrane (1999, pp. 341-342; 2011, pp. 580-581) repeatedly insists on the identification problems of Canzoneri, Cumby, and Diba’s approach to finding the fiscal

regime, a critique he also extends to their monetary precursor, the study of Clarida, Galí, and Gertler (2000).

8.3. Further Applications, Episodes, and Stories

Wartime US Woodford (2001, pp. 672-674, 684, 687-689) interprets Fed policy from the 1940s until 1951 as an interest-rate peg with the aim to stabilize the market value of treasury debt and as an example of autonomous monetary policy in the sense of Section 4.2.2. In line with the fiscalist explanation of price determination, and accounting for wage and price controls in place initially, the wartime deficits lead to accumulated inflation pressure which eventually materialized in 1946-1947, whereas the postwar budget surpluses in 1948-1950 resulted in deflation. With the Korea war putting upward pressure on prices again, the 'bond-price support regime' was ended. During this phase, non-existing interest-rate fluctuations can not explain price movements, and Woodford adds that the causality between *money* and prices is opposite to the quantity theory in that fiscal policy is taken to have determined prices while "growth of the monetary base under this regime was purely a general-equilibrium phenomenon" (p. 674), that is, an accommodation of money demand in order to maintain interest rates. (Woodford also uses \nearrow *Long-Term Debt* in his deliberations about this phase, see my p. 180 below.)

Further interpretations of postwar U.S. data also beyond 1951 can be found in Cochrane (1999) and Woodford (1999), for instance.

Brazil Loyo (1999) explains the 1980-1994 hyperinflation in Brazil²⁷ by a change in the macroeconomic policy regime: While fiscal policy is assumed to have been active all along, monetary policy changed from passive to active in 1980. Like this, relatively harmless (considering the experience of the 1970s) supply shocks suddenly induced explosive behavior. Reconciling this analysis with the model developed so far is hard, of course, since Chapter 4.3.3.1 tends to consider explosive paths as non-equilibria. Loyo (1999, p. 27) seems to see the solution of the puzzle in the behavior of the central bank which "may be the one expected to blink [i.e., give in and adjust policy] in the future, and in the meantime inflation explodes all the same" (emphasis added).

Japan Many authors struggle with the case of Japan, which saw an increase in debt relative to GDP from 56% in 1993 to 197% in 2014. At the same time, inflation rates averaged 0.14%. The interbank rate lay at 0.44% on average (1993-2014, 0.16% on average

²⁷ Inflation rates lay at roughly 100% in 1980, peaked at almost 3000% in 1989, and experienced a second peak at over 2000% in 1993 according to the \nearrow FPCPITOTLZGBRA data series from FRED (go to \nearrow <https://fred.stlouisfed.org>).

between 1996 and 2014), indicating passive monetary policy.²⁸ How does this match the fiscal theory? Leeper and Leith (2017, pp. 2393-2394) call Japan a “poster child for inconsistency in macroeconomic policies.” Policymakers failed to anchor expectations, especially with regard to fiscal policy. Leeper and Leith suggest that this might not even constitute fiscal dominance. After such a brief examination, two explanations are conceivable: either fiscal dominance with monetary policy being passive and fiscal policy doing a very poor job of being active (fostering a perception of fiscal sustainability which inhibits price-level increases via the usual wealth channel) or a system of monetary dominance stuck at the zero lower bound (see Chapter 5.2.1).

The US During the ‘Great Recession’ In the United States, the Federal Funds Rate target range has been near zero from December 2008 until December 2015, which could be interpreted as an interest-rate peg. Analyzing this situation through fiscalist eyes, the price level should then be determined by the present-value budget equation. Assume for simplicity that surpluses are constant and focus on treasury debt only, at least in the beginning. In terms of Equation (4.4), a constant right-hand side requires the same of the left-hand side or, in the specific example, that

$$\frac{B_{2008}}{P_{2009}} \stackrel{!}{=} \frac{B_{2014}}{P_{2015}} \Leftrightarrow \frac{B_{2014}}{B_{2008}} \stackrel{!}{=} \frac{P_{2015}}{P_{2009}}.$$

Since P_{2015} denotes the end-of-year-2015 price level, B_{2014}/P_{2015} is used instead of B_{2015}/P_{2016} in order to steer clear of rising interest rates in 2016 (i.e., of a departure from a peg).

Real-world data is also influenced by GDP growth. In order to solve this issue, we use logarithms to simplify the calculation (cf. *↗Gross and Net Rates*, p. 231) and deconstruct growth in (lagged debt)/GDP. Further, we append the above requirement that the ratio be constant:

$$\ln\left(\frac{B_{2014}}{P_{2015}Y_{2015}}\right) - \ln\left(\frac{B_{2008}}{P_{2009}Y_{2009}}\right) = \ln\left(\frac{B_{2014}}{B_{2008}}\right) - \ln\left(\frac{P_{2015}}{P_{2009}}\right) - \ln\left(\frac{Y_{2015}}{Y_{2009}}\right) \stackrel{!}{=} 0$$

During the assumed interest-rate peg, federal debt in the hands of the public increased by 91%; this corresponds to the first term in the middle. Real GDP growth, corresponding to the last term, was 13%. Hence, in order for the ratio to stay constant, P_{2015}/P_{2009} would have had to increase by about 69% (use $\exp[\ln 1.91 - \ln 1.13]$ to find this), which translates into an annual inflation rate of 9.1%. Actual inflation between 2009 and 2015

²⁸ *↗FRED* (cf. Footnote 27) data series *↗DEBTTLJPA188A*, *↗FPCPITOTLZGJPN*, and *↗IRSTCI01JPM156N*, respectively. Leeper and Leith (2017, p. 2392) report debt-to-GDP figures of 75% for 1993 and 230% for an unspecified ‘recent’ point in time; their average of inflation rates lies at 0.21%.

was 9% in total or 1.4% in average yearly rates.²⁹ (Including monetary measures would not ameliorate the ‘missing inflation’ result; since it increased by 132% over the same timespan, it would rather exacerbate it.)

How should this divergence be explained? Is the fiscal theory pointless after all? Not quite. As Leeper and Leith (2017, pp. 2392-2393) rightfully argue, the present-value of surpluses not only depends on *their* sequence but also on discount factors. Kiley (2015) and Holston, Laubach, and Williams (2017), among many others, find that the real interest rate decreased markedly during and after the financial crisis, even moving into negative territory, which increases the present value on the right-hand side of the budget equation and thus allows for higher (instead of constant) B_{t-1}/P_t , that is, less (potentially ‘missing’) inflation.

8.4. Concluding Remarks

Defeatism As a DSGE model, the fiscal theory describes—and within this world view, observables correspond to—equilibria. Who or what (which agent, which variable) adjusts so as to move from out of equilibrium into it can not be observed (Sims 1994, p. 399: “The value of fiat money always depends on public beliefs about fiscal policy under circumstances that are never observed in equilibrium.”). This applies to both stable regimes within the fiscal-monetary model developed so far because the equations (*inter alia* the present-value budget equation (2.43)) are always the same; an empirical ‘regime test’ is therefore not possible (cf. Canzoneri, Cumby, and Diba 2001b, p. 1234). The situation is similar to the analysis of a market for an arbitrary product: Quantities and the prices at which they are traded are observable, but the underlying supply and demand curves are not. Vector autoregressions do not remedy this condition because the “sequence of price levels, surplus[es], and debt [...] is a *single* equilibrium” (Cochrane 1999, p. 338).

Pragmatism Leeper and Leith (2017, pp. 2832-2833) are less beat down about this *observational equivalence* for they seem to consider it a fact of life (in their words: a “truism”). As already indicated above (↗*Defeatism*), observables are always considered equilib-

²⁹ Data on the Federal Funds Rate stems from <https://www.federalreserve.gov/monetarypolicy/openmarket.htm> (↗*link*). The target range of 0.00%-0.25% announced on December 16, 2015 was in force until the first increase to 0.25%-0.50% announced on December 17, 2015.

Data on federal debt in the hands of the public, base money, GDP, and consumer prices/inflation stems from ↗*FRED* (cf. Footnote 27, p. 159; time series identifiers ↗*FYGFDPUN*, ↗*AMBSL*, ↗*GDPC1*, and ↗*CPIAUCSL*, respectively). Year-end values are represented by January 1 datapoints of the subsequent years; for instance, P_{2015} corresponds to the 2016-01-01 datapoint in the CPIAUCSL series. Be cautioned that most series are seasonally adjusted, only federal debt in the hands of the public (FYGFDPUN) is not; this is innocuous in constructing ballpark figures as the ones above, but any detailed analysis should of course be based on more carefully constructed time series.

rium outcomes, so the underlying equilibrium conditions of the model behave like identities in empirical work, rearrangements of which “do not impose enough structure to distinguish between regimes.” Another way to put this is based on the ‘correlation does not imply causation’ mantra, leading to the same conclusion that observed relationships between surpluses and consolidated-government liabilities, for instance, can not discern which regime is currently at play. Without additional *identifying assumptions*, it can only be tested how well the model replicates factual relationships, but not which variant of the model—conventional or fiscalist—is responsible for this.

In this respect, the fiscal is no different from the quantity theory: According to Leeper and Leith (2017, p. 2383), Sims 1972 shows that the results of Friedman and Schwartz (1963; for instance, that monetary policy shocks are a prime reason for volatility in nominal income) are based on additional assumptions that identify “exogenous” money-supply shocks. In short: Identifying restrictions turns the quantity equation into a quantity theory; analogously, they can transform a fiscal equation (namely, the present-value budget relation) into a fiscal theory.

Cochrane (1999, p. 325) sets out to “construct a plausible story for the time series rather than pursue a test.” That is a constructive approach, but of course, it is questionable whether it represents a satisfactory be-all-end-all verification strategy.

Key Takeaways from Chapter 8

Both simple as well as more advanced empirical approaches suffer from observational equivalence. Finding regimes in real-world data might therefore be a matter of arguing about plausibility. A fiscalist story can be argued theoretically, but its assumptions do not seem to be given in reality.

9. Discussion

I discuss the results obtained in the previous chapters along different criteria and compare ‘my’ monetary-fiscal to traditional theory. Furthermore, I touch on the desirability of fiscal vis-à-vis conventional price determination and list possible extensions of the model.

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9.1. Debt Interpretations

9.1.1. Real vs. Nominal Debt

One implication of Chapter 4 not mentioned so far regards the distinction between real and nominal debt. In principle, debt contracts are always entered in nominal terms in the fiscal theory. The fiscal-dominance regime (see Section 4.3.2) demonstrates the

consequence: The treasury does not adjust its surpluses to changes in real debt but lets its budget equation (be it the flow version $\langle 4.3 \rangle$ or present-value version $\langle 4.4 \rangle$) be satisfied through changes in the price level.

By way of contrast, monetary dominance represents an environment in which debt can be considered real because the treasury has no way of debasing it via the price level. In fact, its passive policy rule (Equation $\langle 4.7 \rangle$ with $\gamma_b^T > r$) is what ensures that debt does not change in present-value terms. To clear up semantics, it might not be entirely correct to speak of ‘indexed’ debt in this context because the present model features no indexation in the most common sense of the word: It is not B_{t-1} (in the consolidated-government budget equation $\langle 4.4 \rangle$) that changes in reaction to a rising price level but the sequence of expected surpluses. In other words, households are not compensated directly by an increase in the principal but indirectly by a expected future policy path that recovers their wealth position.

9.1.2. The Stock-Price Analogy

A popular analogy in the fiscal-theory literature is the valuation of stocks. It is told by Woodford (2001), Cochrane (2005b), and Sims (1997; 1999a), among others, but is already indicated in Sargent (1982a).

The Analogy Consider the case of a firm which does not pay dividends but passes its earnings on to investors by buying back shares. The market capitalization of the company is given by the standard asset valuation equation

$$\begin{aligned} \{\text{share price}\}_t \cdot \{\text{amount of shares}\}_t \\ = \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \cdot \{\text{earnings used for share repurchases}\}_{t+j} \end{aligned} \quad \langle 9.1 \rangle$$

which is uncontroversial and, as Woodford (2001, pp. 694-696) notes, similar in form to the present-value budget equation $\langle 2.43 \rangle$ (or any of its follow-up variants used elsewhere in this thesis). The amount of shares in the stock-price example corresponds to the amount of outstanding liabilities of the consolidated government in the theory of price determination. While the analogy is often discussed in relation to a cashless economy with just debt, adding money to liabilities (as in definition $\langle 2.28 \rangle$) is unproblematic because each single debt title promises payment of a single unit of currency, so both components have the same dimension and unit of account. Without going into much detail about the details of their generation (firms and governments have different ‘business models’ after all: revenues minus cost versus tax volume minus public spending), earnings used for share repurchases are likened to surpluses. Finally, stock prices are analog to the inverse of the price level.

Promising What? Calling the price level by its alias—purchasing power of money—turns up a reference to the respective claim in each case. This is an important parallel between both stories: Each entity promises to deliver a commodity that it can produce itself. The company passes earnings on to investors by buying back its own shares. The consolidated government, if it does not roll over or issue additional debt, can run surpluses and redeem its debt or simply deliver money to the public. Its debt is “a promise only to deliver *more of its own liabilities*” (Woodford 2001, p. 693).

Stock Splits and Currency Reforms A company doubling the amount of shares in Equation (9.1) without adjusting earnings projections is performing a stock split; the commonly expected only result is that the stock price halves. Assuming for simplicity that the consolidated government only changes the monetary part of outstanding liabilities, a similar increase of money balances without any other change in policy is a currency reform: Replacing the Deutsche Mark by the Euro brings down consolidated-government liabilities and the price level alike. (Cf. Cochrane 2005b, pp. 504/515.)

Historical Example While the mechanics behind this could also be explained differently (namely, with securitization), it is interesting to note that there is an example which not only likens public debt to equity shares, but actually transformed the former into the latter: the Bank of England shortly after its inception. In the beginning, it was a vehicle to collectively hold public debt issuing shares to individual creditors of the British crown (cf. Sims 1997, p. 2).

9.2. Budget Identities vs. Equilibrium Conditions

9.2.1. The Critique

Equilibrium Conditions To begin with a clarifying counterexample, consider the first-order equilibrium conditions from an optimization problem: Their violation means there is room for improvement that the respective economic agents will be eager to use, but the mere fact that the economy is out of equilibrium is not impossible (even though large parts of the literature, and obviously general-*equilibrium* models, focus exclusively on equilibrium). Alternatively, take goods-market clearing, the imposition of which is often a prerequisite for equilibrium. Nonetheless, its violation is not an unrealistic circumstance—actually, slack or tightness in many markets can be observed frequently, if not more frequently than perfect market clearing.

Identities Budget constraints are different. Be it in or out of equilibrium, the consolidated government has to honor its liabilities or default has to be modeled explicitly.

Either way, it all happens ‘within’ the budget constraint, which is always satisfied, using the variables it contains. Buiter (2002) states that fiscal price determination may violate this *identity*: As argued above, equilibrium conditions can be met or not. But if the price level as the last remaining free variable takes an off-equilibrium value, this would be tantamount to a violation of the budget identity.

The Critics For this reason, Buiter (2002, p. 459) asserts that the fiscal theory is “fatally flawed.” See also Marimon (1999).

9.2.2. Rebuttals

Currency Reforms Cochrane (2005b, pp. 504/515) argues that if the present-value budget equation were a constraint instead of a valuation equation, the isolated currency reform described in Section 9.1.2 would be impossible: Pulling up Equation (2.36), for instance, holding all other policy variables (the amount of debt B_t , real surpluses s_t , as well as the nominal interest rate i_{t+1}) constant as well as accepting the price level P_t and of course all predetermined variables as given, there is no way to, say, strike three zeros from M_t in order to bring the Italian Lira on par with the Deutsche Mark.

Admittedly, this line of reasoning is debatable since currency reforms, even more so when announced early and performed cleanly, seem like something different than gradual changes in one or several of the variables involved. Therefore, one could argue that the described reform would actually divide the entire equation by 1.000, including the price level P_t .

Stock-Price Analogy Taking a hint from the analogy in Section 9.1.2 also leads to the conclusion that the present-value budget equation is an equilibrium condition rather than a constraint on consolidated-government behavior: If something drives up the share price in Equation (9.1), the respective company does not automatically have to increase its earnings used for share repurchases. Aside from price management to avoid hostile takeovers, for which there is no direct analogon in the public-sector case, firms are more or less unconcerned with their share price. Similarly, the consolidated government does not have to increase surpluses should prices drift off the equilibrium path. (Cf. Cochrane 2005b, p. 515.)

Derivation It can be checked in Section 2.3.2 how the present-value budget equation of the consolidated government (2.43) is constructed: by combining the household budget constraint (2.3) with (*inter alia*) goods-market clearing (2.27), which is an equilibrium condition (cf. Canzoneri, Cumby, and Diba 2001b, p. 1224). Market clearing may not be satisfied out of equilibrium so the same is true for the consolidated-government present-value budget equation (2.43) or its logical complement, the transversality condition

⟨2.42⟩. (Cf. Cochrane 2005b, pp. 518-520. By contrast, the budget constraint of atomistic households without market power, even when bundled into the ‘representative household’ construct, is an identity that must hold under all circumstances, but this is part of the reason why the government is ‘special,’ see Section 9.2.3.) Leeper and Leith (2017, p. 2316) support this derivation argument and add that “[t]he valuation equation imposes no restrictions on the government’s choices of future surpluses, in the same way that the Fisher relation does not limit the central bank’s choices of the nominal interest rate.”

Equilibrium Conditions vs. Identities, Part II Extending on the above argument, Cochrane (2005b, p. 518) further notes that “[b]udget constraints do not respect market clearing conditions,” which can be underpinned by a simple example: If the budget constraint reads ‘price · quantity = 200’ and the price turns out to be 100, feasible demand is two units. If, however, there is only one unit of the good in question, satisfaction of both market clearing and the budget constraint can only occur at a price of 200.

Kölsch Consider the following scenario: In the terminal period, outstanding nominal consolidated-government liabilities in the amount of 100 € are institutionally backed by 100 bottles of Kölsch beer (the only good of this cheerful economy), stored in a treasury warehouse or the central-bank vault. Obviously, the equilibrium price level is 1 € per bottle.

If the auctioneer instead announced the off-equilibrium price of 0.50 € per bottle, the final exchange would be 100 beers against 50 € of nominal liabilities. Households can either passively accept that they are left with 50 € which are useless to them or signal that they would be willing to trade these liabilities against Kölsch, possibly also at prices less favorable to them. This is “money chasing goods” and might make the auctioneer reconsider the announced price. Conversely, if the auctioneer announced 2 € per bottle, the final exchange would stop after 50 beers, which is when households run out of nominal consolidated-government liabilities to trade with. The government is left with 50 bottles which are useless to it.

Markets do not clear and households could increase utility via the appropriate trades, so this is not an equilibrium. However, nothing forces—or *constrains*—the government to engage in these beneficial exchanges. The fact that either households or the consolidated government keep something that is worthless to them is not impossible, it “must be reflected in preferences, not constraints.” (The author of these quotes is a bit more extensive in this argument, see Cochrane 1999, Section 2.3.4; 2005b, Section 3.2. A combination of this example and the above reasoning with respect to *Derivation* also corresponds to an explanation given by Obstfeld and Rogoff 1996, p. 65.)

9.2.3. End of Discussion

Taking Sides It may be clear from the way the previous section is written, but in my opinion, the argument about the \nearrow *Derivation* of the present-value budget equation overcomes concerns about identities being violated. It is an equilibrium condition.

Additional Voices Critics and proponents of the fiscal theory are already named in the respective subsections. However, there are some authors who argue in such a differentiated fashion that it is hard to assign them to either group. For instance, Daniel (2001b, p. 298) posits that the present-value budget equation “is an independent equilibrium condition only if policy is non-Ricardian.” Given all of the above, however, this seems like an acknowledgement that it actually is an equilibrium condition combined with the oversight to realize that, just because Ricardian fiscal policy ‘willingly’ sees to its satisfaction, it is not less of an equilibrium condition given such a fiscal policy (cf. Woodford 1996, p. 24, who writes something very similar to Daniel 2001b, but his surrounding explanations clarify the matter).

Is the Government Special, and Why? There are several reasons why the government could be considered special with respect to the discussion above. One is that it is a large agent with the power to change prices instead of just taking them as given (cf. Woodford 2001, p. 693). The crucial point, however, is best made in two steps (cf. Woodford 2001, pp. 693, 695-696; Cochrane 1999, p. 337; 2005b, pp. 516-517):

- As mentioned before (\nearrow *Promising What?*, p. 165), and insofar as it does not forego this possibility for some reason, the consolidated government in a fiat-money economy relies entirely on liabilities it can freely create itself by issuing new to roll over old debt or by honoring the commitment that nominal treasury debt stands for and exchanging it for money—“[t]here is thus no possible doubt about the government’s technical ability to deliver what it has promised; this is not an implausible reason for financial markets to treat government debt issues in a different way than the issuance of private debt obligations.” (Woodford 2001, p. 693) This is why the consolidated government does not necessarily have to adjust its plans to a constraint.
- Of course, by the stock analogy of Section 9.1.2, something similar holds for a company that finances itself with equity, at least in principle. The difference between a firm and the consolidated government is that the ultimate liability of the latter—money or, in a cashless economy, nominal debt—is typically also the unit of account for the broader economy. If prices were expressed in terms of a certain company’s stock or a certain household’s IOU, we would arrive at Buiter’s (2002, p. 477) “[company or] household intertemporal budget constraint theory of the

price level.” But since they are not, only the budget equation of the consolidated government has the potential to determine the price level.

One of the reasons to deprive themselves of the convenience described in the first point is of course connected to the ramifications of mistaking it for endless possibilities: Consolidated governments debasing nominal debt too much too often might see themselves forced to ‘go real’ by issuing debt in, or even altogether adopting, a foreign currency.

Regarding the unit of account, a somewhat circular argument can be added: If the consolidated governments is special enough to be able to tax, it can choose to accept only its own fiat money. This deters company shares or some random household’s liabilities from becoming unit of account, reinforcing ‘specialness.’ (Cf. Cochrane 2005b, p. 502; see also *Frictions: An Argument About Form*, p. 172.)

As a final note: Insofar as a (consolidated) government relying on nominal debt is special, one has to be very careful in applying the Modigliani and Miller (1958) theorem to the macroeconomy (cf. Marimon 1999, for instance) because there is no analogue to corporate debt.³⁰

9.3. What Distinguishes the Fiscal from ‘Traditional’ Theory?

9.3.1. How to Achieve Determinacy

Multiple Equilibria Buiter (2002) gives a very explicit account in the context of a finite-horizon model with money and comes to the conclusion that there are “multiple equilibria for the general price level sequence” (p. 470). Splitting up the money-demand equation into two parts—one for the final period J (after which no money is held anymore so that inflation drops out of the equation) and one for all previous periods $t < J$ (in which inflation is relevant)—he finds that *one* equilibrium first determines P_J as a function of M_J in the final period J and then moves back through time to determine all other P_t given the respective M_t . Be that as it may, there are up to J additional equilibria in which money is not valued (because of an infinite price level) in some of the final periods.

Equilibrium Selection Although Buiter (2002) does not state this explicitly, it seems obvious that the existence of a ‘meaningful’ (that is, for instance, finite) price-level sequence P_t depends on the characteristics and the public’s knowledge of the money-supply process M_t . Woodford (1995) is more explicit in asserting that one can “solve

³⁰ Since it might spring to mind immediately: Wallace’s 1981 paper titled “A Modigliani-Miller Theorem for Open-Market Operations”, and especially its modern-day application, ‘Wallace neutrality,’ emphasize the existence of a *consolidated* government sector, a notion that is taken as given here. In any case, they do not argue whether the present-value budget equation is a constraint or an equilibrium condition.

for P_t given the *sequence* M_t " (p. 15, emphasis added, variables in my notation). More generally, he associates with quantity-theoretic works the notion to add some additional requirement that helps select the 'correct' one out of a multitude of possible solutions (pp. 14, 23). These deliberations hint at the possibility to distinguish two slightly different approaches in quantity-theoretic literature:

- One is to equip the respective institution with a policy rule for the variable in question. To give an example also referred to by Woodford, McCallum (1989, pp. 148-155) analyzes a rational-expectations version of the Cagan (1956) hyperinflation model and shows that P_t can be uniquely determined if the money-supply process M_t follows a rule that agents can include in their expectation-formation scheme formally to eliminate the $\mathbb{E}_t P_{t+1}$ component and thus obtain a closed-form solution.
- Another option is to exclude solutions in which real money either grows without bound or asymptotically approaches zero (cf. Woodford 1995, p. 14). In the rational-expectations context proposed here, it seems that this approach hinges on the coordination of expectations towards the simplest, or 'most convenient,' equilibrium. (At least this is a possible reading of Woodford 2001, who often avoids strong assertions or adds some qualification to the proposed expectation coordination, see his pp. 701, 710, 712, 717, 719. It has to be noted that he argues in favor of fiscal as opposed to quantity-theoretic price determination, but the 'argument of simplicity' could just as well be used as a selection device only within the smaller subset of quantity-theoretic equilibria.)

The main difference between traditional (quantity) and fiscal theory is therefore that the former selects "the 'correct' solution on the basis of *some* criterion that does not involve reference to the behavior of variables other than the money supply and the price level" (Woodford 1995, p. 23, emphasis added), i.e., which is "not derived from money demand, optimization, or any other principle" (Cochrane 1999, p. 349), while the latter recognizes that the 'correct' equilibrium can be found by also taking fiscal variables into account.

9.3.2. Interest-Rate Pegs

Sargent and Wallace (1975) provide the "familiar result" (Buiter 2002, p. 472) that interest-rate pegs lead to indeterminacy. McCallum (1981) shows that this outcome depends on autonomous policy; as soon as the interest rate is made to depend (even in the slightest) on other endogenous variables, it vanishes. By contrast, the results obtained in Section 5.1.5 lead to determinacy even if the interest-rate peg is "pure" (Woodford 1995, p. 33), that is, even if the rate is not changed *at all*. (Much earlier, Begg and Haque 1984

as well as Auernheimer and Contreras 1990 are credited with obtaining similar results; unfortunately, their papers are hard to obtain nowadays.)

9.3.3. The Proportionality of Money and Prices

General Comparison In Sargent and Wallace (1981), there is a strong connection between money and prices. Monetary tightening is temporary at best because after some (or even no) time, monetization takes over (potentially casting a long shadow in the form of 'inflation already now', see *↗Tighter Money Now, Higher Inflation Now* in Section 3.5).

By contrast, the fiscal theory of price determination severs this connection. The case studies in Section 5.1.3 demonstrate that movements of money and prices in opposite directions (as in the *↗Tighter Money Now, Higher Inflation Now* case of the unpleasant monetarist arithmetic, Section 3.5) can be induced by *one-off* yet *permanent* changes in money supply. What is more, the price level can change even if there is no deviation in money supply at all, as exemplified by the 'helicopter debt' case study (Section 5.1.3.4). Ultimately, all that is necessary to exert pressure on prices is a mere change in *expectations* about the right-hand side of the present-value budget equation $\langle 2.43 \rangle / \langle 5.4 \rangle$ (cf. Woodford 2001, p. 684).

So, to clearly separate the Fiscal Theory of the Price Level from 'traditional' results, McCallum and Nelson (2005, p. 566) suggest that "only cases in which the price-level path veers away from the path of the money stock should be regarded as reflecting a *bona fide* FTPL." The notion of a "strong-form [fiscal theory]" (Carlstrom and Fuerst 2000, p. 23) goes in the same direction.

Hyperinflation Cochrane (1999, pp. 356-357) compares alternative explanations of hyperinflation, noting first that a rapid increase in money supply evokes this phenomenon in the fiscal just as in the quantity theory. Then he goes on to a thought experiment in search of the differences: In case the consolidated government issued (ultra-)short-term debt instead of money (which is assumed to be kept constant) to finance its budget deficits, the quantity theory predicts price stability whereas the fiscal theory still predicts hyperinflation. Conversely, the explosive creation of *inside* money by private issuers such as banks in a situation of solid public finances as well as stable base money and debt would cause a hyperinflation in the quantity but not in the fiscal theory.

9.3.4. How Strong Is the Influence of Expectations on Current Prices?

Expanding on his examination of *↗Hyperinflation*, Cochrane (1999, p. 357) argues that the influence of expectations about future budgets has different strength in the quantity and the fiscal theory.

Weak(er) in the Quantity Theory With transmission running through seigniorage in the quantity theory, they mainly drive future price levels and inflation. The current price level is only affected insofar as expected inflation curtails “Cagan-style” money demand, which is associated with a rather low discount factor of around 0.15.

To make this clear, I borrow his money-demand equation (30)

$$\ln M_t = \ln P_t + \ln y - O_{VII} (\ln r + \ln \mathbb{E}_t P_{t+1} - \ln P_t).$$

(His p. 349, my notation. The reason for doing so is that replicating Cochrane’s formal part of the argument with the specifications used in the corresponding part of this thesis—Chapter 3, in particular utility function (3.12) and the resulting money-demand function (3.13)—is impossible because they would not yield a closed-form solution.) Rearranging this into a difference equation in P (not shown) then leads to the solution

$$\ln P_t = \sum_{j=0}^{\infty} \left(\frac{O_{VII}}{1 + O_{VII}} \right)^j \frac{1}{1 + O_{VII}} (\ln \mathbb{E}_t M_{t+j} - \ln y - O_{VII} \ln r).$$

The solution is finite for typical interest elasticities of money demand O_{VII} , which he places at the mentioned value of 0.15. Goldfeld (1987) and Hoffman and Rasche (1991) report values of 0.1-0.2 and 0.4-0.5, respectively. See also Lucas (1988) for, in part, higher values.

Strong(er) in the Fiscal Theory By comparison, the stochastic discount factor v on future budget deficits on the right-hand side of the present-value budget equation (2.43) is rather high, usually around 0.95 or more depending on the interest rate. Future events can therefore have a much stronger influence on current price levels in the fiscal theory.

9.3.5. Private Transactions vs. Tax Payments: The Value of Money

Frictions: An Argument About Form This remark might be denigrated as representing the opposite of the ‘form follows function’ credo, but giving money or monetary policy a role in the models that matches their perceived importance in real-world affairs is a complicated thing in modern economics. As Cochrane (2005b, p. 503) puts it: “Throughout economics, frictionless competitive models are the benchmark, the foundation upon which we add interesting frictions. Yet monetary economics has so far crucially relied on a big friction at the short end of the yield curve in order even to start talking about a price level.” Following his argument, the fiscal-theory literature offers a nice solution as it is able to determine the price level even without money demand or without money at all (cf. Cochrane 2005b, p. 506). Cochrane (1999, pp. 324, 329, 348, 354-355) deems this a better representation of the actual (U.S.) economy than a model with very strict

separation between money (used in transactions) and bonds (used for saving).

Taxes for Money Nonetheless, if money is to be there, he seems to favor the simple approach of giving it value by making it a way to pay taxes (cf. Starr 1974, tracing it back to Adam Smith later in Starr 2003, p. 470) over the quantity theory. This property seems to make a fiscal theory of price determination especially convenient in the face of financial innovation that threatens (or blurs) the “special demand for transactions-facilitating assets” (i.e., money; Cochrane 2005b, p. 502). “The crucial change [...] is that an excess of cash is measured relative to tax liabilities that soak it up, not relative to a transactions-based demand” (Cochrane 1999, p. 331).

Backing The ideal commodity standard promises a fixed parity between the means of payment and a certain commodity ‘backing’ it (gold for instance). When a commodity standard is dropped, the promise to exchange currency for the backing assets vanishes. Following Cochrane (1999, pp. 332-333), the quantity theory does have an explanation for the (surprisingly?) stable price level in the United States after the gold standard had been dropped: The supply of money was already in line with the need for transactions-facilitating assets of the public. The implicit question is whether this monetarist explanation eventually disqualifies (if not then, maybe now or some time soon given the pace of financial innovation).

The alternative interpretation is that the intertemporal fiscal policy stance had already been such that the price level did not have to change dramatically. The fiscal theory relies on the backing of government liabilities—with great resolve: It is not the stock of a single arbitrary commodity that guarantees the (real) value of consolidated-government liabilities but the entire (prospective) income stream of the public sector.

Using the example of exchange-rate pegs, this leads Cochrane (1999, p. 330) to assert that they “do not fall to speculative attack when the government ‘runs out of reserves’; they fall apart when the government becomes unable or unwilling to *buy* reserves.” Solid consolidated governments can support a fixed exchange rate with very little reserves. By contrast, if the taxing capacity is considered too small, the consolidated government might not be able to acquire the foreign reserves necessary to defend the peg anymore; this is even with a filled ‘war chest.’ Of course, this insight is not the genuine contribution of Cochrane; following Sargent (1982b, pp. 45, 91), it goes back at least to Keynes (1924; 1925).

The latter interpretation proposed by Cochrane (1999) leads to the conclusion that fiat money must be backed by fiscal policy as well, in the sense that its real value will decline otherwise (cf. also Sims 1997, p. 4).

9.3.6. The Fiscal Theory vs. the Unpleasant Monetarist Arithmetic in Particular

Real vs. Nominal Debt Sargent and Wallace (1981) assume real debt whereas the fiscal theory considers nominal debt. The obvious qualification for the latter is that it *can* behave as if it were real (in the monetary-dominance regime, cf. Section 9.1.1; cf. also Leeper and Walker 2012, pp. 14-15).

Regime In the unpleasant monetarist arithmetic, the fiscal authority sets budget deficits/surpluses autonomously while the central bank conducts money-supply policy (cf. Section 3.5). The core result is that the latter might not be able to control inflation for too long: There is either a trade-off between less inflation now and more inflation later or an even more unpleasant constellation in which tighter monetary policy (read: a lower money growth rate) leads to higher inflation rates immediately. This setup as well as the outcome resemble the passive-monetary/active-fiscal regime laid out first in Chapter 4.

Contradicting the Grumpy Economist Cochrane (2005b, p. 523) claims that “Sargent and Wallace’s indexed debt is equivalent to nominal debt and a Ricardian regime.” Considering the above notes about real vs. nominal debt, policy regimes, and the functioning of the model, it is hard to support this statement: In Sargent and Wallace (1981), ‘contractually real’ (indexed) debt is financed by generating seigniorage through money creation because the fiscal authority is behaving actively, that is, refusing to adjust surpluses. Applying the *↗The Bohn-Woodford Criterion* (Section 4.4.1), this clearly constitutes non-Ricardian fiscal policy. In contrast, the fiscal theory can only ‘simulate’ real debt by assuming Ricardian policy, but it then has to be paired with active monetary policy—read: no (fiscally induced) inflation—in order to yield sensible results. Producing a similar outcome as the unpleasant monetarist arithmetic (exogenous surpluses and a strong movement in prices) is only possible through non-Ricardian fiscal and passive monetary policy in the fiscal theory, and then it is achieved through a one-time wealth effect rather than an attempt to generate seigniorage (cf. Leeper and Leith 2017, p. 2321). The difference in transmission is also emphasized by Ljungqvist and Sargent (2012, pp. 1060-1062) who call the unpleasant monetarist arithmetic a “fiscal theory of *inflation*” and the modern literature (referencing Woodford 1995 and Sims 1994) a “fiscal theory of the *price level*.”

Abandoning Seigniorage Picking up the above argument, Chapters 4 and 5.1.3, respectively, demonstrate that the fiscal theory still works if there is no seigniorage to begin with or if the government rebates it back to households. This is also what makes it more appealing from a practical standpoint because, as many commentators note, seigniorage revenues play a miniscule role in developed countries (cf. King 1995, p. 171; Woodford 2001, pp. 670-672, 684; Leeper and Walker 2012, pp. 14-15, for instance; see also Section

3.4.2).

9.3.7. On Ricardian Equivalence

In Itself The notion that financing a budget deficit by issuing bonds or reducing it by raising taxes are equivalent and, therefore, that the choice between the two options is irrelevant for real variables (such as consumption, real rates, or wealth) of course traces back to Ricardo (1821, Ch. 17). Barro (1974) is often credited with kicking off the discussions that are relevant nowadays, while the term 'Ricardian equivalence' stems from Buchanan (1976, cf. Abel 2008).

In the 'Unpleasant Monetarist Arithmetic' With the main findings of Sargent and Wallace (1981) in mind, Sargent (1982a, p. 385) calls fiscal policy Ricardian if "the issuing of additional interest-bearing government securities is always accompanied by a planned increase of explicit tax collections just sufficient to repay the debt," whereas the opposite regime (which is not explicitly called 'non-Ricardian' yet) consists of financing additional government debt via seigniorage revenues. The same distinction is also made by Aiyagari and Gertler (1985).

In the Fiscal Theory In the younger approach, Ricardian fiscal policy is classified by the criterion that it ensures satisfaction of the transversality condition for all paths of the goods and asset price levels (cf. Sections 4.4, 5.1.4, and Appendix C.2). Woodford (1995, p. 26) effectively eliminates money from the consolidated-government budget equation by giving seigniorage revenues back to the household via tax rebates; therefore, the respective transversality condition actually only applies to treasury bonds B . However, Woodford (2001, pp. 690-691) clarifies that the 'broader' case in which the transversality condition applies to total government liabilities (as in Equation (2.42)) is "conceptually preferable." This is because, given a Ricardian fiscal policy, the present-value budget equation (2.43) holds in any case, so the transversality condition necessarily holds as well and adds nothing to equilibrium determination anymore. Naturally, behavior of the treasury is similar to the Sargent quote above: if, for whatever reason, real consolidated-government liabilities increase, the treasury (at some point in time) reacts by raising taxes.

9.3.8. Precursors

The importance of fiscal policy for prices presented itself to economists not only when Sargent and Wallace (1981) did some unpleasant monetarist arithmetic or even with the literature emerging in the 1990s (*inter alia* Leeper, Woodford), but much earlier. However, principles that are similar to the 'modern' fiscal theory were often discarded as special or unrealistic cases. For instance, Brunner and Meltzer (1972, pp. 953, 973) are

said to foreshadow the fiscal theory by showing that deficits can drive up prices even (or especially) if money is kept constant (which resembles the ‘helicopter debt’ case study in Section 5.1.3.4) and, at the same time, do not really believe in it. Wallace (1981) is given as a second example: Like the monetary-fiscal theory, his paper also describes a situation in which the effects of a policy move of one agency (the monetary authority in this case) depends on the policy design of the other. (Cf. Leeper and Leith 2017, pp. 2309-2310.) Similarly, Woodford (1995, pp. 13, 30-31) already sees some of the arguments about wealth effects and the (consolidated) government not being constrained by given prices in Patinkin (1965) and Brunner and Meltzer (1976), although not in a form as ‘pure’ as the theory presented by him (and here).

9.4. Convenience and Desirability of Fiscal Price Determination

9.4.1. Cochrane’s Fundamental Critique of the Conventional Wisdom

Focus Cochrane (2011) argues that one of the generally accepted notions about equilibrium determinacy in the monetary DSGE universe is fatally flawed. However, pretty much in the beginning, on pp. 569-570, he concedes that his arguments are mostly negative, and indeed, he does not lay out a clear alternative with the same confidence found in his objections to conventional wisdom, which is why this summary of his remarks ends up here as a note rather than in Chapter 4 as a ‘main attraction.’ Nonetheless, this does not take away anything from the gravity of his assertions. I focus on the theoretical challenges of determinacy, but his paper also puts forward some interesting and arguable issues concerning empirical identification.

Nominal Explosions At the heart of his critique lies the fact that the Taylor principle induces explosive behavior in inflation. Following the prevailing doctrine, it was assumed in Section 4.2.3 that a diverging inflation path would lead agents to abstain from considering it as an equilibrium, but Cochrane rightly argues that there are no *economic* grounds for *picking* the unique *stable* equilibrium out of the multitude of candidate equilibria, all of which are valid for they completely satisfy the household optimality conditions: “Transversality conditions can rule out *real* explosions but not *nominal* explosions” (p. 566). Accepting this, inflation can not be determined by excluding explosive paths of Equation (4.6) and is hence indeterminate. In particular, he contests the ‘future-expectations-to-current-realizations’ direction of equilibrium formation promoted by Woodford (2003b):

“The equations of the model do not specify a causal ordering. They are just equilibrium conditions. [...] *If you see a small change today in an unstable dynamic system, your expectations of the future may well change by a large amount. If you see the waiter trip, it is a good bet that the stack of plates he is carrying*

will crash. In new-Keynesian models, agents might well see a disturbance, know the Fed will feed back on its past mistakes, think ‘oh no, here we go,’ and radically change their expectations of the future. They do not need to wake up and think ‘gee, I think there will be a hyperinflation’ before reading the morning paper.” (Cochrane 2011, p. 582, emphasis added.)

Stabilization vs. Equilibrium Selection What follows reads like a fundamental critique of the entire New-Keynesian DSGE literature, or at least its mainstream interpretations set out in the conventional regime with active monetary and passive fiscal policy. One of the common threads of his paper is the distinction between inflation stabilization and equilibrium selection. In short, the former is associated with sensible policy while the latter amounts to ‘blowing up the economy.’ Equilibria in which unbounded inflation can be stopped by (sensible) policy are *not* ruled out, so indeterminacy prevails. If this is to be avoided, policy must prevent the respective equilibria from materializing instead of stopping explosive processes—that is, it must threaten to *create* explosive processes and thus blow up the economy.

Sensible Policy & Market Forces Cochrane argues that many of the proposals in this regard are inconsistent or outright impossible. One such example is introducing a commodity standard in the face of high inflation while still trying to adhere to an active Taylor rule. (Cf. pp. 566–568, 583–586).

At the same time, he suggests there *are* ways to achieve determinacy which rely on market forces and plausible policies. In (plain) commodity standards or the fiscalist regime of modern DSGE models, for instance, market clearing can only obtain under the circumstances implied by consolidated-government policy, which “gives a strong supply-demand force toward the equilibrium price” (p. 568, see also pp. 570, 578–579, 580 and ↗Kölsch, my p. 167; cf. Obstfeld and Rogoff 1983, pp. 684–685; 1986, p. 354, for the commodity standard). By contrast, no such market-forces argument can be made in the conventional active-monetary/passive-fiscal regime (cf. pp. 567, 580).

Old- vs. New-Keynesian Logic Another distinction important to Cochrane is that between what he calls “old-Keynesian” and “new-Keynesian” models and logic (cf. pp. 566, 572). Much of this is mere repetition by now: The old-Keynesian story argues via stabilization, that is, central banks raise nominal interest rates more than one-for-one in response to increasing inflation so as to push up the real interest rate, depress aggregate demand, and eventually bring inflation back to target. By contrast, in new-Keynesian models, increasing the nominal interest rate would bring about even higher inflation over time and lead the economy on an explosive path (cf. Section 4.2.3/Figure 4.1). Ruling out such explosions (even if there are no economic grounds for doing so) only leaves a single value that remains stable, to which the inflation rate then jumps.

Taking the model of Taylor (1999) as an old-Keynesian example, however, Cochrane (2011, pp. 601-604) reveals that one has to be careful in associating models with stories. In Taylor's model, there are no forward-looking terms, which is why the dynamics change completely: A sufficiently large Taylor coefficient ($\gamma_{\pi}^C > 1$) ensures *stability* in a *backward-looking* solution procedure, whereas new-Keynesian models rely on *instability* of the system so as to obtain a *forward-looking* solution (recall the terminology from *↗Saddle-Path(-Like) Stability*/Footnote 12, p. 80). The difference is that models like that of Taylor are always determined and 'only' have to avoid spiraling out of balance after shocks, whereas these shocks make new-Keynesian models jump to an entirely new (and, following Cochrane's argument, undetermined) equilibrium. In addition, backward-looking solutions that solely depend on exogenous disturbances do not seem satisfactory when "the whole point of the new-Keynesian enterprise is to microfound behavioral relationships [...] driven by expectations of the future, not memory of the past" (p. 603). For these reasons, Cochrane cautions not to use old-Keynesian stories to rationalize new-Keynesian models.

Ricardian Asymmetry To summarize with regard to the 'convenience and desirability of fiscal price determination,' Cochrane's (2011) paper more or less explicitly tends to favor the fiscalist over the conventional regime. The principal cause of this "Ricardian Asymmetry" (p. 580) is that fiscal price determination works through the transversality condition (as a representative of the entire set of optimality conditions) whereas achieving determinacy in the active-monetary/passive-fiscal regime relies on arguable additional assumptions. In this light, one might even begin to consider the fiscal theory to be more straightforward than the conventional rationale.

Reception In a short reply, Obstfeld and Rogoff (2017) cast doubt on Cochrane's conclusions, especially with respect to the fiscal theory as a possible solution, noting that multiple equilibria could also arise from fiscal policy rules. Whether the consolidated-government budget equation really "*depends* on seigniorage revenue, which in turn is driven by the demand for money and hence [...] by expectations" (p. 13, emphasis added), making it prone to multiple equilibria as well, seems arguable considering the possibility of a seigniorage rebate (cf. Chapter 5.1 and 9.3.6 in Section 9.3.6). Then again, "given the reality of seigniorage from money creation" (p. 15), this argument can not be easily discarded either. McCallum (2009a) utters critique based on an argument about *↗Learning* (p. 179 in Section 9.5).

9.4.2. Further Issues

Gateway to Profligacy? Allowing for the theoretical possibility of a fiscalist regime in modern DSGE models is not tantamount to having a profligate consolidated-government

that sends the economy into hyperinflation any minute now. Maybe this notion is best explained by the fact that episodes of dramatical (hyper-)inflation are often tied to fiscal roots. Be that as it may, “[b]ad economic policies can produce bad economic outcomes in any policy regime” (Leeper and Leith 2017, p. 2391)—that is, also in the conventional regime with active monetary and passive fiscal policy. Turning to a counterexample, Cochrane (1999) presents an explanation of postwar U.S. data that rests on fiscal price determination and a consolidated government which uses its liabilities to stabilize inflation (cf. *↗Inflation Smoothing* on p. 157).

Technical Similarities Both the fiscal and quantity-theoretic approaches to equilibrium determination critically depend on a specific ‘technical’ requirement: the definition of an entire sequence of a policy variable, either as an exogenous process or (more realistically) as a policy rule. While Sections 5.1.2-5.1.3 demonstrate that the budget surplus process can indeed exert a strong influence on the price level, the fiscal approach is quite similar to the quantity-theoretic one in that it also requires the public to analyze the policy process with the ensuing conclusion that it leads to a unique equilibrium. Simply put: Just as the representative household has to know the entire money-supply sequence in the quantity theory (cf. the first bullet point on p. 170), it has to know the entire sequence of budget surpluses in the fiscal theory. This begs the question which policy process is easier to understand and forecast for the general public.

9.5. Omissions and Possible Extensions

Learning There is widespread approval of the notion that rational-expectations equilibria are only plausible if they are learnable in the sense of Evans and Honkapohja (1999; 2001), that is, in an iterative process of adaptive expectation formation that converges to the rational-expectations solution. However, opinions as to which of the two polar regime satisfies this criterion (typically with the accompanying result that the respective other does not) are divided and discussed intensively.

“To save the conventional wisdom,” as Canzoneri, Cumby, and Diba (2011, p. 947) put it (hence implicitly concurring), McCallum (2009a) counters Cochrane’s (2011) arguments about nominal explosions (cf. Section 9.4.1) by stating that the respective equilibria are not learnable and should therefore be discarded, leaving only the stable solution. This exchange goes into a second round: Cochrane (2009) argues that agents can not obtain the information required for McCallum’s learning process (the Taylor coefficient γ_{π}^C , cf. Section 8.1) and that the threat of the Fed to embark on an ‘unlearnable’ path would not “‘coordinate expectations’ on anything other than confusion” (p. 1113). Finally, the “rejoinder” of McCallum (2009b) is not as forceful as the title might suggest, offering a rather conciliatory tone. Still, consensus is not achieved.

MSV & Learning A related discussion revolves around the learnability of different equilibrium concepts. Since rational expectations on their own often lead to multiple equilibria, McCallum (1983; 1999) argues for a ‘minimal state variable’ solution which relies on the method of undetermined coefficients (Lucas 1972) and is constructed so as to lead to a unique solution. In McCallum (2001), he describes a model with constant money supply and fiscal policy similar to that of Chapter 5.1 and finds that the fiscalist does not correspond to the minimal-state-variable solution; then, in McCallum (2003a,b), he extends this analysis by showing that the fiscalist solution is not learnable in the money-instrument model (while the ‘monetarist’ minimal-state-variable solution is).

As Evans and Honkapohja (2007) show, however, the learnability criterion mirrors the determinacy results presented in this thesis (see Chapter 4.3) when a setup such as Leeper’s (1991) with the nominal interest rate replacing money supply as the central bank’s policy instrument is adopted: If a unique rational-expectations equilibrium exists—be it conventional or fiscalist—it is learnable; if there are multiple equilibria, none of them are learnable. This puts the above result from McCallum (2001) into context: Considering typical money-supply policy as active policy in the sense of Chapter 4.2.2 (Sims 1999b, p. 419, does so) makes the model doubly active, which is often associated with explosions (see Section 4.3.3.1). The latter are often not learnable.

McCallum (2003b, pp. 1171–1172) also obtains these results, but chooses to believe only in the regime with active monetary policy; in other (his own) words, he discards the fiscal theory “[f]rom a practical perspective.” Woodford (2003a) voices strong disagreement. While learnability is not the same as determinacy (cf. also Bullard and Mitra 2002), he claims that it supports the latter much more than McCallum’s minimal-state-variable solution. Another severe accusation is that McCallum (2003b) rules out learnable effects of public debt on the price level via his choice of a forecasting algorithm, that is, *by assumption*.

Indexation Loyo (1999, p. 16) argues that, in practice, indexed bonds are not really real bonds. Since price indices are not available in real time but only with certain lags, indexed bonds should rather be seen as nominal bonds whose value is adjusted to lagged inflation. Determination of the price level then occurs in the same way as for normal nominal bonds.

However, this does not even seem to be the most important point. As discussed, *inter alia*, in Section 5.1.4, it is the total amount of all consolidated-government liabilities that is relevant in the present-value budget equation. Therefore, even if ‘really real’ bonds had a share of 100% in the *bond* portfolio, the existence of nominal money M could still save the fiscalist approach.

Long-Term Debt First, note that the maturity structure of debt is irrelevant for inflation under monetary dominance because the latter implies Ricardian equivalence (cf. Leeper

and Leith 2017, p. 2326, see also Section 4.4).

According to Woodford (2001, p. 685), the main difficulty in including longer-term bonds into the model (aside from greater algebraic complexity in general) is that the level of outstanding nominal consolidated-government liabilities Z is not predetermined in the familiar way anymore because it also depends on the current bond price Q . In the context of his paper—the Fed’s bond-price support of the 1940s (see Section 8.3)—however, the latter is constant via Equation (2.8) because the nominal interest rate i is pegged so that fiscal price determination through the present-value budget equation works in the usual way.

More generally, long-term debt interacts with monetary policy in determining bond prices as well as price-level and inflation sequences. This happens via maturity and the Taylor coefficient, respectively. In a rather verbal than formal description, the present-value budget equation (4.4) can be reformulated to include longer-maturity debt as

$$\frac{\{\text{price of bond portfolio}\}_t \cdot \{\text{bond portfolio}\}_t}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \quad (9.2)$$

(the choice of this specific present-value budget equation is for notational simplicity, the argument extends to all other variants as well). In this equation, there appears another variable on the left-hand side that can absorb exogenous changes of surpluses on the right-hand side, namely, the bond-price expression (as a ‘price variable;’ the bond portfolio as the ‘quantity variable’ is still predetermined even though it carries a time index of the current period t).

How are effects from the right-hand side (such as a lower present value of surpluses) distributed to the two adjusting variables? By a combination of two effects:

- The size of the Taylor coefficient determines expected inflation: Setting $\gamma_{\pi}^C = 0$ anchors inflation expectations at the target rate ($\mathbb{E}_t \pi_{t+j} = \pi^* \forall j$ in Equation (4.6) with $\varepsilon_t^i = 0$). Note that this also pegs bond prices. At the other end, setting the Taylor coefficient to the maximal value for which monetary policy remains passive, $\gamma_{\pi}^C = \beta^{-1}$, anchors expectations at the current rate ($\mathbb{E}_t \pi_{t+j} = \pi_t \neq \pi^* \forall j$, it increases in the case of lower surpluses indicated before).
- Prices of bonds with maturity longer than one period react not only to current but also to expected nominal rates or, equivalently, future inflation. At this, maturity acts like an inverse discount factor so prices of portfolios with shorter maturity react less strongly than those with higher maturity.³¹

³¹ From a technical perspective, long-term debt is often modeled in the form of perpetuities with geometrically decaying coupons, which allows to vary the duration of the portfolio. Helpful explanations are

Ceteris paribus, a given increase in current inflation $\Pi_t \equiv P_t/P_{t-1}$ can lead to different decreases in bond prices, depending on the Taylor coefficient and maturity. So for any decrease of the right-hand side of the above equation, it is monetary policy which, by choosing the Taylor coefficient γ_π^C , determines how much weight is carried by bond prices in the numerator and the price level P_t in the denominator of the above equation, respectively. To restate, pegging the interest rate and thus bond prices rate puts all of the pressure on the current price level (on current inflation) whereas increasing rates in the strongest passive way possible decreases bond prices and thus spreads out inflation more evenly. To restate again, longer maturities and a higher Taylor coefficient lead to lower bond prices and trade-off lower current for higher future inflation. The ‘present-value’ effect is the same, of course, as the impulse coming from the right-hand side of the equation is also the same in both polar extremes.³² (Cf. Leeper and Leith 2017, pp. 2326–2328. Further references on long-term debt in the fiscal theory are Cochrane 1999; 2001; 2014.)

Regime Switching Regimes are not cast in stone. A strand of the literature that recognizes this awards policies with the possibility to change (and change back) stochastically over time. The contributions of Davig and Leeper 2006; 2011; Chung, Davig, and Leeper 2007; Bianchi 2012; Bianchi and Ilut 2014 are important, but also tend to muddle the results. At this point, I conclude by pointing to the related argument by Canzoneri, Cumby, and Diba (2001b) that adjusting surpluses to debt ‘every once in a while’ may be sufficient to implement a Ricardian policy (see *The Bohn-Woodford Criterion*, p. 77); something similar seems to hold in the case of regime switching.

Optimal Policy I only consider simple *ad-hoc* rules for monetary and fiscal policy, but everything discussed here could of course also be cast within optimal-policy frameworks. These quickly develop a larger scope both with regard to subject matters and in their formal representation. Often, a social welfare (or loss) function for policy is derived from the utility function of the representative household (Woodford 2003b, Chapter 6; Walsh 2010, Chapter 8.6.2, for example, describe this approach using second-order approximations). The question of optimality also relates to the degree of price stability; in particular, inflation is put into the context of other distortionary taxes. This literature on

given in Woodford (2001, p. 685), Bhattarai, Eggertsson, and Gafarov (2014, pp. 38–39), and Leeper and Leith (2017, pp. 2325–2326), for instance.

³² These findings provide for an interesting side note on the “Operation Twist” carried out by the Fed in 2011, which aimed at depressing long-term rates on treasury bonds by selling short-term in exchange for long-term bonds in the hands of the public: From a fiscalist perspective, such a maturity shortening should lower price of bond portfolio_t and thus require P_t to *fall* in order to keep the left-hand side constant in the face of an unchanged present-value of surpluses—exactly the opposite of the intended effect. (Cf. Leeper and Leith 2017, p. 2324.)

the ‘optimal inflation tax’ (or ‘optimal quantity of money’) spans from Friedman (1969) over Phelps (1973) and Lucas and Stokey (1983) to Chari, Christiano, and Kehoe (1991), to name only a few. More recent variants, in part with sticky prices or a greater focus on fiscal policy, include Benigno and Woodford (2004), Schmitt-Grohé and Uribe (2011), and Correia, Nicolini, and Teles (2008).

Strategic Games The present thesis is concerned with what happens *given* a certain regime. But how does it arise? Answers to this question could be derived from modeling the coordination between monetary and fiscal policy as strategic games, that is, as outcomes of optimal mutual reactions of two non-cooperative policymakers (which distinguishes this strand of the literature from ‘simple’ optimal policy mentioned above, which typically considers *jointly optimal*, or cooperative, behavior). Work headed in this direction is done by Bassetto (2002) and, more generally, Bassetto (2005). Dixit and Lambertini extend the Barro and Gordon (1983) model to monetary and fiscal policies in general (2000, 2003) and in monetary unions (2000, 2001, 2003); this line of research does not directly originate from the fiscal-theory strand, but is closely related and aware of the latter. An important distinction, however, is that they argue mainly in terms of policy targets (the similarity of preferences of both authorities) while most of the discussion echoed here revolves around reaction parameters in policy rules that in most cases only feature ‘their respective’ target variables (inflation but no liabilities for the central bank and liabilities but not, at least not directly, inflation for the treasury).

Default Several authors criticize that typical renderings of the fiscal theory, which is related to debt after all, do not model default (cf. Bohn 1999, p. 388; Obstfeld and Rogoff 2017, p. 13, for instance). And indeed, while it is often voiced that a devaluation of debt through inflation is economically equivalent to partial default, many of the initial landmark contributions remain silent on its explicit occurrence. One exception is Cochrane (2005b, p. 516), who briefly explains that outright default would not imply any changes.

To the same effect as \nearrow Long-Term Debt (p. 180) in Equation (9.2), Buiter (2002) introduces a “public debt revaluation factor” which has the potential to take some adjustment pressure off the price level. Since non-Ricardian fiscal policy is overdetermined in his account, this would not make the model indeterminate but only allow it to function properly; in this way, “non-Ricardian regimes become Ricardian regimes and the fiscal theory of the price level vanishes” (p. 478).

The issue is especially eminent in the context of monetary unions and fixed-exchange-rate regimes. Uribe (2006) studies a model with active fiscal policy (and both active as well as passive monetary policy as subcases) in which default is explicitly included. Sokolova (2015) describes the trade-off between inflation and outright default.

More Critique A point of criticism not mentioned so far relates to ‘time-zero’ liabilities. According to Niepelt (2004), it is always simply assumed that the economy begins with consolidated-government liabilities already outstanding, but if this assumption is dropped, fiscal price determination fails. In a more or less direct reply, Daniel (2007) suggests that the condition of the fiscal theory is not as bleak as purported by Niepelt. Recently, Buiter also reiterated his critique of the fiscal theory (cf. Buiter 2017a,b; Buiter and Sibert 2017).

More Analysis Of course, there are more sophisticated approaches to the solution of rational-expectations DSGE models beyond the basic ‘verbally formal’ style of this thesis. Tan and Walker (2015) and Tan (2017) are recent papers with a focus on monetary-fiscal policy interaction.

Whether this would necessarily be associated with less formal effort in the end is questionable, but another alternative is to abandon the infinitely-lived representative household and switch to Samuelson’s (1958) model of overlapping generations. It is used by Barro (1974) and Wallace (1981), for instance, to discuss Ricardian equivalence and what is now called Wallace neutrality, respectively. It should be noted that this approach is clearly less popular nowadays (going by the mere number of works utilizing the respective modeling strategies).

More Applications Applications for the monetary-fiscal theory abound. As indicated in Chapter 3, the consolidated-government budget equation offers several vantage points for further study. Many recent discussions revolve around the central-bank balance sheet (and sometimes even sense a “mystique” surrounding it, cf. Reis 2013): What happens if potential central-bank losses are not covered by ‘inverse remittances’ from the treasury T^{TC} anymore? (Cf. Benigno and Nisticò 2017.) Are terms like ‘central-bank capital’ (or ‘equity’) meaningful in a fiat-debt world? Can a central bank ‘go bankrupt?’ (Cf. Hellwig 2014.) Does helicopter money work? Always? (Cf. Buiter 2014.)

Cut-Off Point The list of omissions and possible extensions could be continued for quite some time. It would be interesting to study monetary-fiscal interactions from a political-economy perspective. Or to consider the interplay with macroprudential regulation in a macroeconomic policy triangle. Or to go into more detail on policy mixes, that is, the efficacy of different policy branches in achieving certain objectives (the present thesis is mostly concerned with the influence of debt or consolidated-government liabilities in surplus rules, but a typical treasury in a democratic country is probably just as concerned with unemployment, for instance). At some point, however, one has to come to an end and in the present thesis, the above pointers aside, this point is reached on page 179.

Key Takeaways from Chapter 9

The present-value budget equation of the consolidated government is an equilibrium condition, not a constraint on policy. The reason for its potential to determine the price level in a backing-based story is that the ‘ultimate’ public liability—money—is also the economy’s unit of account. By contrast, seigniorage does not play a major role in modern theory. Whether a fiscal theory of the price level is desirable also depends on an assessment of the viability of conventional theory.

10. Conclusion

Short Answers An extreme summary of the answers to the initial questions raised in the introduction reads like this:

1. How do modern DSGE models work?—They employ forward-looking, rational expectations and therefore have to select a unique equilibrium by ‘trimming’ all other candidate paths.
2. What is the role of monetary and fiscal policy in this?—If chosen properly, they generate an infinite amount of explosive paths which the selection device (the transversality condition) rejects as equilibria and only a single candidate path which passes.

Polar Regimes The thesis shows that the conventional way to achieve determinacy requires specific circumstances. For one, the assumption of Ricardian fiscal policy is often made only implicitly; sometimes its description is foregone altogether. More critically, the conventional approach might not be able to establish determinacy at all, namely, if nominal explosions are not considered sufficient to rule out the respective equilibrium candidates. This latter position is controversial even though the transversality condition, which pertains to real variables, is the only such requirement that emerges from *within* the optimization problem. The fiscal theory of the price level is a ‘new,’ alternative way to achieve uniqueness. It does not suffer from the above problem because its pivotal point, the consolidated-government present-value budget equation, is derived with the transversality condition already imposed.

Refusal Rather than repeat numerous other findings from the previous chapters, I want to make some suggestions as to how they could be understood in their entirety. One bone of contention is the stand-out role of fiscal elements, and some reactions to fiscal price determination have a flavor of ‘that which must not, can not be.’³³ Subscribing to this logic would be misguided in my opinion—it can (could) be. Fiscalist price

³³ There is a consensus about the role of fiscal and monetary policy (cf. Goodfriend 2007; Bean et al. 2010 for anecdotal and Klein and Stern 2006; Fuller and Geide-Stevenson 2014 for survey-based descriptions): The latter stabilizes the business cycle and, by a “divine coincidence” (Blanchard and Galí 2007), also prices. Fiscal policy is considered to be mostly about real variables because it has discretion about (re-)distribution and can be a determinant of long-term growth, but it should keep out of short- and medium-term business-cycle management, which it would only complicate. However, it is important to

determination is associated with the same set of equations as the workhorse model that is considered to be the current ‘industry standard,’ the only difference being that the former is not blind in one eye.

Appeasement But just because something *can* happen does not necessarily mean that it *will* (or *should*, for that matter). The possibility of a fiscal regime does not automatically imply profligacy. And fiscal price determination is not perfect either, it can also fail to achieve determinacy if fiscal policy is misspecified. Admittedly, some authors vividly advocate the fiscal theory as possibly the only truly viable approach to price determination, but a less extreme, more nuanced position holds that it is not here to *replace* conventional wisdom but to *extend* it.

Focus on Policy A related strategic advantage of a joint monetary-fiscal theory is that it places more emphasis on policy than on modeling: Instead of calling certain models flawed because they (supposedly inherently) lead to indeterminacy or explosive behavior, we can describe the resulting phenomena as undesired outcomes of unfavorable policy combinations. Of course, this comprehensiveness is not entirely new—Sargent and Wallace did the same for monetarism with their ‘unpleasant monetarist arithmetic,’ for example—but it is relatively new within the current vintage of monetary models.

A Case for No Change The downside of extending ‘purely monetary’ models by a more explicit description of fiscal policy is that it also broadens their (already extensive) formal scope. Whether jointly-monetary-fiscal models can incorporate further complications (think of frictions on financial markets, for instance) and remain manageable analytically is arguable, to say the least.

Combining this argument with observational equivalence in the data could make a strong case for choosing one approach and recognizing the respective other merely as a valid alternative. Since empirical evidence seems slightly in favor of the conventional ‘Taylor-principle’ wisdom rather than pressing for a dogmatic overthrow, nothing much could (or would have to) change in monetary economics.

Time Will Tell Then again, things do change sometimes: While McCallum (1981, p. 328) contended that “the use of an interest rate instrument is feasible, not that it is desirable,” almost 30 years later, Bullard (2010, p. 339) notes that “[a]ctive Taylor-type rules are so commonplace in present-day monetary policy discussions that they have ceased to be controversial.” Maybe the fiscal (part of the) theory will some day also experience what

recognize that, while ‘true,’ these notions are separate. This is where dogma might lead to premature conclusions: prices are seen as a short-term issue whereas the permissible forms of fiscal policy are long-term; hence, they should not mingle.

the nominal interest rate has gone through in the past as a tool of monetary policy and eventually overcome 'theory inertia.'

Appendix

A. Appendix to Chapter 3

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A.1. Money Demand as a Determinant of Seigniorage

A.1.1. The Model of Calvo and Leiderman (1992)

Money (Super-)Neutrality In Calvo and Leiderman (1992, pp. 180-181), endowments and thus also the real interest rate are constant (cf. Section 2.4). Consequently, gross inflation Π_{t+1} adjusts to a given amount of money so as to satisfy Equation $\langle 2.14 \rangle$ (for instance if the central bank constantly increases the nominal money supply at a certain rate). Neither the stock nor the growth rate of money have any effect on real variables, that is, consumption or the real interest rate. (In Walsh 2010, p. 155, this is simply “assumed.” Indeed, using a sophisticated utility function like $\langle 3.12 \rangle$ complicates a formal derivation profoundly, which is why I settle for the the ‘intuitive argument,’ going through equations and determining endogenous variables.)

Money Demand Given the specific utility function $\langle 3.12 \rangle$

$$u(c_t, m_t) = \ln c_t + m_t (O_I - O_{II} \ln m_t),$$

money demand $\langle 2.22 \rangle$ takes the form of Equation $\langle 3.13 \rangle$:

$$O_I - O_{II} - O_{II} \ln m_t = \frac{i_{t+1}}{1 + i_{t+1}} \cdot \frac{1}{c_t} \quad \langle A.1 \rangle$$

$$\Leftrightarrow m_t = O_{\text{III}} \exp\left(-\frac{1}{O_{\text{II}}c_t} \cdot \frac{i_{t+1}}{1+i_{t+1}}\right)$$

$O_{\text{III}} \equiv \exp[(O_{\text{I}}/O_{\text{II}}) - 1]$ is a shorthand.

Seigniorage Laffer Curve Combining this with the definition of interest-saving seigniorage $\langle 3.11 \rangle$ leads to Equation $\langle 3.14 \rangle$ (and beyond):

$$\begin{aligned} \varsigma_t^i &= \left(\frac{i_t}{1+\pi_t}\right) O_{\text{III}} \exp\left(-\frac{1}{O_{\text{II}}c_{t-1}} \cdot \frac{i_t}{1+i_t}\right) \\ &= (1+r_t) \left(\frac{i_t}{1+i_t}\right) O_{\text{III}} \exp\left(-\frac{1}{O_{\text{II}}c_{t-1}} \cdot \frac{i_t}{1+i_t}\right) \\ &= (1+r_t) \dot{I}_t O_{\text{III}} \exp\left(-\frac{1}{O_{\text{II}}c_{t-1}} \cdot \dot{I}_t\right) \end{aligned}$$

At this, using the definition of \dot{I} $\langle 2.23 \rangle$ makes finding the seigniorage-maximizing inflation rate somewhat more convenient for it allows to write

$$\begin{aligned} \frac{\partial \varsigma_t^i}{\partial \pi_t} &= \frac{\partial \varsigma_t^i}{\partial \dot{I}} \cdot \frac{\partial \dot{I}}{\partial i_t} \cdot \frac{\partial i_t}{\partial \pi_t} \stackrel{!}{=} 0 \\ \Leftrightarrow \left[(1+r_t) O_{\text{III}} \exp\left(-\frac{1}{O_{\text{II}}c_{t-1}} \cdot \dot{I}_t\right) \left(1 - \frac{1}{O_{\text{II}}c_{t-1}} \dot{I}_t\right) \right] \cdot \left[\frac{1}{(1+i_t)^2} \right] \cdot [1+r_t] &= 0. \end{aligned}$$

The only way this can hold is if the last term in parentheses in the first bracket equals zero:

$$1 - \frac{1}{O_{\text{II}}c_{t-1}} \dot{I}_t = 0 \quad \Leftrightarrow \quad \dot{I}_t = O_{\text{II}}c_{t-1}$$

Rewinding the definition of \dot{I}_t and rearranging then yields the aspired end result $\langle 3.15 \rangle$:

$$\begin{aligned} O_{\text{II}}c_{t-1} &= \frac{i_t}{1+i_t} = \frac{(1+r_t)(1+\pi_t) - 1}{(1+r_t)(1+\pi_t)} = 1 - \frac{1}{(1+r_t)(1+\pi_t)} \\ \Leftrightarrow \frac{1}{1+\pi_t} &= (1+r_t)(1 - O_{\text{II}}c_{t-1}) \\ \Leftrightarrow \pi_t^{\max} &= \frac{1}{(1+r_t)(1 - O_{\text{II}}c_{t-1})} - 1 \end{aligned}$$

It should be noted that this amounts to a steady-state analysis in which expected and actual inflation rates are equal so that the expectation operator can be dropped.

A.1.2. Traditional Assumptions vs. Microfoundations for Seigniorage Laffer Curves

Forms of Money Demand Appendix A.1.1 demonstrates the derivation of the money-demand function $\langle 3.13 \rangle$. So far, neither money demand nor seigniorage have been expressed explicitly in terms of inflation but only in terms of the nominal interest rate (Figure 3.1 implicitly used the Fisher equation to plot seigniorage as a function of inflation). Catching up on this, Equation $\langle 3.13 \rangle$ eventually becomes Equation $\langle 3.17 \rangle$

$$\begin{aligned} m_t &= O_{III} \exp\left(-\frac{1}{O_{II}c_t} \cdot \frac{i_{t+1}}{1+i_{t+1}}\right) = O_{III} \exp\left[-\frac{1}{O_{II}c_t} \cdot \frac{(1+\mathbb{E}_t r_{t+1})(1+\mathbb{E}_t \pi_{t+1})-1}{(1+\mathbb{E}_t r_{t+1})(1+\mathbb{E}_t \pi_{t+1})}\right] \\ &= O_{III} \exp\left\{-\frac{1}{O_{II}c_t} \left[1 - \frac{1}{(1+\mathbb{E}_t r_{t+1})(1+\mathbb{E}_t \pi_{t+1})}\right]\right\} = O_{VI} \exp\left(O_V \frac{1}{\mathbb{E}_t \Pi_{t+1}}\right), \end{aligned}$$

where

$$O_{VI} \equiv O_{III} \exp\left(-\frac{1}{O_{II}c_t}\right) = \exp\left(\frac{O_I}{O_{II}} - 1 - \frac{1}{O_{II}c_t}\right) \quad \langle A.2 \rangle$$

and

$$O_V \equiv \frac{1}{O_{II}c_t(1+\mathbb{E}_t r_{t+1})}$$

are used as shorthands.

By contrast, Cagan (1956) uses the form $m_{C56,t} = \exp(-O_{IV}\mathbb{E}_t \pi_{t+1})$ (O_{IV} is yet another coefficient, see Equation $\langle 3.16 \rangle$), which cannot be constructed within the present setup. Figure 3.2 in the main text demonstrates that both forms have similar shapes in the positive domain—at least at first sight. Several remarks on this ensue right below.

1. Sketches I have used more general forms (as stated in the figure itself) instead of the actual money demand functions in order to abstract from the various coefficients. While this does not take anything of importance away, it should be clear that the plots in Figure 3.2 are sketches rather than exact representations of Equations $\langle 3.16 \rangle$ and $\langle 3.17 \rangle$.

2. Positive Limits While the dashed line representing the Cagan (1956) specification converges to zero as inflation goes to infinity, the solid microfoundations line converges to unity in Figure 3.2. Correspondingly, in Equation $\langle 3.17 \rangle$, money demand converges to

the shorthand O_{VI} . A closer inspection of its definition $\langle A.2 \rangle$ requires taking the limits as c_t goes to zero and infinity, respectively:

$$\lim_{c_t \rightarrow 0} O_{VI} = \exp\left(\frac{O_I}{O_{II}} - 1 - \frac{1}{0}\right) = \exp(-\infty) = 0$$

$$\lim_{c_t \rightarrow \infty} O_{VI} = \exp\left(\frac{O_I}{O_{II}} - 1 - 0\right) = \begin{cases} \gg 0 & \text{for } O_I \gg O_{II} \\ \approx 1 & \text{for } O_I \approx O_{II} \\ \exp(-1) \approx 0.368 & \text{for } O_I \ll O_{II} \end{cases}$$

The case in which c_t increases towards infinity is a bit more complicated as it is unclear *a priori* to what value the shorthand O_{VI} converges. It depends on the two coefficients O_I as well as O_{II} : If $O_I \gg O_{II}$, the shorthand converges to a possibly large positive number, decreasing the similarity between specifications. As O_I and O_{II} approach each other, the shorthand goes towards unity. If $O_I \ll O_{II}$, taking O_I/O_{II} in the vicinity of zero, the shorthand approaches $\exp\{-1\} \approx 0.368$.

3. Discontinuity It is also apparent from Figure 3.2 that there is a discontinuity in the microfounded money demand function $\langle 3.17 \rangle$, namely at $\Pi_{t+1} = 0$, which corresponds to a net inflation rate of -100% . However, the same problem *always* arises with seigniorage $\langle 3.11 \rangle$, which features gross inflation in the denominator as well (cf. Figure A.1, to which I also return at the end of this section). Therefore, said discontinuity can hardly be seen as an impediment to the use of utility function $\langle 3.12 \rangle$.

4. Strong Deflation Finally, the behavior beyond this point (i.e., for deflation rates greater than 100%) also differs. Consider the limit of money demand $\langle 3.17 \rangle$ as inflation goes to minus infinity:

$$\lim_{\mathbb{E}_t \Pi_{t+1} \rightarrow -\infty} \left[O_{VI} \exp\left(O_V \frac{1}{\mathbb{E}_t \Pi_{t+1}}\right) \right] = O_{VI} \exp\left(O_V \frac{1}{-\infty}\right) = O_{VI} \frac{1}{\exp(0)} = O_{VI}$$

Money demand converges to the shorthand O_{VI} . This might be considered a peculiarity brought about by the underlying mathematics, yet it also fits economic intuition: If one is willing to consider infinite (or at least very high) deflation, any positive amount of money yields infinite (or at least very great) purchasing power. Inflation can also approach the discontinuity from below:

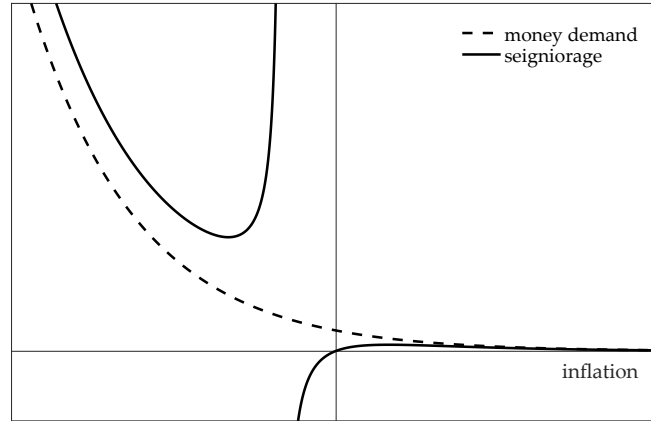


Figure A.1: *Money Demand and Seigniorage Using the Cagan (1956) Specification.* \diamond Source: Own illustration based on Cagan (1956). \diamond Explanations: Even with a ‘well-behaved’ money demand function, seigniorage exhibits a discontinuity and peculiar behavior for (very) high deflation rates.

$$\begin{aligned} \lim_{\mathbb{E}_t \Pi_{t+1} \rightarrow 0^-} \left[O_{VI} \exp \left(O_V \frac{1}{\mathbb{E}_t \Pi_{t+1}} \right) \right] \\ = \lim_{\mathbb{E}_t \Pi_{t+1} \rightarrow 0^+} \left[O_{VI} \exp \left(O_V \frac{1}{-\mathbb{E}_t \Pi_{t+1}} \right) \right] = O_{VI} \frac{1}{\exp(\infty)} = 0 \end{aligned}$$

Money demand goes to zero (Figure 3.2 confirms this). Arguing this case economically seems prohibitively more difficult.

Returning to Figure A.1, it stands out that the Cagan (1956) specification of money demand (3.16) also leads to curious behavior of seigniorage beyond the discontinuity. I abstain from explicitly stating the limits here and rather proceed to a solution (or a ‘quick fix,’ to be more modest).

Zero Lower Bound The arguments in Remarks 3 and 4 neglect the fact that nominal interest rates can typically not fall below the zero lower bound, an omission that is also embedded in the derivation of Equation (3.17) and, more generally, in many simple optimizing models (such as the one of Calvo and Leiderman 1992 used here). This often seems to happen knowingly because, for one, incorporation of the zero lower bound complicates computation and simulation (cf. Section 5.2.1 and Footnote 20 on p. 126), but also when the focus lies on positive or hyperinflation and the associated *high* nominal interest rates. Following this line of reasoning, Figures A.2 and A.3 reproduce Figures A.1 and 3.1, respectively, allowing the nominal interest rate i_t to take only positive

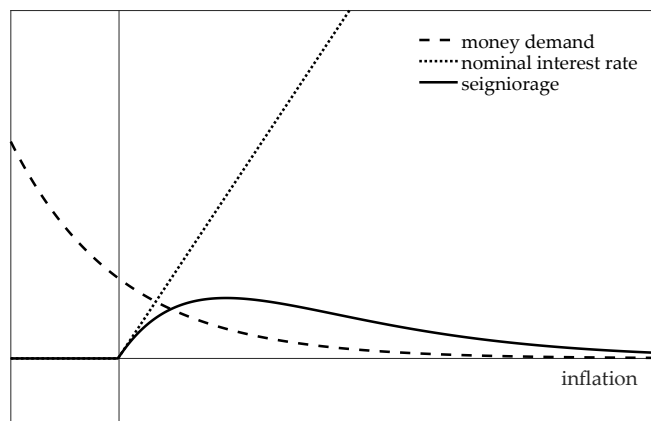


Figure A.2: Money Demand, the Nominal Interest Rate and Seigniorage Using the Cagan (1956) Specification, Incorporating the Zero Lower Bound. \diamond Source: Own illustration based on Cagan (1956).

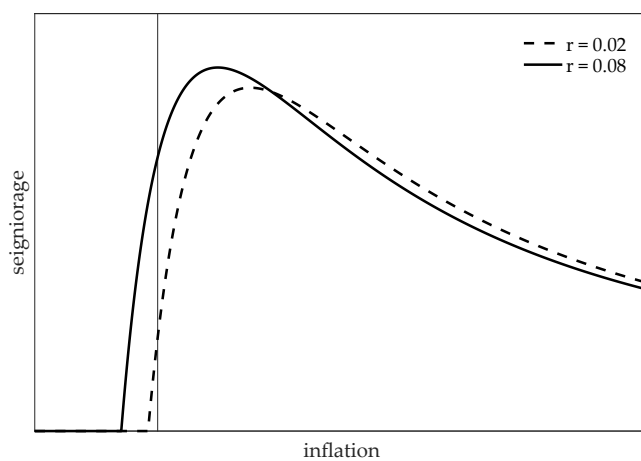


Figure A.3: The Microfounded Seigniorage Laffer Curve, Incorporating the Zero Lower Bound. \diamond Source: Own illustration replicating and expanding on Figure 4.1 in Walsh (2010, p. 155).

values:

$$i_t = \max [(1 + r_t) (1 + \pi_t) - 1, 0]$$

Note again that the actual underlying model does not make this correction (for the arguments given above).

coefficient	context	first appearance	
O_I	utility function of Calvo and Leiderman (1992)	$\langle 3.12 \rangle$	3.4.3, A.1.1
O_{II}	utility function of Calvo and Leiderman (1992) model	$\langle 3.12 \rangle$	3.4.3, A.1.1
O_{III}	shorthand, money demand of Calvo and Leiderman (1992)	$\langle 3.13 \rangle$	3.4.3, A.1.1
φ'	'simple' alternative utility function	$\langle A.3a \rangle$	A.1.3
φ''	CES utility function	$\langle A.3b \rangle$	A.1.3
φ'''	CES utility function	$\langle A.3b \rangle$	A.1.3
O_{IV}	money demand of Cagan (1956)	$\langle 3.16 \rangle$	3.4.3, A.1.2
O_V	shorthand, money demand of Calvo and Leiderman (1992)	$\langle 3.17 \rangle$	3.4.3, A.1.2
O_{VI}	shorthand, money demand of Calvo and Leiderman (1992)	$\langle 3.17 \rangle$	3.4.3, A.1.2

Table A.1: Overview of Coefficients in the Context of Seigniorage Laffer Curves.

A.1.3. Alternative Specifications without Seigniorage Laffer Curves

The result of a Laffer curve for seigniorage critically depends on the chosen utility function. Replacing $\langle 3.12 \rangle$ with a 'simple' logarithmic utility function

$$u(c_t, m_t) = \ln c_t + \varphi' \ln m_t \quad \langle A.3a \rangle$$

or a constant-elasticity-of-substitution (CES) function

$$u(c_t, m_t) = \left[\varphi'' c_t^{1-\varphi'''} + (1 - \varphi'') m_t^{1-\varphi'''} \right]^{\frac{1}{1-\varphi'''}} , \quad \langle A.3b \rangle$$

for example, yields markedly different results. Note that symbols with primes do *not* signify derivatives but some variables or coefficients that are not specified in greater detail. Furthermore, let equation numbering be a guide for the two subcases presented here. Then, combining Equations $\langle A.3a \rangle$ and $\langle A.3b \rangle$, respectively, with $\langle 2.22 \rangle$ yields

money-demand functions for both alternative cases:

$$\begin{aligned}\frac{\varphi'}{m_t} &= \left(\frac{i_{t+1}}{1+i_{t+1}} \right) \frac{1}{c_t} \\ \Leftrightarrow m_t &= \varphi' \left(\frac{1+i_{t+1}}{i_{t+1}} \right) c_t\end{aligned}\quad \langle \text{A.4a} \rangle$$

(Note that for the CES specification, parts of the outer derivatives can also be written as $[u(c_t, m_t)]^{\varphi''}$, which shortens the first line a little:)

$$\begin{aligned}(1 - \varphi'') [u(c_t, m_t)]^{\varphi'''} m_t^{-\varphi'''} &= \left(\frac{i_{t+1}}{1+i_{t+1}} \right) \varphi'' [u(c_t, m_t)]^{\varphi'''} c_t^{-\varphi'''} \\ \Leftrightarrow m_t &= \left(\frac{\varphi''}{1 - \varphi''} \cdot \frac{i_{t+1}}{1+i_{t+1}} \right)^{-\frac{1}{\varphi'''}} c_t,\end{aligned}\quad \langle \text{A.4b} \rangle$$

Plugging $\langle \text{A.4a} \rangle$ and $\langle \text{A.4b} \rangle$, respectively, into the seigniorage expression $\langle 3.11 \rangle$ and using the Fisher equation eventually yields seigniorage expressions that depend on inflation:

$$\begin{aligned}\varsigma_t^i &= \left(\frac{i_t}{1+\pi_t} \right) \varphi' \left(\frac{1+i_t}{i_t} \right) c_{t-1} = \varphi' \left(\frac{1+i_t}{1+\pi_t} \right) c_{t-1} \\ \Leftrightarrow \varsigma_t^i &= \varphi' (1+r_t) c_{t-1}\end{aligned}\quad \langle \text{A.5a} \rangle$$

$$\begin{aligned}\varsigma_t^i &= \left(\frac{i_t}{1+\pi_t} \right) \left(\frac{\varphi''}{1-\varphi''} \cdot \frac{i_t}{1+i_t} \right)^{-\frac{1}{\varphi'''}} c_{t-1} \\ \Leftrightarrow \varsigma_t^i &= \left(\frac{\varphi''}{1-\varphi''} \right)^{-\frac{1}{\varphi'''}} (i_t)^{\frac{\varphi'''-1}{\varphi'''}} \left(\frac{1}{1+\pi_t} \right) \left[\frac{1}{(1+r_t)(1+\pi_t)} \right]^{-\frac{1}{\varphi'''}} c_{t-1} \\ \Leftrightarrow \varsigma_t^i &= \left(\frac{\varphi''}{1-\varphi''} \right)^{-\frac{1}{\varphi'''}} \left(\frac{i_t}{1+\pi_t} \right)^{\frac{\varphi'''-1}{\varphi'''}} (1+r_t)^{\frac{1}{\varphi'''}} c_{t-1} \\ \Leftrightarrow \varsigma_t^i &= \left(\frac{\varphi''}{1-\varphi''} \right)^{-\frac{1}{\varphi'''}} \left[\frac{(1+r_t)(1+\pi_t)-1}{1+\pi_t} \right]^{\frac{\varphi'''-1}{\varphi'''}} (1+r_t)^{\frac{1}{\varphi'''}} c_{t-1} \\ \Leftrightarrow \varsigma_t^i &= \left(\frac{\varphi''}{1-\varphi''} \right)^{-\frac{1}{\varphi'''}} \left[(1+r_t) - \frac{1}{1+\pi_t} \right]^{\frac{\varphi'''-1}{\varphi'''}} (1+r_t)^{\frac{1}{\varphi'''}} c_{t-1}\end{aligned}\quad \langle \text{A.5b} \rangle$$

As shown in the left-hand panel of Figure A.4, the ‘simple’ result (A.5a) is a horizontal line, that is, not affected by the inflation rate—at least as long as the zero lower bound for the net nominal interest rate, which is incorporated into the figure but not explicitly into the equations, does not bind. In the CES case (A.5b) (right-hand panel; also adhering to the zero lower bound), seigniorage asymptotically approaches a certain value determined by the coefficients. In short, none of these alternative specifications leads to a seigniorage Laffer curve or a unique seigniorage-maximizing inflation rate (save for border solutions).

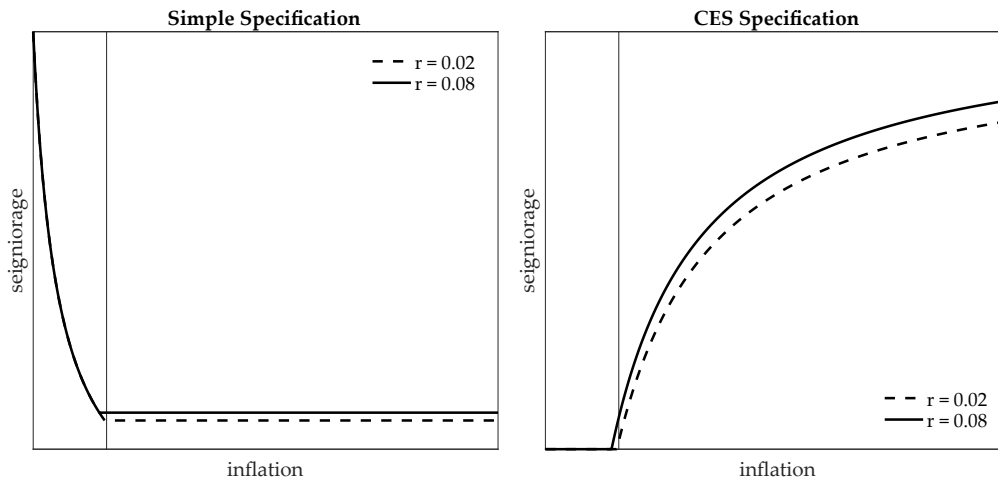


Figure A.4: *Non-Laffer Curves for Seigniorage.* ◇ Source: Own illustration.
 ◇ Explanations: The two panels’ ordinates range across different co-domains so that seigniorage levels from the two specifications cannot be compared visually in this figure.

A.1.4. The Model of Bruno and Fischer (1990), Translated Into Discrete Time

A.1.4.1. Non-Negative Budget Positions with Logarithmic Form

Because of its logarithmic form, the very-right-hand side of Equation (3.29)

$$\mathbb{E}_t \pi'_{t+1} = \frac{1}{O_{IV}} \ln \left(\frac{\mu'_t}{d_t} \right) = \frac{\ln \mu'_t}{O_{IV}} - \frac{\ln d_t}{O_{IV}}$$

could be seen to imply that balanced budgets and budget surpluses are problematic or even impossible, respectively: The logarithm of $d_t = 0$ is minus infinity and that of $d_t < 0$ is not defined. However, a look at the term in the middle reveals a solution (or yet another ‘quick fix’) at least for the latter problem: If money growth is negative whenever the budget is in surplus, the fraction as a whole remains positive and can thus serve as an argument for the logarithm.

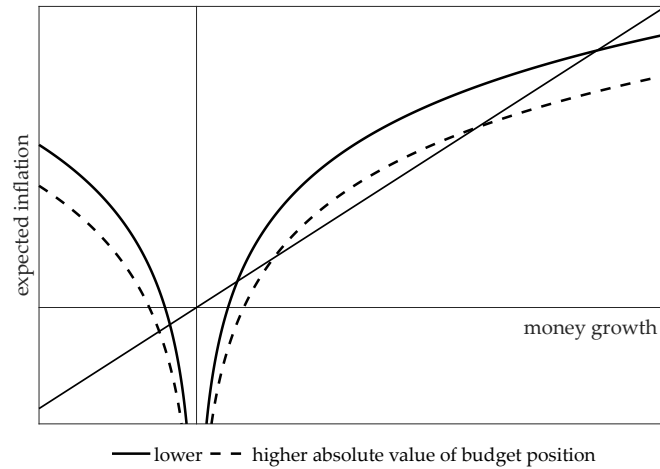


Figure A.5: *Inflation as a Function of Money Growth, Accounting for the Possibility of Budget Surpluses.* ♦ Source: Own illustration based on Bruno and Fischer (1990, p. 355) and Walsh (2010, p. 157). ♦ Explanations: Plot of Equation (3.29) (the positively sloped straight line is the equilibrium condition (3.27)). As in the basic case of Figure 3.3, the budget deficit enters as a negative intercept. In case of a budget surplus, money growth has to be negative; similarly, in case of negative money growth, there has to be a budget surplus.

As a matter of plotting the function (see Figure A.5), if money growth turns negative, a given value for the budget deficit is multiplied by -1 so as to make it a budget surplus of the same size. Similarly, given a budget surplus, one would simply multiply the range of money growth rates by -1 as well. Especially the latter procedure should make obvious that the 'original' graphs associated with budget deficits are mirrored on the ordinate. As shown by the figure, the result will be a unique equilibrium with negative money growth rates in any case.

Of course, this does away with the *ceteris paribus* assumption because budget deficits and money growth rates can not be analyzed independently of the respective other variable over the whole range of values anymore (only locally, i.e., in either the positive or negative domain).

Is it realistic that the consolidated government *always* and *automatically* coordinates its policy stance in the way described above? Put differently, of how much use is a model excluding these at least halfway realistic scenarios (the combination of a slightly decreasing money supply and a budget deficit does not seem completely unthinkable)? Again, there will probably be different opinions about whether one is willing to accept that specific functional forms weigh on the model's results so heavily.

Finally, bear in mind that the other, probably more severe problem has not been solved: Even with mirroring, expected inflation still goes to minus infinity if the ab-

solute value of the budget position approaches zero (and then normalizes again as the budget position becomes larger). This is highly unrealistic.

A.1.4.2. Changes in Inflation Expectations Depending on Money Growth

Required $\Delta \mathbb{E}_t \pi'_{t+1}$: Alternative Derivation The budget constraint (3.29) can be rearranged such that the consolidated government's budget deficit is 'explained' (see below) by functions of money growth and expected inflation:

$$\frac{\ln d_t}{O_{IV}} = \frac{\ln \mu'_t}{O_{IV}} - \mathbb{E}_t \pi'_{t+1} \quad \langle A.6 \rangle$$

If the economy has been in steady state until $(t-1)$ and the government now decides to increase money growth,

$$\mu'_t = \mu'_{t-1} + \Delta \mu'_t,$$

this also affects the above budget constraint. However, the budget deficit is not endogenously determined by the other variables because the consolidated government considers it a policy choice and rather sets it exogenously. (To be precise, the consolidated government is repeatedly set back in its pursuit of a certain real budget deficit because of ever-increasing price levels; this is the crucial point.) To achieve this, the induced effects of a change in money growth on both terms on the right-hand side of Equation (A.6) have to cancel out:

$$\begin{aligned} \Delta \left(\frac{\ln d_t}{O_{IV}} \right) &= \Delta \left(\frac{\ln \mu'_t}{O_{IV}} \right) - \Delta \mathbb{E}_t \pi'_{t+1} \stackrel{!}{=} 0 \\ \Leftrightarrow \Delta \mathbb{E}_t \pi'_{t+1} &= \Delta \left(\frac{\ln \mu'_t}{O_{IV}} \right) = \left[\frac{\ln(\mu'_{t-1} + \Delta \mu'_t)}{O_{IV}} - \frac{\ln \mu'_{t-1}}{O_{IV}} \right] = \frac{1}{O_{IV}} \ln \left(\frac{\mu'_{t-1} + \Delta \mu'_t}{\mu'_{t-1}} \right) \end{aligned}$$

As a fraction of the 'old value over new value' kind, the argument of the logarithm on the far right is a gross growth rate. Knowing that the logarithm of a gross growth rate approximately equals the respective net growth rate, the last equation above can be rewritten as

$$\Delta \mathbb{E}_t \pi'_{t+1} \approx \frac{1}{O_{IV}} \cdot \frac{(\mu'_{t-1} + \Delta \mu'_t) - \mu'_{t-1}}{\mu'_{t-1}} = \frac{1}{O_{IV} \mu'_{t-1}} \Delta \mu'_t,$$

which is the same result as Equation (3.33) in the main text.

B. Appendix to Chapter 4

B.1. The Basic FTPL Model of Leeper and Leith (2017)

B.1.1. Laws of Motion and Solutions for Inflation and Real Debt

B.1.1.1. Inflation

Law of Motion Expanding the Taylor rule (4.5) to

$$1 + i_{t+1} = \beta^{-1}\Pi^* + \gamma_{\pi}^C(\pi_t - \pi^*) + \varepsilon_t^i$$

and equating it to the Fisher equation (2.47)

$$1 + i_{t+1} = \beta^{-1}\mathbb{E}_t\Pi_{t+1}$$

results in the difference equation in inflation (4.6):

$$\beta^{-1}\mathbb{E}_t\Pi_{t+1} = \beta^{-1}\Pi^* + \gamma_{\pi}^C(\pi_t - \pi^*) + \varepsilon_t^i$$

$$\Leftrightarrow \mathbb{E}_t(\pi_{t+1} - \pi^*) = \beta\gamma_{\pi}^C(\pi_t - \pi^*) + \beta\varepsilon_t^i$$

Solution Rearranging the difference equation and iterating forward via $(\pi - \pi^*)$ then leads to the solution (4.9):

$$\begin{aligned}\pi_t - \pi^* &= \frac{1}{\beta\gamma_{\pi}^C}\mathbb{E}_t(\pi_{t+1} - \pi^*) - \frac{1}{\gamma_{\pi}^C}\varepsilon_t^i \\ &= \frac{1}{\beta\gamma_{\pi}^C}\left[\frac{1}{\beta\gamma_{\pi}^C}\mathbb{E}_t(\pi_{t+2} - \pi^*) - \frac{1}{\gamma_{\pi}^C}\mathbb{E}_t\varepsilon_{t+1}^i\right] - \frac{1}{\gamma_{\pi}^C}\varepsilon_t^i \\ &\vdots \\ \Leftrightarrow \pi_t &= \pi^* - \frac{1}{\gamma_{\pi}^C}\mathbb{E}_t\sum_{j=0}^{\infty}\left(\frac{1}{\beta\gamma_{\pi}^C}\right)^j\varepsilon_{t+j}^i\end{aligned}$$

At this, it is implicitly assumed that

$$\lim_{T \rightarrow \infty} \left(\frac{1}{\beta \gamma_{\pi}^C} \right)^T \mathbb{E}_t(\pi_{t+T} - \pi^*) = 0. \quad \langle \text{B.1} \rangle$$

Additional Derivations Plugging the solution into the Fisher equation $\langle 2.47 \rangle$ further yields the equilibrium nominal interest rate $\langle 4.11 \rangle$:

$$1 + i_{t+1} = \beta^{-1} \left[\Pi^* - \frac{1}{\gamma_{\pi}^C} \mathbb{E}_t \sum_{j=0}^{\infty} \left(\frac{1}{\beta \gamma_{\pi}^C} \right)^j \varepsilon_{t+1+j}^i \right] = 1 + i_{SS} - \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{\beta \gamma_{\pi}^C} \right)^j \varepsilon_{t+j}^i$$

B.1.1.2. Real Debt

Law of Motion Equate the fiscal policy rule $\langle 4.7 \rangle$ with the flow budget constraint of the treasury $\langle 4.3 \rangle$ (solved for s_t and with $Q_t = 1/[1 + i_{t+1}]$ from Equation $\langle 2.8 \rangle$):

$$\frac{B_{t-1}}{P_t} - \frac{b_t}{1 + i_{t+1}} = s_{SS} + \gamma_b^T \left(\frac{b_{t-1}}{1 + i_t} - \frac{b_{SS}}{1 + i_{SS}} \right) + \varepsilon_t^s$$

Shifting forward one period and exchanging terms gives:

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1 + i_{t+2}} \right) = \frac{B_t}{\mathbb{E}_t P_{t+1}} - s_{SS} - \gamma_b^T \left(\frac{b_t}{1 + i_{t+1}} - \frac{b_{SS}}{1 + i_{SS}} \right) - \mathbb{E}_t \varepsilon_{t+1}^s \quad \langle \text{B.2} \rangle$$

An intermediate step is to replace s_{SS} . Rewrite the flow budget constraint of the treasury as

$$s_t = \frac{B_{t-1}}{P_t} - \frac{b_t}{1 + i_{t+1}} = \frac{b_{t-1}}{\Pi_t} - \frac{b_t}{1 + i_{t+1}}$$

Exchanging time- t expressions \square_t with their steady-state equivalents \square_{SS} (or Π^* , respectively) and applying the steady-state version of the Fisher equation $\langle 2.47 \rangle$,

$$s_{SS} = \frac{b_{SS}}{\Pi^*} - \frac{b_{SS}}{1 + i_{SS}} = b_{SS} \left(\frac{1 + r}{1 + i_{SS}} - \frac{1}{1 + i_{SS}} \right). \quad \langle \text{B.3} \rangle$$

Plugging this into Equation $\langle \text{B.2} \rangle$ gives

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1 + i_{t+2}} - \frac{b_{SS}}{1 + i_{SS}} \right)$$

$$= \frac{B_t}{\mathbb{E}_t P_{t+1}} - (1+r) \frac{b_{SS}}{1+i_{SS}} - \gamma_b^T \left(\frac{b_t}{1+i_{t+1}} - \frac{b_{SS}}{1+i_{SS}} \right) - \mathbb{E}_t \varepsilon_{t+1}^s.$$

Applying the Fisher equation again to find that

$$\frac{B_t}{\mathbb{E}_t P_{t+1}} = \frac{b_t}{\mathbb{E}_t \Pi_{t+1}} = (1+r) \frac{b_t}{1+i_t}$$

finally leads to Equation [\(4.8\)](#):

$$\mathbb{E}_t \left(\frac{b_{t+1}}{1+i_{t+2}} - \frac{b_{SS}}{1+i_{SS}} \right) = (1+r - \gamma_b^T) \left(\frac{b_t}{1+i_{t+1}} - \frac{b_{SS}}{1+i_{SS}} \right) - \mathbb{E}_t \varepsilon_{t+1}^s$$

Solution Again, rearrange the difference equation and iterate forward:

$$\begin{aligned} \frac{b_t}{1+i_{t+1}} - \frac{b_{SS}}{1+i_{SS}} &= \frac{1}{1+r - \gamma_b^T} \mathbb{E}_t \left(\frac{b_{t+1}}{1+i_{t+2}} - \frac{b_{SS}}{1+i_{SS}} \right) + \frac{1}{1+r - \gamma_b^T} \mathbb{E}_t \varepsilon_{t+1}^s \\ &= \frac{1}{1+r - \gamma_b^T} \mathbb{E}_t \left[\left(\frac{b_{t+2}}{1+i_{t+3}} - \frac{b_{SS}}{1+i_{SS}} \right) + \mathbb{E}_t \varepsilon_{t+2}^s \right] + \frac{1}{1+r - \gamma_b^T} \mathbb{E}_t \varepsilon_{t+1}^s \\ &\vdots \\ \Leftrightarrow \frac{b_t}{1+i_{t+1}} &= \frac{b_{SS}}{1+i_{SS}} + \mathbb{E}_t \sum_{j=1}^{\infty} \left(\frac{1}{1+r - \gamma_b^T} \right)^j \varepsilon_{t+j}^s \end{aligned}$$

As before, this is also under the implicit condition that

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r - \gamma_b^T} \right)^T \mathbb{E}_t \left(\frac{b_{t+T}}{1+i_{t+T+1}} - \frac{b_{SS}}{1+i_{SS}} \right) = 0.$$

Accounting for the fact that $\gamma_b^T = 0$, the solution for real debt takes the form of Equation [\(4.13\)](#):

$$\frac{b_t}{1+i_{t+1}} = \frac{b_{SS}}{1+i_{SS}} + \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \varepsilon_{t+j}^s$$

Additional Derivations: Inflation The solution can also be plugged into the rearranged flow budget constraint of the treasury $\langle 4.3 \rangle$ to extract equilibrium inflation:

$$\frac{b_t}{1+i_{t+1}} + s_t = \frac{b_{t-1}}{\Pi_t} \Leftrightarrow \Pi_t = b_{t-1} \left(\frac{b_t}{1+i_{t+1}} + s_t \right)^{-1}$$

With the solution for real debt $\langle 4.13 \rangle$ as well as the non-Ricardian surplus rule $\langle 4.12 \rangle$, this becomes

$$\Pi_t = b_{t-1} \left(\frac{b_{SS}}{1+i_{SS}} + \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \varepsilon_{t+j}^s + s_{SS} + \varepsilon_t^s \right)^{-1}. \quad \langle B.4 \rangle$$

Rearranging the steady-state version of the treasury's flow budget equation $\langle B.3 \rangle$ gives

$$s_{SS} = \frac{b_{SS}}{\Pi^*} - \frac{b_{SS}}{1+i_{SS}} = b_{SS} \left(\frac{1+r}{1+i_{SS}} - \frac{1}{1+i_{SS}} \right) \Leftrightarrow \frac{b_{SS}}{1+i_{SS}} = \frac{s_{SS}}{r} = \frac{\beta}{1-\beta} s_{SS}. \quad \langle B.5 \rangle$$

Substituting this into Equation $\langle B.4 \rangle$ then leads to Equation $\langle 4.14 \rangle$:

$$\Pi_t = b_{t-1} \left(\frac{s_{SS}}{1-\beta} + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varepsilon_{t+j}^s \right)^{-1}$$

Additional Derivations: Differential Equation in the Nominal Interest Rate Plugging the inflation rate $\langle 4.14 \rangle$ into the Taylor rule gives

$$i_{t+1} = i_{SS} + \gamma_{\pi}^C \left(\frac{b_{t-1}}{\frac{s_{SS}}{1-\beta} + \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varepsilon_{t+j}^s} - \Pi^* \right) + \varepsilon_t^i.$$

Adding the assumption of i.i.d. policy shocks ($\varepsilon_t^s \neq 0$, $\mathbb{E}_t \varepsilon_{t+j}^s = 0 \forall j \geq 1$) and utilizing the Fisher equation $\langle 2.47 \rangle$ turns the above equation into $\langle 4.15 \rangle$:

$$\begin{aligned} i_{t+1} &= i_{SS} + \gamma_{\pi}^C \left(\frac{b_{t-1}}{\frac{s_{SS}}{1-\beta} + \varepsilon_t^s} - \Pi^* \right) + \varepsilon_t^i \\ \Leftrightarrow i_{t+1} - i_{SS} &= \beta \gamma_{\pi}^C \left[\frac{b_{t-1}}{\frac{\beta}{1-\beta} s_{SS} + \beta \varepsilon_t^s} - (1+i_{SS}) \right] + \varepsilon_t^i \end{aligned}$$

Shifting this forward one period, the policy shock term at t , ε_t^s , drops out:

$$\mathbb{E}_t i_{t+2} - i_{ss} = \beta \gamma_\pi^C \left[\frac{b_t}{\frac{\beta}{1-\beta} s_{ss}} - (1 + i_{ss}) \right] + \mathbb{E}_t \varepsilon_{t+1}^i$$

This can be simplified further by using b_t from the solution [\(4.13\)](#) as well as substituting for s_{ss} from Equation [\(B.5\)](#), resulting in Equation [\(4.16\)](#):

$$\begin{aligned} \mathbb{E}_t i_{t+2} - i_{ss} &= \beta \gamma_\pi^C \left[\frac{(1 + i_{t+1}) \frac{b_{ss}}{1 + i_{ss}}}{\frac{\beta}{1-\beta} \frac{1-\beta}{\beta} \frac{b_{ss}}{1 + i_{ss}}} - (1 + i_{ss}) \right] + \mathbb{E}_t \varepsilon_{t+1}^i \\ &= \beta \gamma_\pi^C (i_{t+1} - i_{ss}) + \mathbb{E}_t \varepsilon_{t+1}^i \end{aligned}$$

C. Appendix to Chapter 5

C.1. Legacy Models

C.1.1. Existence of Stationary State in Woodford (1995, Section 2)

The starting point for all three cases in Section 5.1.3 is a stationary state with constant policy variables and price levels. Obviously, with zero inflation, nominal and real interest rates coincide. Solving the stationary-state version of the law of motion $\langle 5.5 \rangle$ for real outstanding liabilities, we have

$$Z_{SS} = \frac{1}{\beta} \left[Z_{SS} + P_{SS} (g_{SS} - \tau_{SS}^{TH}) \right] \Leftrightarrow \frac{Z_{SS}}{P_{SS}} = \frac{\tau_{SS}^{TH} - g_{SS}}{1 - \beta}. \quad \langle C.1 \rangle$$

Further, money demand $\langle 2.22 \rangle$ reads

$$u_m(\cdot) = \frac{i_{SS}}{1 + i_{SS}} u_c(\cdot) = \frac{1 + r_{SS} - 1}{1 + r_{SS}} u_c(\cdot) = \frac{\beta^{-1} - 1}{\beta^{-1}} u_c(\cdot) = (1 - \beta) u_c(\cdot).$$

What are the arguments of $u_m(\cdot)$ and $u_c(\cdot)$? Starting with the latter, goods-market clearing $\langle 2.27 \rangle$ implies $c = y - g$. Regarding the former, in the simple case of only monetary liabilities ($B_{SS} = 0$), $M_{SS}/P_{SS} = Z_{SS}/P_{SS}$. Then, with Equation $\langle C.1 \rangle$,

$$u_m \left(\frac{\tau_{SS}^{TH} - g_{SS}}{1 - \beta} \right) = (1 - \beta) u_c(y_{SS} - g_{SS}).$$

This implies that assumption $\langle 5.1 \rangle$ does not preclude a stationary state: $(1 - \beta) \in (0, 1)$ and $y_{SS} - g_{SS}$ is usually smaller than y^{\max} so $u_c(y_{SS} - g_{SS}) > u_c(y^{\max})$. Only the rather hypothetical limiting case of $\beta = g_{SS} = 0$ would lead to a violation. Finally, the same is true in a situation with a positive amount of bonds; the argument of u_m would be reduced by b_{SS} , but this does not affect the satisfaction of assumption $\langle 5.1 \rangle$, which hinges on the right-hand side of the above equation. (Cf. Woodford 1995, p. 20.)

C.1.2. Change in Nominal Consolidated-Government Liabilities (Helicopter Money without Bonds)

Dividing the deviating and initial variants of Equation (5.5) gives

$$\frac{\dot{Z}_t}{Z_t} = \frac{(1 + i_{t+1}) (Z_{t-1} + \dot{P}_t \tau_t^{TH})}{(1 + i_{t+1}) (Z_{t-1} + P_t \tau_t^{TH})}.$$

With the help of Equations (5.6) and (5.7) in the numerator and $\tau_t^{TH} = \tau_{SS}^{TH}$ for the initial variant in the denominator, this can be rearranged to

$$\begin{aligned} \frac{\dot{Z}_t}{Z_t} &= \frac{(1 + i_{t+1}) \left[Z_{t-1} + \dot{P}_t \left(\tau_{SS}^{TH} + \dot{\phi} \frac{M_t}{\dot{P}_t} \right) \right]}{(1 + i_{t+1}) (Z_{t-1} + P_t \tau_{SS}^{TH})} \\ &= \frac{(1 + i_{t+1}) (Z_{t-1} + \dot{P}_t \tau_{SS}^{TH} + \dot{\phi} M_t)}{(1 + i_{t+1}) (Z_{t-1} + P_t \tau_{SS}^{TH})} \\ &= \frac{(1 + i_{t+1}) [Z_{t-1} + (1 + \dot{\phi}) P_t \tau_{SS}^{TH} + \dot{\phi} M_t]}{(1 + i_{t+1}) (Z_{t-1} + P_t \tau_{SS}^{TH})}. \end{aligned}$$

Since the initial situation was a stationary state, it holds that $Z_{t-1} = M_{t-1} = M_{SS} = M_t = Z_t$. Turning both Z_{t-1} on the right-hand side into M_t and, in the second step, the Z_t in the denominator on the left-hand side into Z_{t-1} , we arrive at Equation (5.8):

$$\begin{aligned} \frac{\dot{Z}_t}{Z_t} &= \frac{(1 + i_{t+1}) (1 + \dot{\phi}) (P_t \tau_{SS}^{TH} + M_t)}{(1 + i_{t+1}) (P_t \tau_{SS}^{TH} + M_t)} \\ &\Leftrightarrow \dot{Z}_t = (1 + \dot{\phi}) Z_{t-1} \end{aligned}$$

C.2. Ricardian Fiscal Policy with Money

Similarly to the procedure in Section 4.4, combine the surplus rule (5.9) with the respective flow budget equation of the consolidated-government (2.40):

$$\begin{aligned} \frac{Z_{t-1}}{P_t} &= \mathbb{E}_t \left(v_{t,t+1} \frac{Z_t}{P_{t+1}} \right) + \dot{I}_{t+1} m_t + s_t = \mathbb{E}_t \left(v_{t,t+1} \frac{Z_t}{P_{t+1}} \right) + \gamma_Z^T \frac{Z_{t-1}}{P_t} \\ &\Leftrightarrow \mathbb{E}_t \left(v_{t,t+1} \frac{Z_t}{P_{t+1}} \right) = (1 - \gamma_Z^T) \frac{Z_{t-1}}{P_t} \end{aligned}$$

$$\Rightarrow \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}}{P_{t+J+1}} \right) = \left(1 - \gamma_Z^T \right)^{J+1} \frac{Z_{t-1}}{P_t}$$

This is analogous to Equation (4.21) and thus implies that the respective transversality condition

$$\lim_{J \rightarrow \infty} \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}}{P_{t+J+1}} \right) = 0$$

is always satisfied.

Also, rule (5.9) makes the present-value budget equation (2.43) hold identically:

$$\begin{aligned} \frac{Z_{t-1}}{P_t} &= \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} (\dot{I}_{t+j+1} m_{t+j} + s_{t+j}) = \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \gamma_Z^T \frac{Z_{t+j-1}}{P_{t+j}} \\ &= \gamma_Z^T \frac{Z_{t-1}}{P_{t+j}} \sum_{j=0}^J \left(1 - \gamma_Z^T \right)^j = \gamma_Z^T \frac{Z_{t-1}}{P_{t+j}} \frac{1}{1 - (1 - \gamma_Z^T)} = \frac{Z_{t-1}}{P_{t+j}} \quad \langle \text{C.2} \rangle \end{aligned}$$

C.3. The Zero Lower Bound

C.3.1. The Null of the Taylor Rule

Adding unity on both sides of the Taylor rule (4.5) (or rather the respective part of (5.13)),

$$1 + i_{t+1} = (1 + i^*) + \gamma_\pi^C (\pi_t - \pi^*) + \varepsilon_t^i,$$

and then setting $i_{t+1} = 0$ gives

$$1 = (1 + r) (1 + \pi^*) + \gamma_\pi^C (\pi_t - \pi^*) + \varepsilon_t^i$$

(note that we still consider a constant-endowment economy as described in Section 2.4). This can be solved for π_t with a few rearrangements:

$$\begin{aligned}
 \gamma_{\pi}^C (\pi_t - \pi^*) &= 1 - (1+r) (1 + \pi^*) - \varepsilon_t^i \\
 &= - (1+r) \pi^* - r - \varepsilon_t^i \\
 \Leftrightarrow \pi_t &= \pi^* - \frac{(1+r) \pi^* + r + \varepsilon_t^i}{\gamma_{\pi}^C} \\
 &= \frac{\gamma_{\pi}^C - (1+r)}{\gamma_{\pi}^C} \pi^* - \frac{r}{\gamma_{\pi}^C} - \frac{\varepsilon_t^i}{\gamma_{\pi}^C}
 \end{aligned}$$

Since we are interested in ‘regular’ reactions to inflation instead of discretionary action, we can discard the monetary policy shock ε_t^i here and obtain the critical value for inflation π^{ZLB} as reported in Equation [5.14](#):

$$\pi^{\text{ZLB}} = \frac{\gamma_{\pi}^C - (1+r)}{\gamma_{\pi}^C} \pi^* - \frac{r}{\gamma_{\pi}^C}$$

C.3.2. An Alternative Scenario for the Inflation Law of Motion

In Figure 5.2 of Section 5.2.1, the horizontal part of the phase line $\mathbb{E}_t \pi_{t+1} = \mathbb{F}(\pi_t)$ and the 45° line intersect, resulting in an alternative steady-state inflation rate $\pi'_{\text{SS}} < 0$ (a stable deflationary trap).

This appendix describes an alternative scenario in which the phase and 45° lines do not intersect. Figure C.1 provides an illustration. As in the main text, the inflation process starts with an initial $\pi_0 < \pi^*$ and progresses to point A, which corresponds to $\mathbb{E}_t \pi_{t+4}$, over time. Via point B, the process reaches $\mathbb{E}_t \pi_{t+5}$ in point C on the horizontal part of the phase line. Again, moving to the next value $\mathbb{E}_t \pi_{t+6}$ requires to first move the ‘current’ value $\mathbb{E}_t \pi_{t+5}$ from the ordinate to the abscissa, that is, a horizontal move to the 45° line (point D). In contrast to the scenario of the main text, however, the zero lower bound does not bind here, so the central bank is actually expected to abide by the Taylor rule [5.13](#) and set a nominal interest rate $\mathbb{E}_t i_{t+5} > 0$. Consequently, the inflation process does not remain at the level π'_{SS} but ‘takes another lap’ via E, F, and G. Of course, it does not remain there now but repeats this cycle (D, E, F, G, D, ...) indefinitely; in terms of inflation rates, this shows up as infinite fluctuation between π'_{SS} and π''_{SS} . (Such a *limit cycle* is consistent with the findings of Benhabib, Schmitt-Grohé, and Uribe 2001.)

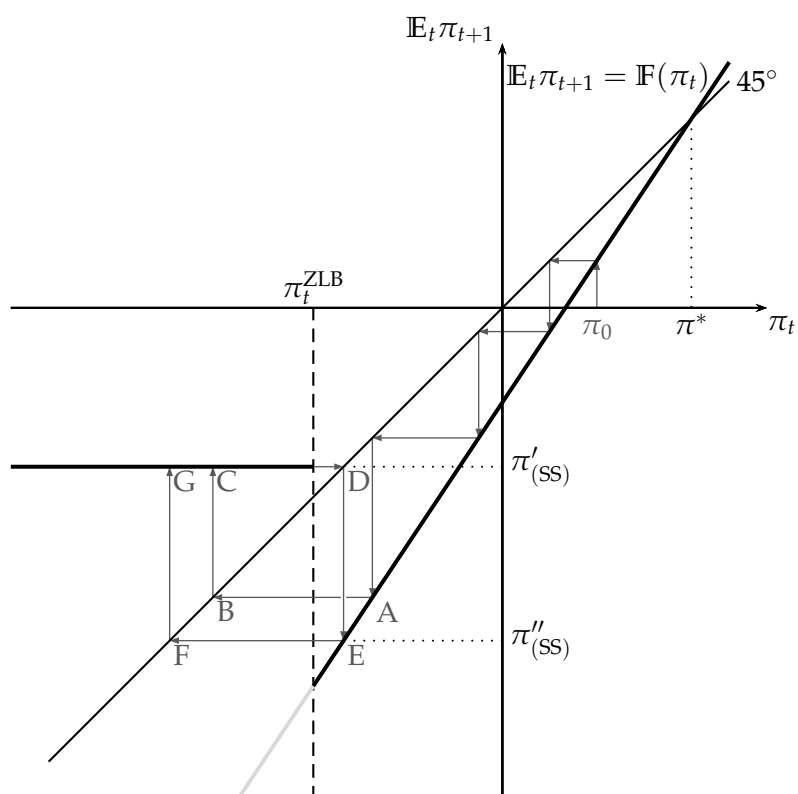


Figure C.1: *Inflation Dynamics in Consideration of the Zero Lower Bound on Nominal Interest Rates (Alternative Scenario).* ♦ Source: Own illustration.

C.3.3. A Surplus Rule for Liability Growth Targeting

Solving the liability growth rule (5.18) for outstanding liabilities gives

$$Z_t = (1 + \mu_Z) Z_{t-1}.$$

Similarly, solving the consolidated-government budget equation (2.36) for outstanding liabilities (dropping the ‘dis’ superscripts and using definition (2.28)) yields

$$\begin{aligned} \frac{1}{1 + i_{t+1}} Z_t + \dot{I}_{t+1} M_t &= Z_{t-1} - S_t \\ \Leftrightarrow Z_t &= (1 + i_{t+1}) (Z_{t-1} - S_t - \dot{I}_{t+1} M_t). \end{aligned}$$

Equating the above expressions for Z_t then leads to Equation (5.19):

$$\begin{aligned} (1 + \mu_Z) Z_{t-1} &= (1 + i_{t+1}) (Z_{t-1} - S_t - \dot{I}_{t+1} M_t) \\ \Leftrightarrow S_t &= Z_{t-1} - \frac{1 + \mu_Z}{1 + i_{t+1}} Z_{t-1} - \dot{I}_{t+1} M_t \\ &= \frac{i_{t+1} - \mu_Z}{1 + i_{t+1}} Z_{t-1} - \dot{I}_{t+1} M_t \end{aligned}$$

C.3.4. Law of Motion for Real Balances

Solve the implicit money demand function (2.22) for i_{t+1} (under the continued assumption that utility is additively separable):

$$\begin{aligned} \frac{i_{t+1}}{1 + i_{t+1}} &= \frac{u_m(m_t)}{u_c(c_t)} \quad \Leftrightarrow \quad i_{t+1} = (1 + i_{t+1}) \frac{u_m(m_t)}{u_c(c_t)} \\ \Leftrightarrow i_{t+1} \left[1 - \frac{u_m(m_t)}{u_c(c_t)} \right] &= \frac{u_m(m_t)}{u_c(c_t)} \quad \Leftrightarrow \quad i_{t+1} = \frac{u_m(m_t)}{u_c(c_t)} \left/ \left[1 - \frac{u_m(m_t)}{u_c(c_t)} \right] \right. \end{aligned}$$

Then solve the Fisher equation (2.47) for i_{t+1} as well,

$$i_{t+1} = \beta^{-1} \mathbb{E}_t \Pi_{t+1} - 1,$$

and equalize both results:

$$\begin{aligned}\beta^{-1}\mathbb{E}_t\Pi_{t+1} - 1 &= \frac{u_m(m_t)}{u_c(c_t)} \Big/ \left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right] \\ \Leftrightarrow \beta^{-1}\frac{\mathbb{E}_t P_{t+1}}{P_t} &= \left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right]^{-1} \\ \Leftrightarrow \left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right] \frac{1}{P_t} &= \beta \frac{1}{\mathbb{E}_t P_{t+1}}\end{aligned}$$

Multiply through with $M_t \mathbb{E}_t M_{t+1}$ and note that $\mathbb{E}_t M_{t+1} / M_t = \mu_M$ to arrive at Equation [⟨5.23⟩](#):

$$\begin{aligned}\left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right] m_t \mathbb{E}_t M_{t+1} &= \beta M_t m_{t+1} \\ \Leftrightarrow \left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right] m_t &= \frac{\beta}{\mu_M} \mathbb{E}_t m_{t+1} \\ \Leftrightarrow \mathbb{E}_t m_{t+1} &= \frac{\mu_M}{\beta} \left[1 - \frac{u_m(m_t)}{u_c(c_t)}\right] m_t\end{aligned}$$

C.4. Debt Limits

C.4.1. Satisfaction of the Transversality Condition Under a Liability Limit

Finiteness of Planned Expenditure The following preparatory argument is a streamlined version of the respective deliberations presented in Woodford (2003b, pp. 64-70). Consider again the iterated version [⟨2.33⟩](#) ($J \rightarrow \infty$) of the present-value budget constraint, which is repeated here with the transversality condition [⟨2.34⟩](#) already in place (the rearrangements in the second line also use Equation [⟨2.17⟩](#)):

$$\begin{aligned}Z_{t-1} &= \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} \left[P_{t+j} c_{t+j} + \dot{I}_{t+j+1} M_{t+j} - P_{t+j} (y_{t+j} + t_{t+j}^{TH}) \right] \\ \Leftrightarrow \frac{Z_{t-1}}{P_t} &= \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left[c_{t+j} + \dot{I}_{t+j+1} m_{t+j} - (y_{t+j} + t_{t+j}^{TH}) \right] \quad \langle C.3 \rangle\end{aligned}$$

By assumption [⟨2.31⟩](#), the present value of disposable income (the term after the minus sign on the right-hand side) is finite. Rewriting it in real terms so as to make it

compatible with Equation ⟨C.3⟩ gives

$$\mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(y_{t+j} + t_{t+j}^{TH} \right) < \infty$$

(dividing through by P_t and using Equation ⟨2.17⟩ does not invalidate Equation ⟨2.31⟩). By implication, the difference between planned expenditure and disposable income—i.e., the entire sum on the right-hand side of Equation ⟨C.3⟩—can only be infinite if Z_{t-1}/P_t is infinite as well. Woodford does not mention this explicitly in his treatment, but it seems reasonable that this is not the case (to put it bluntly: why study economics when there is infinite wealth initially?). Hence, the present value of planned expenditure must also be finite:

$$\mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} (c_{t+j} + \dot{I}_{t+j+1} m_{t+j}) < \infty$$

By implication, the individual components of the sum (the present values of planned consumption and money holdings, respectively) must be finite, too. Furthermore, to achieve this over an infinite time horizon, individual summands must converge to zero over time:

$$\lim_{J \rightarrow \infty} \mathbb{E}_t (v_{t,t+J} \dot{I}_{t+J+1} m_{t+J}) = 0 \quad \langle C.4 \rangle$$

Technical Preparations Multiplying both sides of the implicit money demand equation ⟨2.22⟩ by $-\mathbb{E}_t[\beta^J m_{t+J}/u_c(c_{t+J-1})]$ and, in the second step, using Equation ⟨2.16⟩ gives

$$\begin{aligned} -\mathbb{E}_t \left[\beta^J \frac{u_m(m_{t+J})}{u_c(c_{t+J-1})} m_{t+J} \right] &= -\mathbb{E}_t \left[\beta^J \frac{u_c(c_{t+J})}{u_c(c_{t+J-1})} \frac{i_{t+J+1}}{1+i_{t+J+1}} m_{t+J} \right] \\ &= -\mathbb{E}_t \left[v_{t,t+J} \frac{i_{t+J+1}}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} \right]. \end{aligned}$$

In order to be able to make an argument below, it has to be shown that the far-right-hand side is smaller than

$$\mathbb{E}_t \left[v_{t,t+J} \left(\frac{1}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} + \frac{Q_{t+J} B_{t+J}}{P_{t+J}} \right) \right],$$

which can be done by writing down the respective inequality and rearranging:

$$\begin{aligned}
 -\mathbb{E}_t \left(v_{t,t+J} \frac{i_{t+J+1}}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} \right) &\leq \mathbb{E}_t \left[v_{t,t+J} \left(\frac{1}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} + \frac{Q_{t+J} B_{t+J}}{P_{t+J}} \right) \right] \\
 \Leftrightarrow -\mathbb{E}_t \left(\frac{i_{t+J+1}}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} \right) &\leq \mathbb{E}_t \left(\frac{1}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} + \frac{Q_{t+J} B_{t+J}}{P_{t+J}} \right) \\
 \Leftrightarrow 0 &\leq \mathbb{E}_t \left(\frac{M_{t+J} + Q_{t+J} B_{t+J}}{P_{t+J}} \right)
 \end{aligned}$$

Since the last line repeats the first part of assumption $\langle 5.24 \rangle$, the posited inequality holds.

Automatic Satisfaction of the Transversality Condition Consider the following chain of (in)equations:

$$\begin{aligned}
 -\mathbb{E}_t \left[\beta^J \frac{u_m(m_{t+J})}{u_c(c_t)} m_{t+J} \right] &= -\mathbb{E}_t \left(v_{t,t+J} \frac{i_{t+J+1}}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} \right) \\
 &\leq \mathbb{E}_t \left[v_{t,t+J} \left(\frac{1}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} + \frac{Q_{t+J} B_{t+J}}{P_{t+J}} \right) \right] \\
 &\leq \mathbb{E}_t \left(v_{t,t+J} \frac{M_{t+J} + Q_{t+J} B_{t+J}}{P_{t+J}} \right) \\
 &\leq \mathbb{E}_t v_{t,t+J} \bar{z}
 \end{aligned}$$

The first two lines are explained above (see \nearrow *Technical Preparations*). The third line is true if net nominal interest rates are non-negative, either because of an explicit zero lower bound (as in Section 5.2.1) or because of the added assumption that policy simply does not implement negative nominal interest rates. The fourth line is part of assumption $\langle 5.24 \rangle$.

As stated by Equation $\langle C.4 \rangle$, the right-hand side of the first line approaches zero as $J \rightarrow \infty$. Further, since \bar{z} is a finite value, the term in the last line also approaches zero (assuming $\mathbb{E}_t u_c[c_{t+J}] < \infty$, which is the case for positive consumption levels, cf. Footnote 2 on p. 30).

It is important to note that the term in the second line can be rearranged into the term in the transversality condition $\langle 2.42 \rangle$ (using several equations from Chapter 2):

$$\mathbb{E}_t \left[v_{t,t+J} \left(\frac{1}{1+i_{t+J+1}} \frac{M_{t+J}}{P_{t+J}} + \frac{Q_{t+J} B_{t+J}}{P_{t+J}} \right) \right] = \mathbb{E}_t \left(v_{t,t+J} \frac{1}{1+i_{t+J+1}} \frac{M_{t+J} + B_{t+J}}{P_{t+J}} \right)$$

$$\begin{aligned}
&= \mathbb{E}_t \left[v_{t,t+J} \left(v_{t+J,t+J+1} \frac{P_{t+J}}{P_{t+J+1}} \right) \frac{M_{t+J} + B_{t+J}}{P_{t+J}} \right] \\
&= \mathbb{E}_t \left(v_{t,t+J+1} \frac{M_{t+J} + B_{t+J}}{P_{t+J+1}} \right) = \mathbb{E}_t \left(v_{t,t+J+1} \frac{Z_{t+J}}{P_{t+J+1}} \right)
\end{aligned}$$

Since it is always bounded above and below by zero, and the whole chain of inequations stems from an equilibrium condition (implicit money demand (2.22)), the transversality condition (2.42) is always satisfied.

C.4.2. Steady States with a Liability Limit

This section also assumes constant endowments (cf. Section 2.4).

The ‘Normal’ Steady State The unique steady state in the absence of a liability limit can be found simply by setting $\mathbb{E}_t Z_{t+1}/P_t = Z_t/P_{t-1} \equiv z_{SS}$ in the consolidated-government budget equation (2.40) (immediately substituting β for the stochastic discount factor v):

$$z_{SS} = \beta z_{SS} + \dot{m}_{SS} + s_{SS} \quad \Leftrightarrow \quad z_{SS} = \frac{1}{1-\beta} (\dot{m}_{SS} + s_{SS}) > 0$$

Additional Steady States Implied by the Liability Limit With the help of Equations (2.8), (2.23), and (2.28), the numerator of the central term in (5.24) can be rearranged to

$$\begin{aligned}
M_t + Q_t B_t &= M_t + \frac{1}{1+i_{t+1}} B_t + (\dot{I}_{t+1} - \dot{I}_{t+1}) B_t \\
&= M_t + B_t - \dot{I}_{t+1} B_t = Z_t - \dot{I}_{t+1} (Z_t - M_t) = \frac{1}{1+i_{t+1}} Z_t + \dot{I}_{t+1} M_t.
\end{aligned}$$

Combining it with the denominator again and substituting out i_{t+1} by use of Equations (2.47) as well as (2.6) then yields

$$\frac{M_t + Q_t B_t}{P_t} = \frac{1}{1+i_{t+1}} \frac{Z_t}{P_t} + \dot{I}_{t+1} m_t = \left(\beta \frac{P_t}{\mathbb{E}_t P_{t+1}} \right) \frac{Z_t}{P_t} + \dot{I}_{t+1} m_t = \beta \frac{Z_t}{\mathbb{E}_t P_{t+1}} + \dot{I}_{t+1} m_t. \quad \langle C.5 \rangle$$

The liability limit places two bounds on Equation (C.5). Starting with the lower one (in steady-state notation):

$$\beta z_{SS} + \dot{I}_{SS} m_{SS} = 0 \quad \Leftrightarrow \quad z'_{SS} = -\beta^{-1} \dot{I}_{SS} m_{SS} < 0$$

Similarly, the higher one is given by

$$\beta z_{ss} + \dot{I}_{ss} m_{ss} = \bar{z} \quad \Leftrightarrow \quad z''_{ss} = \beta^{-1} (\bar{z} - \dot{I}_{ss} m_{ss}) > z^*,$$

which lies above the ‘normal’ steady state z^* if it is assumed that

$$\bar{z} > \frac{1}{1-\beta} (\beta s_{ss} + \dot{I}_{ss} m_{ss}) > 0$$

(cf. Woodford 2001, p. 699).

D. Appendix to Chapter 6

D.1. Present-Value Budget Constraints

D.1.1. Country-Specific Present-Value Budget Constraints

The Domestic household flow budget constraint (6.1) can be rearranged to

$$\left(B_{t-1}^{D,D} + B_{t-1}^{D,F}\right) = Q_t \left(B_t^{D,D} + B_t^{D,F}\right) + P_t c_t^D - P_t y_t^D - P_t t_t^{DTH} + M_t^D - M_{t-1}^D$$

and iterated forward via the total amount of bonds $(B^{D,D} + B^{D,F})$,

$$\begin{aligned} \left(B_{t-1}^{D,D} + B_{t-1}^{D,F}\right) &= \mathbb{E}_t \left\{ V_{t,t+1} \left[V_{t+1,t+2} \left(B_{t+1}^{D,D} + B_{t+1}^{D,F} \right) + P_{t+1} c_{t+1}^D - P_{t+1} y_{t+1}^D - P_{t+1} t_{t+1}^{DTH} \right. \right. \\ &\quad \left. \left. + M_{t+1}^D - M_t^D \right] + P_t c_t^D - P_t y_t^D - P_t t_t^{DTH} + M_t^D - M_{t-1}^D \right\}, \\ &\vdots \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} \left(P_{t+j} c_{t+j}^D - P_{t+j} y_{t+j}^D - P_{t+j} t_{t+j}^{DTH} + M_{t+j}^D - M_{t+j-1}^D \right), \end{aligned} \tag{D.1}$$

where the bond price Q is replaced by the nominal stochastic discount factor V via Equation (2.19) and the transversality condition holds:

$$\lim_{j \rightarrow \infty} \mathbb{E}_t \left[V_{t,t+j+1} \left(B_{t+j}^{D,D} + B_{t+j}^{D,F} \right) \right] = 0$$

Section D.1.2 just below demonstrates how the two adjacent terms $M_{t+j}^D - M_{t+j-1}^D$ in the infinite sum can be rearranged such that

$$B_{t-1}^{D,D} + B_{t-1}^{D,F} = \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} \left(P_{t+j} c_{t+j}^D - P_{t+j} y_{t+j}^D - P_{t+j} t_{t+j}^{DTH} + M_{t+j}^D \right) - M_{t-1}^D.$$

Bringing M_{t-1} to the left-hand side and applying the definition of Z_{t-1}^{DH} (6.6) then gives

$$Z_{t-1}^{DH} = \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} \left(C_{t+j}^D - Y_{t+j}^D - T_{t+j}^{DTH} + \dot{I}_{t+j+1} M_{t+j}^D \right).$$

By utilizing the properties of \nearrow Stochastic Discount Factors (p. 26), one arrives at Equation (6.5) (proceed analogously for Foreign):

$$\begin{aligned} \frac{Z_{t-1}^{DH}}{P_t} &= \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(c_{t+j}^D - y_{t+j}^D - t_{t+j}^{DTH} + \dot{I}_{t+j+1} m_{t+j}^D \right) \\ \Leftrightarrow \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(c_{t+j}^D + \dot{I}_{t+j+1} m_{t+j}^D \right) &= \mathbb{E}_t \sum_{j=0}^{\infty} v_{t,t+j} \left(y_{t+j}^D + t_{t+j}^{DTH} \right) + \frac{Z_{t-1}^{DH}}{P_t} \end{aligned}$$

D.1.2. Two Ways of Including Money in the Present-Value Budget Constraint

The (unpolished) present-value budget constraint (D.1) features money creation ($M_{t+j}^D - M_{t+j-1}^D$) on the right-hand side. Limiting focus to this term, the infinite sum can be expanded and rearranged:

$$\begin{aligned} \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} (M_{t+j} - M_{t+j-1}) &= M_t - M_{t-1} + \mathbb{E}_t \left(\frac{M_{t+1} - M_t}{1 + i_{t+1}} \right) + \mathbb{E}_t \left[\frac{M_{t+2} - M_{t+1}}{(1 + i_{t+1})(1 + i_{t+2})} \right] + \dots \\ &= -M_{t-1} + M_t \left(\frac{i_{t+1}}{1 + i_{t+1}} \right) + \mathbb{E}_t \left\{ M_{t+1} \left[\frac{i_{t+2}}{(1 + i_{t+1})(1 + i_{t+2})} \right] \right\} + \dots \\ &= -M_{t-1} + M_t \dot{I}_{t+1} + \mathbb{E}_t \left(M_{t+1} \dot{I}_{t+2} \frac{1}{1 + i_{t+1}} \right) + \dots \\ &= -M_{t-1} + \mathbb{E}_t \sum_{j=0}^{\infty} V_{t,t+j} \dot{I}_{t+j+1} M_{t+j} \end{aligned}$$

At this, country superscripts as well as expectation operators are dropped to save on space. Further, the above rearrangements utilize the properties of \nearrow Stochastic Discount Factors (p. 26).

D.1.3. Union-Wide Present-Value Budget Constraint

Add the individual household budget constraints $\langle 6.1 \rangle$,

$$\begin{aligned} & \left(C_t^D + M_t^D + Q_t B_t^{D,D} + Q_t B_t^{D,F} \right) + \left(C_t^F + M_t^F + Q_t B_t^{F,D} + Q_t B_t^{F,F} \right) \\ &= \left(Y_t^D + T_t^{DTH} + B_{t-1}^{D,D} + B_{t-1}^{D,F} + M_{t-1}^D \right) + \left(Y_t^F + T_t^{FTH} + B_{t-1}^{F,D} + B_{t-1}^{F,F} + M_{t-1}^F \right), \end{aligned}$$

and use the goods-market-clearing condition $\langle 6.2 \rangle$ to substitute out Y_t^\square as well as C_t^\square , leaving only G_t^\square :

$$\begin{aligned} & M_t^D + Q_t B_t^{D,D} + Q_t B_t^{D,F} + M_t^F + Q_t B_t^{F,D} + Q_t B_t^{F,F} \\ &= G_t^D + T_t^{DTH} + B_{t-1}^{D,D} + B_{t-1}^{D,F} + M_{t-1}^D + G_t^F + T_t^{FTH} + B_{t-1}^{F,D} + B_{t-1}^{F,F} + M_{t-1}^F \end{aligned}$$

Then, use Equation $\langle 2.10 \rangle$ to substitute in surpluses S_t^\square ,

$$\begin{aligned} & \left(M_t^D + Q_t B_t^{D,D} + Q_t B_t^{D,F} \right) + \left(M_t^F + Q_t B_t^{F,D} + Q_t B_t^{F,F} \right) + S_t^D + S_t^F \\ &= \left(B_{t-1}^{D,D} + B_{t-1}^{D,F} + M_{t-1}^D \right) + \left(B_{t-1}^{F,D} + B_{t-1}^{F,F} + M_{t-1}^F \right), \end{aligned}$$

and apply bond-market-clearing conditions $\langle 6.3 \rangle$ as well as money-market clearing $\langle 6.4 \rangle$ to simplify:

$$\begin{aligned} & M_t + S_t^D + S_t^F + Q_t B_t^D + Q_t B_t^F = M_{t-1} + B_{t-1}^D + B_{t-1}^F \\ & \Leftrightarrow B_{t-1}^D + B_{t-1}^F = Q_t \left(B_t^D + B_t^F \right) + S_t^D + S_t^F + M_t - M_{t-1} \end{aligned}$$

In the next step, use Equation $\langle 2.19 \rangle$ to replace the bond price Q by the nominal stochastic discount factor V :

$$\begin{aligned} & B_{t-1}^D + B_{t-1}^F = \mathbb{E}_t V_{t,t+1} \left(B_t^D + B_t^F \right) + S_t^D + S_t^F + M_t - M_{t-1} \\ & \Leftrightarrow B_{t-1}^D + B_{t-1}^F + M_{t-1} = \mathbb{E}_t V_{t,t+1} \left(B_t^D + B_t^F + M_t \right) + S_t^D + S_t^F + \dot{\mathbf{I}}_{t+1} M_t \end{aligned}$$

Finally, use definition $\langle 6.8 \rangle$ to receive Equation $\langle 6.9 \rangle$:

$$Z_{t-1}^U = \mathbb{E}_t V_{t,t+1} Z_t^U + S_t^D + S_t^F + \dot{\mathbf{I}}_{t+1} M_t$$

Iterating forward in the usual way then yields the respective present-value budget equation (6.10).

D.1.4. Ricardian Policy Rules in the Union Present-Value Budget Constraint

Rewriting the union-wide present-value budget equation (6.10) as

$$\frac{B_{t-1}^D + B_{t-1}^F}{P_t} = -\frac{M_{t-1}}{P_t} + \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \left(s_{t+j}^D + s_{t+j}^F + \frac{\dot{I}_{t+j+1} M_{t+j}}{P_{t+j}} \right)$$

and rearranging the money-related terms similarly to Appendix D.1.2 gives

$$\begin{aligned} \frac{B_{t-1}^D + B_{t-1}^F}{P_t} &= \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \left(s_{t+j}^D + s_{t+j}^F + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) \\ &= \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \left[s_{t+j}^D + s_{t+j}^F + \left(m_{t+j} - \frac{m_{t+j-1}}{\Pi_{t+j}} \right) \right]. \end{aligned}$$

In combination with rule (6.15) for s_t^F , this turns into Equation (6.16):

$$\frac{B_{t-1}^D}{P_t} = \mathbb{E}_t \sum_{j=0}^J v_{t,t+j} \left[s_{t+j}^D + 0.5 \left(m_{t+j} - \frac{m_{t+j-1}}{\Pi_{t+j}} \right) \right]$$

D.2. Variables and Equations in the Monetary-Union Model

no.	Equation	description
<i>'normal' equations</i>		
1-2	$\langle 6.1 \rangle$	household flow budget constraints
3	$\langle 2.5 \rangle / \langle 2.7 \rangle$	Fisher equation
4	$\langle 2.8 \rangle$	no-arbitrage condition for bond prices
5-6	$\langle 2.12 \rangle$	first-order conditions with respect to consumption
7-8	$\langle 2.14 \rangle$	first-order conditions with respect to money
9-10	$\langle 2.15 \rangle$	first-order conditions with respect to bonds
11	$\langle 6.2 \rangle$	goods-market clearing
12-13	$\langle 6.3 \rangle$	bond-market clearing
14	$\langle 6.4 \rangle$	money-market clearing
15-16	$\langle 6.13 \rangle$	treasury flow budget constraints
<i>'mere definitions'</i>		
17	$\langle 2.6 \rangle$	inflation rate
18-19	$\langle 2.10 \rangle$	budget surplus of the treasury
20	$\langle 2.16 \rangle$	real stochastic discount factor
21	$\langle 2.17 \rangle$	nominal stochastic discount factor
22	$\langle 2.23 \rangle$	opportunity cost of holding money
23	$\langle 6.7 \rangle$	union-wide bond supply
24	$\langle 6.8 \rangle$	outstanding liabilities of all public entities

Table D.1a: *Equations Available for Equilibrium Determination in the Monetary-Union Model.* \diamond *Explanations:* See \nearrow *Counting Variables and Equations* on p. 110.

no.	variable		determination	description
	D	F	U	
'truly' exogenous				
1-2	y^D	y^F	exogenous	endowment
quasi-exogenous in the constant-endowment economy				
3-4	g^D	g^F	policy	treasury expenditure
5-6	c^D	g^F	(2.27)	consumption
7-8	λ^D	λ^F	(2.12)	Lagrange parameter for the budget constraint
9			(2.15)	real interest rate
endogenous by definition				
10			Π (2.6)	inflation rate
11-12	s^D	s^F	(2.10)	budget surplus of the treasury
13			v (2.16)	real stochastic discount factor
14			V (2.17)	nominal stochastic discount factor
15			\dot{I} (2.23)	opportunity cost of holding money
16			B^U (6.7)	union-wide bond supply
17			Z^U (6.8)	outstanding liabilities of all public entities
to be determined by equations or set exogenously by policy				
18			M	money supply
19-20	M^D	M^F		money demand
21-22	t^{DTH}	t^{FTH}		net transfers from the treasury to the household
23-24	B^D	B^F		bond supply
25-26	B^{DD}	B^{FF}		bond holdings (internal)
27-28	B^{DF}	B^{FD}		bond holdings (cross-border)
29			i	nominal interest rate
30			P	price level
31			Q	bond price

Table D.1b: *Overview of Variables in the Monetary-Union Model.* \diamond *Explanations:* Two exogenous variables, five determined by policy; see \nearrow *Counting Variables and Equations* on p. 110.

E. Log-Linearization

This part of the appendix gives an overview about the methods used to linearize the model of Chapter 7. It mostly resembles Zietz (2006; 2008), especially with regards to brevity. Alternative references are Campbell (1994), Uhlig (1997; 1999), and Mc-Candless (2008, pp. 95-100).

E.1. Taylor Series Expansion I

General Description A function $\mathbb{F}(\square + \Delta)$ can be expressed by a J^{th} -degree polynomial function, or Taylor series,

$$\mathbb{F}(\square + \Delta) \approx \sum_{j=0}^J \frac{\mathbb{F}^{(j)}(\square)}{j!} \Delta^j$$
$$\text{or } \mathbb{F}(\square + \Delta) \equiv \sum_{j=0}^J \frac{\mathbb{F}^{(j)}(\square)}{j!} \Delta^j + \text{remainder}_J,$$

where the superscript (j) indicates the j^{th} derivative of $\mathbb{F}(\cdot)$. The difference between the approximative Taylor series and the original function is often called the remainder, which is increasing in Δ and decreasing in J —in other words, the approximation is more precise closer to \square and the higher the degree J (Simon and Blume 2006, pp. 827-831).

Specific Application Macroeconomic applications often use first- or second-degree Taylor-series approximations of a discrete-time variable \square_t around its steady state value \square_{ss} :

$$\mathbb{F}(\square_t) \approx \sum_{j=0}^J \frac{\mathbb{F}^{(j)}(\square_{\text{ss}})}{j!} (\square_t - \square_{\text{ss}})^j$$

Since $0! = 1$, this becomes

$$\mathbb{F}(\square_t) \approx \mathbb{F}(\square_{\text{ss}}) + \mathbb{F}'(\square_{\text{ss}}) (\square_t - \square_{\text{ss}})$$

$$\mathbb{F}(\square_t) \approx \mathbb{F}(\square_{\text{ss}}) + \mathbb{F}'(\square_{\text{ss}}) (\square_t - \square_{\text{ss}}) + \frac{1}{2} \mathbb{F}''(\square_{\text{ss}}) (\square_t - \square_{\text{ss}})^2,$$

where the approximation (remainder, if it were stated) of the second line is better (smaller) than that of the first one.

Multivariate Functions Functions of more than one variable are approximated similarly in principle, but the necessary additional computations typically make it more tedious. Just to give an impression, consider the second-degree Taylor series of a function of two variables in the general ‘notation’ (cf. Simon and Blume 2006, pp. 832-833):

$$\begin{aligned} \mathbb{F}(\square + \Delta, \bigcirc + \nabla) &\approx \mathbb{F}(\square, \bigcirc) + \mathbb{F}_{\square}(\square, \bigcirc) \Delta + \mathbb{F}_{\bigcirc}(\square, \bigcirc) \nabla \\ &\quad + \frac{1}{2} [\mathbb{F}_{\square\square}(\square, \bigcirc) \Delta^2 + 2 \mathbb{F}_{\square\bigcirc}(\square, \bigcirc) \Delta \nabla + \mathbb{F}_{\bigcirc\bigcirc}(\square, \bigcirc) \nabla^2] \end{aligned}$$

In a macroeconomic context, the respective first- and second-order Taylor-series approximations are of the following forms:

$$\begin{aligned} \mathbb{F}(\square_t, \bigcirc_t) &\approx \mathbb{F}(\square_{ss}, \bigcirc_{ss}) + \mathbb{F}_{\square}(\square_{ss}, \bigcirc_{ss}) (\square_t - \square_{ss}) + \mathbb{F}_{\bigcirc}(\square_{ss}, \bigcirc_{ss}) (\bigcirc_t - \bigcirc_{ss}) \\ \mathbb{F}(\square_t, \bigcirc_t) &\approx \mathbb{F}(\square_{ss}, \bigcirc_{ss}) + \mathbb{F}_{\square}(\square_{ss}, \bigcirc_{ss}) (\square_t - \square_{ss}) + \mathbb{F}_{\bigcirc}(\square_{ss}, \bigcirc_{ss}) (\bigcirc_t - \bigcirc_{ss}) \\ &\quad + \frac{1}{2} [\mathbb{F}_{\square\square}(\square_{ss}, \bigcirc_{ss}) (\square_t - \square_{ss})^2 \\ &\quad + 2 \mathbb{F}_{\square\bigcirc}(\square_{ss}, \bigcirc_{ss}) (\square_t - \square_{ss}) (\bigcirc_t - \bigcirc_{ss}) \\ &\quad + \mathbb{F}_{\bigcirc\bigcirc}(\square_{ss}, \bigcirc_{ss}) (\bigcirc_t - \bigcirc_{ss})^2] \end{aligned}$$

E.2. Logarithms

Simple Form Recall and rearrange the definition of log deviations ⟨7.29⟩:

$$\hat{\square}_t \equiv \ln \square_t - \ln \square_{ss} = \ln \left(\frac{\square_t}{\square_{ss}} \right) = \ln \left(\frac{\square_t}{\square_{ss}} + 1 - 1 \right) = \ln \left(1 + \frac{\square_t - \square_{ss}}{\square_{ss}} \right) \quad \langle \text{E.1} \rangle$$

The first-order Taylor approximation of the far-right-hand side with respect to the argument \square_t around the steady state \square_{ss} is given by

$$\ln \left(1 + \frac{\square_t - \square_{ss}}{\square_{ss}} \right) \approx \ln 1 + \left[\left(1 + \frac{\square_t - \square_{ss}}{\square_{ss}} \right)^{-1} \frac{1}{\square_{ss}} \right]_{\square_t = \square_{ss}} (\square_t - \square_{ss}) = \frac{\square_t - \square_{ss}}{\square_{ss}}.$$

Plugging this back into definition ⟨7.29⟩/Equation ⟨E.1⟩ yields

$$\hat{\square}_t \approx \frac{\square_t - \square_{ss}}{\square_{ss}} \quad \Leftrightarrow \quad \square_t \approx \square_{ss} (1 + \hat{\square}_t), \quad \langle \text{E.2} \rangle$$

which is the first ‘replacement rule’ that can be applied to log-linearize macroeconomic models.

Gross and Net Rates As a side note, recall that the argument of the last log function in Equation (E.1) constitutes a gross ‘growth’ (or rather: deviation) rate. The fraction states a percentage deviation. Recall further that, for small values of \square (roughly: single-digit percentages), the natural logarithm of a gross rate is approximately equal to the net rate:

$$\ln(1 + \square) \approx \square \quad (\text{E.3})$$

This is the reason for the fact that interest and inflation rates appear in levels instead of deviations in the model (for example, Equations (7.29) and (E.3) imply that $\hat{\Pi}_t \approx \pi_t - \pi^*$).

Products of Log Deviations Because of this relationship between \nearrow Gross and Net Rates, products of log deviations are very small, which is why they are often ignored in order to make the exposition more easily comprehensible.

Exponent Form Another approach (which is more general and thus more useful in many situations) is to rearrange definition (7.29) to $\ln \square_t = \ln \square_{ss} + \hat{\square}_t$ and take exponents:

$$\begin{aligned} \square_t &= \square_{ss} e^{\hat{\square}_t} \\ \Leftrightarrow \frac{\square_t}{\square_{ss}} &= e^{\hat{\square}_t} \end{aligned} \quad (\text{E.4})$$

Equation (E.4) is the second replacement rule. Approximating the right-hand side of the equation below it by forming a first-degree Taylor expansion around the steady state—that is, at $\hat{\square}_t = 0$ —gives

$$e^{\hat{\square}_t} \approx e^0 + e^{\hat{\square}_t} \Big|_{\hat{\square}_t=0} (\hat{\square}_t - 0) = 1 + \hat{\square}_t. \quad (\text{E.5})$$

It follows from Equation (E.4) that

$$\square_t \approx \square_{ss} (1 + \hat{\square}_t),$$

which is equivalent to the first replacement rule (E.2).

E.3. Taylor Series Expansion II

Simple Method The log-linearization procedure can be sped up by changing the order of steps, or more specifically, by carrying out the Taylor-series approximation first and only then applying the results of Section E.2. For future reference, this is called the ‘simple method.’ Explanations for univariate and multivariate expressions ensue in the subsequent paragraphs.

Univariate Case Consider the function

$$\square_{t+1} = \mathbb{F}(\square_t)$$

and its first-degree Taylor-series approximation around the steady state \square_{ss}

$$\square_{t+1} \approx \mathbb{F}(\square_{ss}) + \mathbb{F}'(\square_{ss}) (\square_t - \square_{ss}).$$

Since $\square_{ss} = \mathbb{F}(\square_{ss})$ in steady state, the last equation can be rewritten as

$$\square_{t+1} \approx \square_{ss} + \mathbb{F}'(\square_{ss}) (\square_t - \square_{ss}) \Leftrightarrow \frac{\square_{t+1}}{\square_{ss}} \approx 1 + \frac{\mathbb{F}'(\square_{ss}) (\square_t - \square_{ss})}{\square_{ss}}.$$

Applying the first replacement rule (E.2) on both the left- and right-hand sides then gives

$$\begin{aligned} 1 + \hat{\square}_{t+1} &\approx 1 + \mathbb{F}'(\square_{ss}) \hat{\square}_t \\ \Leftrightarrow \hat{\square}_{t+1} &\approx \mathbb{F}'(\square_{ss}) \hat{\square}_t. \end{aligned}$$

Multivariate Case The function

$$\square_{t+1} = \mathbb{F}(\square_t, \circ_t)$$

can be approximated around the steady state $(\square_{ss}, \circ_{ss})$ by a first-degree Taylor series as follows:

$$\square_{t+1} \approx \mathbb{F}(\square_{ss}, \circ_{ss}) + \mathbb{F}_{\square}(\square_{ss}, \circ_{ss}) (\square_t - \square_{ss}) + \mathbb{F}_{\circ}(\square_{ss}, \circ_{ss}) (\circ_t - \circ_{ss})$$

Again, it holds in steady state that $\square_{ss} = \mathbb{F}(\square_{ss}, \bigcirc_{ss})$, so the above equation can be rewritten as

$$\frac{\square_{t+1}}{\square_{ss}} \approx 1 + \mathbb{F}_{\square}(\square_{ss}, \bigcirc_{ss}) \frac{\square_t - \square_{ss}}{\square_{ss}} + \mathbb{F}_{\bigcirc}(\square_{ss}, \bigcirc_{ss}) \frac{\bigcirc_{ss}}{\square_{ss}} \frac{\bigcirc_t - \bigcirc_{ss}}{\bigcirc_{ss}}.$$

With replacement rule ⟨E.2⟩, we then get

$$\begin{aligned} 1 + \hat{\square}_{t+1} &\approx 1 + \mathbb{F}_{\square}(\square_{ss}, \bigcirc_{ss}) \hat{\square}_t + \mathbb{F}_{\bigcirc}(\square_{ss}, \bigcirc_{ss}) \frac{\bigcirc_{ss}}{\square_{ss}} \hat{\bigcirc}_t \\ \Leftrightarrow \hat{\square}_{t+1} &\approx \mathbb{F}_{\square}(\square_{ss}, \bigcirc_{ss}) \hat{\square}_t + \mathbb{F}_{\bigcirc}(\square_{ss}, \bigcirc_{ss}) \frac{\bigcirc_{ss}}{\square_{ss}} \hat{\bigcirc}_t. \end{aligned}$$

E.4. Specific Expressions

E.4.1. Expectation Terms

The techniques presented so far—namely, forming logs—should not be applied to expectation terms because the logarithm of an expectation formed from a probability distribution is greater than or equal to the expectation of the respective logarithms: $\ln \mathbb{E}[\cdot] \geq \mathbb{E} \ln[\cdot]$ (this is called “Jensen’s inequality” after Jensen 1906). Instead, the individual elements of such an expectational equation can be substituted according to the second replacement rule ⟨E.4⟩.

This issue points to another advantage of the *Simple Method* (p. 232): Executing the Taylor approximation in the first step dissects expectational terms before log terms are introduced.

E.4.2. Multiplicative Terms

The linear approximation of a multiplicative term $\square_t \bigcirc_t$ can be achieved most easily by taking logs,

$$\ln(\square_t \bigcirc_t) = \ln(\square_t) + \ln(\bigcirc_t),$$

and subtracting the respective steady-state expressions (the second line follows from Equation ⟨7.29⟩):

$$\begin{aligned} \ln(\square_t \bigcirc_t) - \ln(\square_{ss} \bigcirc_{ss}) &= \ln(\square_t) - \ln(\square_{ss}) + \ln(\bigcirc_t) - \ln(\bigcirc_{ss}) \\ \Leftrightarrow \widehat{\square_t \bigcirc_t} &= \hat{\square}_t + \hat{\bigcirc}_t \end{aligned} \tag{E.6}$$

E.4.3. Additive Constants

Handling expressions such as $\widehat{(\square_t + \bigcirc)}$, where \bigcirc is a constant, is relatively simple: From Equation (E.2), we know that

$$\widehat{(\square_t + \bigcirc)} \approx \frac{(\square_t + \bigcirc) - (\square_{ss} + \bigcirc)}{(\square_{ss} + \bigcirc)} \Leftrightarrow \square_t - \square_{ss} \approx \widehat{(\square_t + \bigcirc)} (\square_{ss} + \bigcirc)$$

and

$$\hat{\square}_t \approx \frac{\square_t - \square_{ss}}{\square_{ss}} \Leftrightarrow \square_t - \square_{ss} \approx \hat{\square}_t \square_{ss}.$$

Equating both expressions then leads to

$$\begin{aligned} \widehat{(\square_t + \bigcirc)} (\square_{ss} + \bigcirc) &\approx \hat{\square}_t \square_{ss} \\ \Leftrightarrow \widehat{(\square_t + \bigcirc)} &\approx \frac{\square_{ss}}{\square_{ss} + \bigcirc} \hat{\square}_t. \end{aligned} \tag{E.7}$$

E.5. Limitations of Log-Linearization

Positive Domain One obvious drawback of using logarithms is that they are not defined for negative arguments. This is not of great concern in many models insofar as all variables tend to be positive. However, the present one features variables that can become negative and therefore do not lend themselves to log-linearization.

Workaround Treasury debt b , budget surpluses s , transfers from the treasury to households t^{TH} , and consolidated-government liabilities z are therefore linearized via Equation (7.31). $\check{\square}_t \equiv (\square_t - \square_{ss})/|\square_{ss}|$ also yields percentage deviations from steady state. At this, using the modulus of the steady-state value can avoid possibly confusing results. To make this clear, assume a steady-state value of $\square_{ss} = -100$ and an actual period- t realization of $\square_t = 50$. Using the ‘normal’ definition of deviations or growth rates, $(\square_t - \square_{ss})/\square_{ss}$, would lead to a deviation of $[50 - (-100)]/(-100) = -150\%$ in this case. With an absolute value in the denominator, by contrast, the result is $+150\%$. While it might be a matter of personal preference (and adherence to mathematical rigor), it seems more intuitive to me to use positive numbers for values above the steady state and negative numbers for values below, especially in the context of graphical impulse-response functions.

Non-Zero Steady States A further problem is that variables could also become zero. In baseline models which feature positive variables only, this is often avoided by imposing the ‘Inada (1963) conditions.’ Zero net interest and inflation rates do not pose a problem because they imply unit gross factors that are approximated by net levels (so no zero would appear in a denominator; see *Gross and Net Rates*, p. 231).

Things are different for the extended model of this thesis: The four variables listed above can take negative just as well as positive values, and zero is just one right in the middle of the scale. What’s more, it might even be seen as the most reasonable result: Common sense holds that budget surpluses and deficits should compensate each other in the long run, implying a zero steady state ($s_{SS} = 0$). Unfortunately, such zero steady states are not compatible with any technique that expresses the model in percentage deviations and Dynare fails to simulate the model using a combination of relative- and absolute-deviation (e.g., $\check{\square}_t' \equiv \square_t - \square_{SS}$) variables. Therefore, zero steady states have to be avoided here.

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F.1. Derivations for the Sticky-Price Model

F.1.1. The Second Half of Optimal Household Consumption Choice

Dixit and Stiglitz Maximization In the New-Keynesian model variant, households make two-stage consumption decisions in each period:

- One decision is about the ‘total’ level of consumption in relation to other utility-inducing variables (such as real-money holdings m_t , for instance) as governed by the respective first-order conditions; it is largely analogous to Chapter 2.
- *Given* the total amount of consumption, denoted c'_t in real and C'_t in nominal terms here, households further decide on an allocation towards the different varieties $c_t(j)$. This is what the present subsection describes.

Households are guided by Dixit and Stiglitz (1977) preferences, which means they maximize the consumption bundle $\langle 7.4 \rangle$ subject to the constraint

$$\int_0^1 P_t(j) c_t(j) dj = C'_t. \quad \langle F.1 \rangle$$

Equation $\langle F.1 \rangle$ states that nominal expenditure on all varieties must exactly equal C'_t (maximization implies that an original ‘smaller-than-or-equal-to’ inequality becomes a strict equality). Maximization of the respective Lagrangian then takes the form

$$\max_{c(j)} \mathbb{L} = \max_{c(j)} \left\{ \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \lambda_t \left[\int_0^1 P_t(j) c_t(j) dj - C'_t \right] \right\},$$

The respective first-order conditions for all $j \in [0, 1]$ read

$$\begin{aligned} \left(\frac{\theta}{\theta-1} \right) \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\left(\frac{\theta}{\theta-1} - 1 \right)} \left(\frac{\theta-1}{\theta} \right) c_t(j)^{\left(\frac{\theta-1}{\theta} - 1 \right)} - \lambda_t P_t(j) &\stackrel{!}{=} 0 \\ \Leftrightarrow c_t(j)^{-\frac{1}{\theta}} c_t^{\frac{1}{\theta}} &= \lambda_t P_t(j) \\ \Leftrightarrow c_t(j) &= \frac{c_t}{\lambda_t P_t(j)^{\theta}}. \end{aligned}$$

Relating two goods j and k , we therefore have

$$c_t(j) = c_t(k) \left[\frac{P_t(k)}{P_t(j)} \right]^{\theta}. \quad \langle F.2 \rangle$$

Plugging this result for $c_t(j)$ into the expenditure constraint (F.1) leads to

$$\begin{aligned} \int_0^1 P_t(j) c_t(k) \left[\frac{P_t(k)}{P_t(j)} \right]^\theta dj &= C'_t \\ \Leftrightarrow c_t(k) &= C'_t \frac{P_t(k)^{-\theta}}{\int_0^1 P_t(j)^{1-\theta} dj}. \end{aligned} \quad \langle \text{F.3} \rangle$$

Aggregate Price Level Inserting Equation (F.3) into the definition of the consumption bundle (7.4) (with integration index k) gives

$$\begin{aligned} c_t &= \left\{ \int_0^1 \left[C'_t \frac{P_t(k)^{-\theta}}{\int_0^1 P_t(j)^{1-\theta} dj} \right]^{\frac{\theta-1}{\theta}} dk \right\}^{\frac{\theta}{\theta-1}} = \left\{ C'_t \frac{\int_0^1 P_t(k)^{1-\theta} dk}{\left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{\theta-1}{\theta}}} \right\}^{\frac{\theta}{\theta-1}} \\ &= C'_t \frac{\left[\int_0^1 P_t(k)^{1-\theta} dk \right]^{\frac{\theta}{\theta-1}}}{\int_0^1 P_t(j)^{1-\theta} dj} = C'_t \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}}, \quad \langle \text{F.4} \rangle \end{aligned}$$

where the last rearrangement recognizes that integration over the entire interval always yields the same result, no matter whether the integration index is j or k . Noting further that $c'_t \equiv c_t$, which is tautological, and that nominal and real total expenditure on consumption goods must be linked by the aggregate price level P_t in a similarly tautological relationship $C'_t \equiv P_t c'_t$, Equation (F.4) can be rearranged into an expression for the latter (this is Equation (7.6) in the main text):

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Good-Specific Demand Functions Using Equations (F.2) and then (at the equivalence sign) (7.6) in the expenditure constraint (F.1) gives

$$\begin{aligned} C'_t &= \int_0^1 P_t(j) c_t(k) \left[\frac{P_t(k)}{P_t(j)} \right]^\theta dj = c_t(k) P_t(k)^\theta \int_0^1 P_t(j)^{1-\theta} dj = c_t(k) P_t(k)^\theta P_t^{1-\theta} \\ \Leftrightarrow c_t(k) &= C'_t P_t(k)^{-\theta} P_t^{\theta-1} = c'_t P_t(k)^{-\theta} P_t^\theta \end{aligned}$$

Noting again that $c'_t \equiv c_t$ (and switching the integration index from k back to j) then produces the good-specific demand functions $\langle 7.5 \rangle$

$$c_t(j) = \left[\frac{P_t}{P_t(j)} \right]^\theta c_t.$$

Elasticity of Substitution Between Goods Bringing c_t over to the left-hand side in Equation $\langle 7.5 \rangle$ and forming the ratio between two goods j and k , we have

$$\frac{c_t(j)}{c_t(k)} = \left[\frac{P_t(k)}{P_t(j)} \right]^\theta.$$

Thus the elasticity of substitution between j and k with respect to the relative price $P_t(k)/P_t(j)$ reads as follows (this follows the usual form ‘argument times first derivative over function’):

$$\frac{P_t(k)}{P_t(j)} \cdot \frac{\partial \left[\frac{c_t(j)}{c_t(k)} \right]}{\partial \left[\frac{P_t(k)}{P_t(j)} \right]} \cdot \left[\frac{P_t(k)}{P_t(j)} \right]^{-\theta} = \left[\frac{P_t(k)}{P_t(j)} \right]^{1-\theta} \theta \left[\frac{P_t(k)}{P_t(j)} \right]^{\theta-1} = \theta$$

Taking the inverse of the relative price (that is, writing $P_t[j]/P_t[k]$ instead of $P_t[k]/P_t[j]$) yields a substitution elasticity of $-\theta$, which is equal to θ in absolute value.

F.1.2. Aggregation

F.1.2.1. Goods-Market Clearing with Treasury Expenditure

Aggregate Goods-Market Clearing Dixit and Stiglitz (1977) preferences like those in Equation $\langle 7.4 \rangle$ exhibit ‘constant returns to scale’ (i.e., are homogeneous of degree one, cf. Chen 2011, p. 3), which is why reducing consumption of every goods variety by, say, half also reduces the aggregate consumption bundle to half its original size. By implication—since the treasury and households share the same aggregator—the ratio of public and private demand for each goods variety j is the same as the ratio of aggregate public and private demands. Formally (repeating Equation $\langle 7.22 \rangle$),

$$g_t(j) = \Gamma_t c_t(j) \quad \forall j \quad \Rightarrow \quad g_t = \Gamma_t c_t$$

in the treasury-expenditure bundle $\langle 7.20 \rangle$. This property greatly facilitates the handling of goods-market clearing. Defining a third Dixit and Stiglitz bundler for aggregate pro-

duction,

$$y_t \equiv \left\{ \int_0^1 [y_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}},$$

and plugging in the good-specific market-clearing conditions (7.26) then yields aggregate market clearing (7.27):

$$\begin{aligned} y_t &= \left\{ \int_0^1 [c_t(j) + g_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} = \left\{ \int_0^1 [(1 + \Gamma_t) c_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \\ &= (1 + \Gamma_t) \left\{ \int_0^1 [c_t(j)]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} = (1 + \Gamma_t) c_t = c_t + g_t \end{aligned}$$

This approach is indicated more or (rather) less explicitly in Benigno and Woodford (2004, p. 274) and Schmitt-Grohé and Uribe (2005, p. 17), for instance. Another option would be to introduce treasury expenditure as a multiplicative ‘shifter’ or shock term directly (cf. Woodford 2003b, pp. 147-149; Mankiw and Reis 2006, p. 10), which would place a (differently defined) g_t , instead of $(1 + \Gamma_t)$, in front of c_t . I choose the former option as it represents the more commonly used notation. Note that the ratio Γ carries a time index because it varies with different levels of g relative to c (but it could also be understood as the policy instrument itself, which exposes the equivalence of both approaches to capturing treasury expenditure).

Total Demand for Good j Another implication is that the specific goods-demand functions (7.5) and (7.21) can be transferred to $y_t(j)$ as well: Combining said equations with goods-market clearing (7.26) yields

$$y_t(j) = c_t(j) + g_t(j) = \left[\frac{P_t}{P_t(j)} \right]^\theta (c_t + g_t) = \left[\frac{P_t}{P_t(j)} \right]^\theta y_t, \quad \langle \text{F.5} \rangle$$

which is stated in Footnote 21 on p. 127.

Intuition The introduction of the public-expenditure share Γ might be somewhat unintuitive: Usually, one thinks of public expenditure g (G) as an exogenous decision made by the fiscal authority about a real quantity (or nominal expenditure volume in the sense of ‘spending an additional 20 bn. € on infrastructure’), but not as being determined relative to consumption. The former interpretation would set g exogenous and Γ endogenous whereas the latter one tends towards targeting a constant Γ and matching g to c in order to achieve this target.

As shown above, however, having Γ available facilitates certain derivations because it turns goods-market clearing from an additive into a multiplicative relationship. Finding *Labor-Market Equilibrium* in steady state (p. 247, Appendix F.2.2) is another instance in which this is helpful.

F.1.2.2. Aggregate Profits

Aggregate profits have the same form as firm-specific profits, which is useful for some derivations ensuing below (see Sections F.2.2 and F.3.4). To see this, plug total demand for good j (F.5) into the respective profit function (7.13) and note that the cost of production is determined by the nominal wage W and the amount of labor $\ell(j)$. Integrating over j and using Equations (7.6) and (7.28) then establishes Equation (7.14):

$$\begin{aligned}\Psi_t &\equiv \int_0^1 \Psi_t(j) \, dj = \int_0^1 P_t(j) y_t(j) - \text{Cost}_t(j) \, dj \\ &= \int_0^1 P_t(j) \left[\frac{P_t}{P_t(j)} \right]^\theta y_t - W_t \ell_t(j) \, dj \\ &= P_t^\theta y_t \int_0^1 [P_t(j)]^{1-\theta} \, dj - W_t \ell_t = P_t y_t - W_t \ell_t\end{aligned}$$

F.1.3. Firms' Maximization Problem with Staggered Price Setting

Maximization Problem Firms able to reset prices in t will do so with the aim of maximizing the present value of expected profits,³⁴

$$\mathbb{E}_t \sum_{T=t}^{\infty} V_{t,T} \Psi_T(j) = \mathbb{E}_t \sum_{T=t}^{\infty} V_{t,T} [P_T(j) y_T(j) - \text{Cost}_T(j)] \quad (\text{F.6})$$

where the linear production function (7.10) is already substituted. Owing to the Calvo (1983) rigidity, firm j 's time- t expectation of its price in a future period $T > t$ has increasingly many parts:

$$\begin{aligned}\mathbb{E}_t P_{t+1}(j) &= \alpha (\Pi^*) P_t^o(j) + (1 - \alpha) \mathbb{E}_t P_{t+1}^o(j) \\ \mathbb{E}_t P_{t+2}(j) &= \alpha^2 (\Pi^*)^2 P_t^o(j) + \alpha (1 - \alpha) \mathbb{E}_t \mathbb{E}_t P_{t+2}^o + (1 - \alpha) \alpha \Pi^* \mathbb{E}_t P_{t+1}^o + (1 - \alpha)^2 \mathbb{E}_t P_{t+2}^o \\ &\vdots\end{aligned}$$

³⁴ This can be done either in real terms (e.g., Galí 2015, pp. 56-57, 84-85) or in nominal terms (e.g., Walsh 2010, p. 241; Benigno and Nisticò 2015). As Equation (F.6) shows, I take the nominal route.

Figure F.1 sketches this problem for periods t through $t + 2$. Many paths can be either

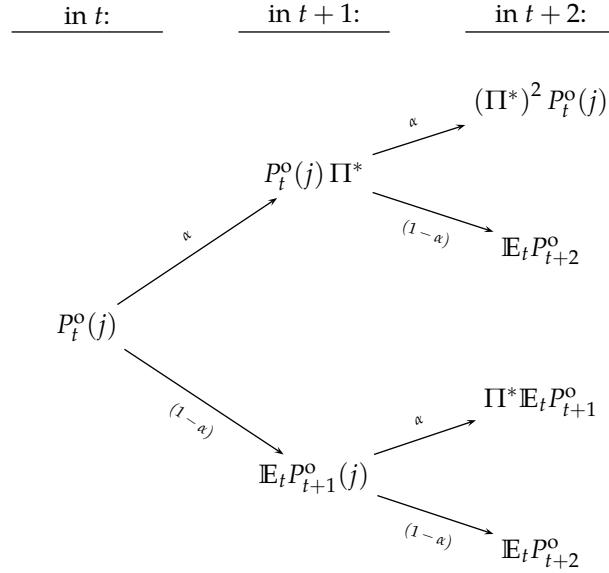


Figure F.1: *Firms' Expectations about Their Future Price Setting with the Calvo (1983) Rigidity.* \diamond Source: Own illustration. \diamond Explanations: Upward-sloping arrows (probability α) indicate no possibility to reset prices optimally, so the previous price is increased at the target inflation rate Π^* . Downward-sloping arrows (probability $1 - \alpha$) indicate the possibility to set an optimal price in the respective period.

cut off at some point or excluded entirely as they do not contain the choice variable $P_t^o(j)$ (anymore). To be precise, this happens every time the respective firm is allotted to the ' $(1 - \alpha)$ group' which is allowed to reset prices optimally; in this instance, $P_t^o(j)$ does not play a role anymore. Therefore, the maximization problem (F.6) simplifies to

$$\max_{P_t^o(j)} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \left[(\Pi^*)^{T-t} P_t^o(j) y_T(j) - \text{Cost}_T(j) \right].$$

Combining this with ‘total demand’ (F.5) then allows to restate the maximization problem as

$$\begin{aligned} & \max_{P_t^o(j)} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \left\{ (\Pi^*)^{T-t} P_t^o(j) y_T \left[\frac{P_T}{(\Pi^*)^{T-t} P_t^o(j)} \right]^{\theta} \right. \\ & \quad \left. - \text{Cost}_T \left\langle y_T \left[\frac{P_T}{(\Pi^*)^{T-t} P_t^o(j)} \right]^{\theta} \right\rangle \right\} \\ & = \max_{P_t^o(j)} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \left\{ P_T y_T \left[\frac{(\Pi^*)^{T-t} P_t^o(j)}{P_T} \right]^{1-\theta} - \text{Cost}_T \left\langle y_T \left[\frac{(\Pi^*)^{T-t} P_t^o(j)}{P_T} \right]^{-\theta} \right\rangle \right\} \end{aligned}$$

First-Order Condition In equilibrium, it holds that

$$\begin{aligned} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \left\{ P_T y_T (1 - \theta) \left[\frac{(\Pi^*)^{T-t} P_t^o(j)}{P_T} \right]^{-\theta} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right] \right. \\ \left. - \text{MC}_T(j) y_T (-\theta) \left[\frac{(\Pi^*)^{T-t} P_t^o(j)}{P_T} \right]^{-(1+\theta)} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right] \right\} \stackrel{!}{=} 0. \end{aligned}$$

Since the inner derivative of the cost term is now explicit in the above equation, we can return to the more parsimonious notation and write $\mathbb{E}_t \text{MC}_T(j)$ instead of $\mathbb{E}_t \text{MC}_T[y_T(j)]$ (or even more complicated substitutes) for the marginal cost of producing $\mathbb{E}_t y_T(j)$. Rearranging further finally gives the optimality condition (7.15):

$$\begin{aligned} & \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} (1 - \theta) P_T y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right] \\ & = -\frac{1}{P_t^o(j)} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \theta \text{MC}_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-(1+\theta)} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right] \\ & \Leftrightarrow \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} (\theta - 1) y_T (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\ & = \frac{1}{P_t^o(j)} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} \theta \text{MC}_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \end{aligned}$$

$$\begin{aligned}
& \Leftrightarrow P_t^o(j) \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} y_T (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
& \quad = \frac{\theta}{\theta-1} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} MC_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
& \Leftrightarrow P_t^o(j) = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} MC_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} y_T (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}} \quad \langle F.7 \rangle \\
& \Leftrightarrow \frac{P_t^o(j)}{P_t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} MC_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} V_{t,T} y_T P_t (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}} \\
& \Leftrightarrow \frac{P_t^o(j)}{P_t} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} v_{t,T} mc_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}{\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} v_{t,T} y_T \frac{P_t}{P_T} (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta}}
\end{aligned}$$

F.1.4. Shutting Down the Frictions

Actual outcomes for the economy are often compared to certain (rather hypothetical or limiting) ‘benchmark cases’ that arise when the two important frictions of the present model variant—price stickiness and monopolistic competition—are shut down. I briefly consider the implications without giving all the formal details that could possibly be expounded here (cf. Galí 2015, Chapters 2 and 3, for instance).

Desired Markup With flexible prices ($\alpha = 0$), the optimal-pricing condition of firm j $\langle F.7 \rangle$ reduces to

$$P_t^o(j) = \frac{\theta}{\theta-1} MC_t(j), \quad \langle F.8 \rangle$$

which demonstrates more tangibly that firms typically demand a price *above* their marginal cost of production or, put differently, that they conduct markup pricing. In the following, the (gross) desired markup over marginal costs $MC_t(j)$ is denoted by

$$\varrho \equiv \frac{\theta}{\theta-1}$$

as defined in Equation (7.16) in the main text. It is also called ‘frictionless markup’ because the Calvo (1983) mechanism does not distort outcomes anymore (cf. Galí 2008, p. 45). However, this can be misleading because firms’ market power as the second supply-side friction is still active and indeed the only driver of positive (net) markups.

Importantly, the same is true in steady state, in which re-optimized and non-re-optimized (indexed) prices move in line and none of the firms has to accept a lower-than-desired mark-up. Rewriting Equation (F.8) accordingly shows that natural and steady-state marginal costs (both expressed by mc here) are equal to the inverse of the desired markup:

$$mc(j) = \frac{MC(j)}{P_t} = \frac{\theta - 1}{\theta} \frac{P_t^o(j)}{P_t} = \varrho^{-1} \quad (\text{F.9})$$

Natural vs. Steady-State Output The concept of defining a flexible-price benchmark also applies to output, where actual output y (with Calvo pricing) is compared to *natural output* y^{nat} . The latter comes about when all firms can set prices optimally ($\alpha = 0$); in this case, the ‘total demand’ equation (F.5) implies that all specific outputs are produced in equal amount ($y[j] = y \forall j$), which maximizes the Dixit and Stiglitz bundler in Equation (7.27) (given the degree of monopoly power of firms expressed by the price elasticity of substitution θ). Given the definition of the preferred output-gap measure \tilde{y} (Equation (7.30)) and noting that $y_{\text{SS}}^{\text{nat}} = y_{\text{SS}}$, we can write

$$\begin{aligned} \hat{y}_t &= (\ln y_t - \ln y_{\text{SS}}) + (\ln y_t^{\text{nat}} - \ln y_t^{\text{nat}}) \\ &= (\ln y_t - \ln y_t^{\text{nat}}) + (\ln y_t^{\text{nat}} - \ln y_{\text{SS}}^{\text{nat}}) \\ &= \tilde{y}_t + \hat{y}_t^{\text{nat}}. \end{aligned} \quad (\text{F.10})$$

The second term in the last line, \hat{y}_t^{nat} , depends only on shocks to technology a (see Section F.3.2 of the appendix below and cf. Walsh 2010, p. 340).

Efficient Output For the sake of completeness, consider the move to perfectly competitive goods markets. Goods become ever closer substitutes ($\theta \rightarrow \infty$), which decreases and eventually abolishes mark-ups altogether (in Equation (7.16), $\varrho = 1$ in the limit). Compared to the natural level, a complete removal of market power further increases output to its maximal or *efficient* level. As long as market power is present, however, actual output will not reach the efficient level.

F.2. Steady State

F.2.1. Preparatory Observations

Prices If the target inflation rate is positive ($\Pi^* > 0$), prices are not constant in steady state. Hence, for the sake of clarity, let $P_{SS}(j, t)$ and $P_{SS}(t)$ denote the steady-state values of the price of the specific good j and the aggregate price level, respectively, both as functions of time t . By implication from Equation (7.12),

$$P_{SS}^o(j, t) = P_{SS}(t) = \Pi^* P_{SS}(t-1) \quad \forall j, \quad (\text{F.11})$$

or in other words: If non-re-optimizing firms index at the target inflation rate Π^* , the achievement of this rate as a defining feature of steady state is only possible if re-optimizing also choose prices consistent with Π^* .

Output Given equal prices, the good-specific demand equation (7.5) as well as the goods-market-clearing conditions (7.26)-(7.27) imply constant values for $c_t(j) = c_t = c_{SS}$ and $y_t(j) = y_t = y_{SS} = y_{SS}^{\text{nat}}$ (see also *Natural vs. Steady-State Output* on p. 246).

Technology Since shock terms equal zero in steady state, the autoregressive process for technology (7.11) (with $\gamma_a \neq 0$) implies that $\ln a_{SS} = 0 \Leftrightarrow a_{SS} = 1$.

F.2.2. The Whole Model in Steady State

Interest Rates In steady state, consumption is constant, so the Euler equation (2.21) (derived from the first-order conditions with respect to consumption c (2.12) and bonds b (2.15)) implies that $(1 + r_{SS}) = \beta^{-1}$. The Fisher equation (2.5)/(2.7) implies that $(1 + i_{SS}) = (1 + r_{SS})\Pi^* = \beta^{-1}\Pi^*$. The bond price, defined in Equation (2.8), therefore is $Q_{SS} = (1 + i_{SS})^{-1} = \beta(\Pi^*)^{-1}$.

Labor-Market Equilibrium The Calvo rigidity has no effect in steady state insofar as all prices are the same and increase at the target inflation rate. In this environment, the shortest path to (aggregate) labor demand is profit maximization via aggregate output y (as an alternative to the procedure described in Appendix F.1.3):

$$\max_{y_{SS}} \psi_{SS} = \max_{y_{SS}} (y_{SS} - w_{SS} \ell_{SS}) = \max_{y_{SS}} \left[y_{SS} - w_{SS} \left(\frac{y_{SS}}{a_{SS}} \right)^{\frac{1}{\zeta}} \right]$$

On the far right-hand side, the production function (7.10) is used to substitute for labor ℓ , where the j -specific and aggregate versions are equivalent since $y_{SS}(j) = y_{SS} \forall j$ in

steady state (see Appendix F.2.1). Noting that $a_{SS} = 1$, the respective first-order condition then gives steady-state labor demand:

$$\frac{\partial \psi_{SS}}{\partial y_{SS}} = 1 - w_{SS} \frac{1}{\zeta} y_{SS}^{\frac{1-\zeta}{\zeta}} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad w_{SS} = \zeta y_{SS}^{\frac{\zeta-1}{\zeta}} \quad \langle F.12 \rangle$$

After equating labor supply $\langle 7.9 \rangle$ and demand $\langle F.12 \rangle$, c_{SS} is replaced via goods-market clearing $\langle 7.27 \rangle$, y_{SS} via the production function $\langle 7.10 \rangle$, and g_{SS} via Equation $\langle 7.22 \rangle$, which yields the labor-market equilibrium ℓ_{SS} :

$$\begin{aligned} \ell_{SS}^\eta \tilde{\zeta}_\ell \left[\frac{1}{1 + \Gamma_{SS}} \ell_{SS}^\zeta \right]^\rho &= \zeta \left(\ell_{SS}^\zeta \right)^{\frac{\zeta-1}{\zeta}} \quad \Leftrightarrow \quad \ell_{SS}^{1+\eta-(1-\rho)\zeta} = \frac{\zeta}{\tilde{\zeta}_\ell} (1 + \Gamma_{SS})^\rho \\ \Leftrightarrow \quad \ell_{SS} &= \left[\frac{\zeta}{\tilde{\zeta}_\ell} (1 + \Gamma_{SS})^\rho \right]^{\frac{1}{1+\eta-(1-\rho)\zeta}} \end{aligned} \quad \langle F.13 \rangle$$

Output, Consumption, and the Role of Treasury Expenditure Plugging ℓ from Equation $\langle F.13 \rangle$ into the production function gives aggregate output in steady state:

$$y_{SS} = (\ell_{SS}^*)^\zeta = \left[\frac{\zeta}{\tilde{\zeta}_\ell} (1 + \Gamma_{SS})^\rho \right]^{\frac{\zeta}{1+\eta-(1-\rho)\zeta}} \quad \langle F.14 \rangle$$

Unwinding Equation $\langle 7.22 \rangle$ and Appendix F.1.2.1, we find that

$$c_{SS} = \frac{1}{1 + \Gamma_{SS}} y_{SS} = \frac{1}{1 + \Gamma_{SS}} \left[\frac{\zeta}{\tilde{\zeta}_\ell} (1 + \Gamma_{SS})^\rho \right]^{\frac{\zeta}{1+\eta-(1-\rho)\zeta}} \quad \langle F.15 \rangle$$

$$\Rightarrow g_{SS} = \Gamma_{SS} c_{SS} = \frac{\Gamma_{SS}}{1 + \Gamma_{SS}} y_{SS} = \frac{\Gamma_{SS}}{1 + \Gamma_{SS}} \left[\frac{\zeta}{\tilde{\zeta}_\ell} (1 + \Gamma_{SS})^\rho \right]^{\frac{\zeta}{1+\eta-(1-\rho)\zeta}}. \quad \langle F.16 \rangle$$

Fiscal policy plays a part in the determination of labor-market equilibrium and thus also on aggregate output. As indicated by Equations $\langle F.13 \rangle$ and $\langle F.14 \rangle$, an increase in Γ_{SS} raises labor input and thus also aggregate output; however, for typical parameterizations (see Table 7.2), ℓ_{SS} and y_{SS} increase less than one-for-one because the respective exponents are smaller than unity.

Money Demand Given steady-state consumption and nominal interest rates, the respective money demand can be obtained from Equations $\langle 2.22 \rangle$ and $\langle 7.3 \rangle$:

$$m_{SS} = \tilde{\zeta}_m^{\frac{1}{\nu}} \tilde{I}_{SS}^{-\frac{1}{\nu}} c_{SS}^\ell \quad \langle F.17 \rangle$$

Budget Constraint Section F.1.2.2 shows that $\Psi_t = Y_t - W_t \ell_t$. Therefore, the household budget constraint (7.7) can be written as

$$C_t + M_t + Q_t B_t = T_t^{TH} + Y_t + B_{t-1} + M_{t-1},$$

which is equivalent to budget constraint (2.36) and can therefore be transformed into

$$\begin{aligned} Z_t &= (1 + i_{t+1}) (Z_{t-1} - S_t - \dot{I}_{t+1} M_t) \\ \Leftrightarrow z_t &= (1 + i_{t+1}) \left(\frac{z_{t-1}}{\Pi_t} - s_t - \dot{I}_{t+1} m_t \right) \end{aligned} \quad \langle \text{F.18} \rangle$$

(see Equation (2.37) and recall the definition of \dot{I} (2.23)). In steady state, this reads

$$\begin{aligned} z_{SS} &= (1 + i_{SS}) \left(\frac{z_{SS}}{\Pi^*} - s_{SS} - \dot{I}_{SS} m_{SS} \right) = \beta^{-1} z_{SS} - (1 + i_{SS}) s_{SS} - i_{SS} m_{SS} \\ \Leftrightarrow z_{SS} &= \frac{(1 + i_{SS}) s_{SS} + i_{SS} m_{SS}}{\beta^{-1} - 1}, \end{aligned}$$

including the surplus rule (7.23) (with $y_{SS} = y_{SS}^{\text{nat}}$ and $\varepsilon_{SS}^s = 0$ already incorporated), it becomes

$$\begin{aligned} z_{SS} &= \beta^{-1} z_{SS} - (1 + i_{SS}) \left(s^{\text{fix}} + \gamma_z^T \frac{z_{SS}}{\Pi^*} - \gamma_m^T \dot{I}_{SS} m_{SS} \right) - i_{SS} m_{SS} \\ &= \beta^{-1} \left(1 - \gamma_z^T \right) z_{SS} - (1 + i_{SS}) s^{\text{fix}} - \left(1 - \gamma_m^T \right) i_{SS} m_{SS} \\ \Leftrightarrow z_{SS} &= \frac{1}{\beta^{-1} (1 - \gamma_z^T) - 1} \left[(1 + i_{SS}) s^{\text{fix}} + \left(1 - \gamma_m^T \right) i_{SS} m_{SS} \right]. \end{aligned} \quad \langle \text{F.19} \rangle$$

F.3. Log-Linearization of the Sticky-Price Model

F.3.1. Piecemeal Linearization and Preparations

F.3.1.1. Fisher Equation

Rewriting the Fisher equation (2.5) in terms of gross rates (denoted by capital letters),

$$R_t = \frac{I_t}{\Pi_t},$$

linearizing (cf. Appendix E.4.2) and recalling the relationship between \nearrow Gross and Net Rates gives

$$\hat{R}_t = \hat{I}_t - \hat{\Pi}_t \Leftrightarrow r_t = i_t - \pi_t + r_{SS} - (i_{SS} - \pi^*).$$

For small values (i.e., single-digit percentages), it follows from the original version (2.5) that $r \approx i - \pi$, so that the last equation can be simplified to

$$r_t = i_t - \pi_t. \quad \langle F.20 \rangle$$

F.3.1.2. Labor Supply

Rearrange labor supply (7.9) to

$$\xi_\ell c_t^\rho \ell_t^\eta = w_t$$

and linearize using the \nearrow Simple Method:

$$\begin{aligned} \xi_\ell \rho c_{SS}^{\rho-1} \ell_{SS}^\eta (c_t - c_{SS}) + \xi_\ell c_{SS}^\rho \eta \ell_t^{\eta-1} (\ell_t - \ell_{SS}) &= (w_t - w_{SS}) \\ \Leftrightarrow \rho \hat{c}_t + \eta \hat{\ell}_t &= \hat{w}_t \end{aligned} \quad \langle F.21 \rangle$$

F.3.1.3. Aggregate Production Function

Approximation of the Aggregate Price-Level Index Rearrange the definition of the aggregate price level (7.6) to

$$\begin{aligned} 1 &= \left\{ \int_0^1 \left[\frac{P_t(j)}{P_t} \right]^{1-\theta} dj \right\}^{\frac{1}{1-\theta}} = \left\langle \int_0^1 \exp\{(1-\theta) [\ln P_t(j) - \ln P_t]\} dj \right\rangle^{\frac{1}{1-\theta}} \\ \Leftrightarrow 1 &= \int_0^1 \exp\{(1-\theta) [\ln P_t(j) - \ln P_t]\} dj \end{aligned}$$

and approximate the second line via a first-order Taylor-series expansion (cf. Appendix E.1; note that the argument is $\ln P_t[j]$, not $P_t[j]$) around the steady state in which $P_{SS}(j, t) =$

$P_{SS}(t) \forall j$ by Equation (F.11):

$$\begin{aligned}
 1 &\approx \int_0^1 \exp\{(1-\theta) [\ln P_{SS}(j, t) - \ln P_{SS}(t)]\} dj \\
 &\quad + \int_0^1 \exp\{(1-\theta) [\ln P_{SS}(j, t) - \ln P_{SS}(t)]\} (1-\theta) [\ln P_t(j) - \ln P_{SS}(t)] dj \\
 &= \int_0^1 e^0 dj + \int_0^1 e^0 (1-\theta) [\ln P_t(j) - \ln P_{SS}(t)] dj \\
 &= 1 - (1-\theta) \ln P_{SS}(t) + (1-\theta) \int_0^1 \ln P_t(j) dj \\
 &\Leftrightarrow \ln P_{SS}(t) \approx \int_0^1 \ln P_t(j) dj \tag{F.22}
 \end{aligned}$$

Aggregate Production Function Combining labor-market clearing (7.28) and the production function (7.10) with the j -specific total-demand function (F.5) gives

$$\ell_t = \int_0^1 \left[\frac{y_t(j)}{a_t} \right]^{\frac{1}{\zeta}} dj = \left(\frac{y_t}{a_t} \right)^{\frac{1}{\zeta}} \int_0^1 \left[\frac{P_t}{P_t(j)} \right]^{\frac{\theta}{\zeta}} dj.$$

Taking logarithms, this becomes

$$\begin{aligned}
 \ln \ell_t &= \frac{1}{\zeta} (\ln y_t - \ln a_t) + \ln \left\{ \int_0^1 \left[\frac{P_t}{P_t(j)} \right]^{\frac{\theta}{\zeta}} dj \right\} \\
 \Leftrightarrow \ln y_t &= \ln a_t + \zeta \ln \ell_t - \zeta \ln \left\{ \int_0^1 \left[\frac{P_t}{P_t(j)} \right]^{\frac{\theta}{\zeta}} dj \right\}. \tag{F.23}
 \end{aligned}$$

Approximate Aggregate Production Function Start by rearranging the argument of the logarithm in the last term on the right-hand side of Equation (F.23):

$$\int_0^1 \left[\frac{P_t}{P_t(j)} \right]^{\frac{\theta}{\zeta}} dj = \int_0^1 \exp \left\{ \frac{\theta}{\zeta} [\ln P_t - \ln P_t(j)] \right\} dj \tag{F.24}$$

The next step is a first-order Taylor-series approximation of Equation (F.24):

$$\begin{aligned}
 \int_0^1 \exp \left\{ \frac{\theta}{\zeta} [\ln P_{SS}(t) - \ln P_{SS}(t)] \right\} dj \\
 + \int_0^1 \exp \left\{ \frac{\theta}{\zeta} [\ln P_{SS}(t) - \ln P_{SS}(t)] \right\} \left(-\frac{\theta}{\zeta} \right) [P_t(j) - P_{SS}(t)] dj
 \end{aligned}$$

$$\begin{aligned}
&= \int_0^1 e^0 dj - \int_0^1 \frac{\theta}{\zeta} e^0 [\ln P_t(j) - \ln P_{SS}(t)] dj \\
&= 1 + \frac{\theta}{\zeta} \ln P_{SS}(t) - \frac{\theta}{\zeta} \int_0^1 \ln P_t(j) dj
\end{aligned}$$

Equation (F.22) implies that the second and third term in the last line cancel out so that the above expression is approximately equal to unity. Therefore (and because $\ln 1 = 0$), Equation (F.23) is equivalent to

$$\ln y_t = \ln a_t + \zeta \ln \ell_t \quad (\text{F.25})$$

up to a first-order approximation, which can also be written in levels as an approximate aggregate production function:

$$y_t = a_t \ell_t^\zeta \quad (\text{F.26})$$

Log Deviations Use the simple method to find that the linear version of Equation (F.26) is

$$\begin{aligned}
(y_t - y_{SS}) &= \ell_{SS}^\zeta (a_t - a_{SS}) + a_{SS} \zeta \ell_{SS}^{\zeta-1} (\ell_t - \ell_{SS}) \\
&\Leftrightarrow \hat{y}_t = \hat{a}_t + \zeta \hat{\ell}_t.
\end{aligned} \quad (\text{F.27})$$

F.3.1.4. Goods-Market Clearing

The aggregate goods-market-clearing condition (7.27) can be linearized by the simple procedure consisting of a Taylor approximation and replacement rule (E.2):

$$\begin{aligned}
(y_t - y_{SS}) &= (c_t - c_{SS}) + (g_t - g_{SS}) \\
&\Leftrightarrow \hat{y}_t = \frac{c_{SS}}{y_{SS}} \hat{c}_t + \frac{g_{SS}}{y_{SS}} \hat{g}_t
\end{aligned} \quad (\text{F.28})$$

F.3.1.5. Marginal Costs

Good-Specific and Aggregate Marginal Costs The marginal cost of increasing the production of good j is given by Equation (7.17). Taking logs,

$$\ln mc_t(j) = \ln W_t - \ln P_t - \ln \zeta - \ln a_t + (1 - \zeta) \ln \ell_t(j), \quad (\text{F.29})$$

and substituting out the labor term via the production function (7.10) (in logs: $\ln y_t[j] = \ln a_t + \zeta \ell_t[j]$) gives

$$\begin{aligned} \ln mc_t(j) &= \ln W_t - \ln P_t - \ln \zeta - \ln a_t + \frac{1-\zeta}{\zeta} [\ln y_t(j) - \ln a_t] \\ &= \ln W_t - \ln P_t - \ln \zeta + \frac{(1-\zeta) \ln y_t(j) + \ln a_t}{\zeta}. \end{aligned} \quad \langle F.30 \rangle$$

Given the approximated *aggregate* production function (F.25) / (F.26), the respective marginal-cost expressions can be derived analogously:

$$MC_t = \frac{W_t}{\zeta a_t \ell_t^{\zeta-1}} \quad \Leftrightarrow \quad mc_t = \frac{w_t}{\zeta a_t \ell_t^{\zeta-1}} \quad \langle F.31 \rangle$$

$$\Rightarrow \ln mc_t = \ln W_t - \ln P_t - \ln \zeta + \frac{(1-\zeta) \ln y_t + \ln a_t}{\zeta} \quad \langle F.32 \rangle$$

The difference between the j -specific and average expressions (Equations (F.30) and (F.32), respectively) is therefore given by

$$\ln mc_t(j) - \ln mc_t = \frac{1-\zeta}{\zeta} [\ln y_t(j) - \ln y_t],$$

in which a logarithmic version of the j -specific total-demand function (F.5),

$$\ln y_t(j) = \theta [\ln P_t - \ln P_t(j)] + \ln y_t,$$

can be used to finally arrive at

$$\ln mc_t(j) = \ln mc_t + \frac{\theta(1-\zeta)}{\zeta} [\ln P_t - \ln P_t(j)]. \quad \langle F.33 \rangle$$

Marginal Costs and the Output Gap Log-linearize the marginal-cost expression (F.31) using the *Simple Method*:

$$\begin{aligned} (mc_t - mc_{SS}) &= \frac{1}{\zeta a_{SS} \ell_{SS}^{\zeta-1}} (w_t - w_{SS}) - \frac{w_{SS}}{\zeta a_{SS}^2 \ell_{SS}^{\zeta-1}} (a_t - a_{SS}) \\ &\quad - (\zeta - 1) \frac{w_{SS}}{\zeta a_{SS} \ell_{SS}^{\zeta-1+1}} (\ell_t - \ell_{SS}) \end{aligned}$$

$$\Leftrightarrow \widehat{mc}_t = \widehat{w}_t - \widehat{a}_t - (\zeta - 1) \widehat{\ell}_t \quad \langle \text{F.34} \rangle$$

Solving for \widehat{w} in this as well as in labor supply $\langle \text{F.21} \rangle$ and equalizing gives

$$\begin{aligned} \rho \widehat{c}_t + \eta \widehat{\ell}_t &= \widehat{mc}_t + \widehat{a}_t + (\zeta - 1) \widehat{\ell}_t \\ \Leftrightarrow \widehat{mc}_t &= \rho \widehat{c}_t - \widehat{a}_t - (\zeta - 1 - \eta) \widehat{\ell}_t. \end{aligned}$$

Substituting out $\widehat{\ell}_t$ via the aggregate production function $\langle \text{F.27} \rangle$ and \widehat{c}_t via goods-market clearing $\langle \text{F.28} \rangle$, we have

$$\begin{aligned} \widehat{mc}_t &= \rho \left(\frac{y_{SS}}{c_{SS}} \widehat{y}_t - \frac{g_{SS}}{c_{SS}} \widehat{g}_t \right) - \widehat{a}_t - \frac{\zeta - 1 - \eta}{\zeta} (\widehat{y}_t - \widehat{a}_t) \\ &= \left(\rho \frac{y_{SS}}{c_{SS}} - \frac{\zeta - 1 - \eta}{\zeta} \right) \widehat{y}_t - \frac{1 + \eta}{\zeta} \widehat{a}_t - \rho \frac{g_{SS}}{c_{SS}} \widehat{g}_t. \end{aligned} \quad \langle \text{F.35} \rangle$$

In case prices are flexible ($\alpha = 0$), the \nearrow Desired Markup (p. 245) is constant and the appropriate measure of aggregate production is natural output. Therefore, the flexible-price analogue to Equation $\langle \text{F.35} \rangle$ comes about by setting $\widehat{mc}_t = 0$ and replacing \widehat{y}_t by $\widehat{y}_t^{\text{nat}}$:

$$0 = \left(\rho \frac{y_{SS}}{c_{SS}} - \frac{\zeta - 1 - \eta}{\zeta} \right) \widehat{y}_t^{\text{nat}} - \frac{1 + \eta}{\zeta} \widehat{a}_t - \rho \frac{g_{SS}}{c_{SS}} \widehat{g}_t \quad \langle \text{F.36} \rangle$$

After repeating the shorthand $\langle 7.36 \rangle$ for convenience,

$$\Theta \equiv \rho \frac{y_{SS}}{c_{SS}} - \frac{\zeta - 1 - \eta}{\zeta},$$

we subtract Equation $\langle \text{F.36} \rangle$ from Equation $\langle \text{F.35} \rangle$ and thereby get

$$\widehat{mc}_t = \Theta (\widehat{y}_t - \widehat{y}_t^{\text{nat}}) = \Theta \widetilde{y}_t. \quad \langle \text{F.37} \rangle$$

(The derivations linking marginal costs to the output gap are closer to Walsh 2010, pp. 336-340 than to Galí 2008; 2015 because the former handles the distinction between output levels more clearly.)

F.3.1.6. Linearized Aggregate Price Dynamics

Slightly rearrange the law of motion of the aggregate price level $\langle 7.12 \rangle$ to

$$P_t^{1-\theta} = \alpha (\Pi^* P_{t-1})^{1-\theta} + (1-\alpha) (P_t^o)^{1-\theta}$$

and linearize it around steady state: For the left-hand side, we have

$$\begin{aligned} P_t^{1-\theta} &\approx [P_{SS}(t)]^{1-\theta} + (1-\theta) [P_{SS}(t)]^{-\theta} [P_t - P_{SS}(t)] \\ &= [P_{SS}(t)]^{1-\theta} + (1-\theta) [P_{SS}(t)]^{1-\theta} \hat{P}_t. \end{aligned} \quad \langle F.38 \rangle$$

Similarly, for the right hand side,

$$\begin{aligned} &\alpha (\Pi^* P_{t-1})^{1-\theta} + (1-\alpha) (P_t^o)^{1-\theta} \\ &\approx \alpha (\Pi^*)^{1-\theta} [P_{SS}(t-1)]^{1-\theta} + (1-\alpha) [P_{SS}^o(t)]^{1-\theta} \\ &\quad + \alpha (\Pi^*)^{1-\theta} (1-\theta) [P_{SS}(t-1)]^{-\theta} [P_{t-1} - P_{SS}(t-1)] \\ &\quad + (1-\alpha) (1-\theta) [P_{SS}^o(t)]^{-\theta} [P_t^o - P_{SS}^o(t)] \\ &= \alpha (\Pi^*)^{1-\theta} [P_{SS}(t-1)]^{1-\theta} + (1-\alpha) [P_{SS}^o(t)]^{1-\theta} \\ &\quad + \alpha (\Pi^*)^{1-\theta} (1-\theta) [P_{SS}(t-1)]^{1-\theta} \hat{P}_{t-1} + (1-\alpha) (1-\theta) [P_{SS}^o(t)]^{1-\theta} \hat{P}_t^o. \\ &= \alpha [P_{SS}(t)]^{1-\theta} + (1-\alpha) [P_{SS}^o(t)]^{1-\theta} \\ &\quad + \alpha (1-\theta) [P_{SS}(t)]^{1-\theta} \hat{P}_{t-1} + (1-\alpha) (1-\theta) [P_{SS}^o(t)]^{1-\theta} \hat{P}_t^o, \end{aligned} \quad \langle F.39 \rangle$$

where the last rearrangement uses Equation $\langle F.11 \rangle$. (Note that, as also stated by Equation $\langle F.11 \rangle$, the \square^o notation in some of the steady-state terms is not necessary; it is just carried over until the respective terms with and without the ‘optimality superscript’ \square^o cancel out later.) Reuniting both sides (already simplified versions of Equations $\langle F.38 \rangle$ and $\langle F.39 \rangle$) yields

$$\begin{aligned} 1 + (1-\theta) \hat{P}_t &= \alpha + (1-\alpha) + \alpha (1-\theta) \hat{P}_{t-1} + (1-\alpha) (1-\theta) \hat{P}_t^o \\ \Leftrightarrow \ln P_t - \ln P_{SS}(t) &= \alpha [\ln P_{t-1} - \ln P_{SS}(t-1)] + (1-\alpha) [\ln P_t^o - \ln P_{SS}^o(t)]. \end{aligned}$$

Using Equation (F.11) again and adding $(\ln P_t - \ln P_{t-1}) = 0$, this finally becomes

$$\begin{aligned}
 \ln P_t - \ln P_{SS}(t) &= [\alpha \ln P_{t-1} - \alpha \ln P_{SS}(t) + \alpha \ln \Pi^*] \\
 &\quad + [\ln P_t^o - \ln P_{SS}^o(t)] - [\alpha \ln P_t^o - \alpha \ln P_{SS}^o(t)] \\
 &\quad + (\ln P_{t-1} - \ln P_{t-1}) \\
 &\Leftrightarrow \ln P_t - \ln P_{t-1} = (1 - \alpha) (\ln P_t^o - \ln P_{t-1}) + \alpha \ln \Pi^* \\
 &\Leftrightarrow \pi_t = (1 - \alpha) (\ln P_t^o - \ln P_{t-1}) + \alpha \pi^*. \tag{F.40}
 \end{aligned}$$

F.3.1.7. Money Demand

Given the specific utility function (7.3), money demand (2.22) becomes

$$\zeta_m m_t^{-\nu} = \dot{I}_{t+1} c_t^{-\rho}.$$

Log-linearization via the *Simple Method* turns this into

$$\begin{aligned}
 -\zeta_m \nu m_{SS}^{-\nu-1} (m_t - m_{SS}) &= \frac{1}{(1 + i_{SS})^2} (i_{t+1} - i_{SS}) c_{SS}^{-\rho} - \dot{I}_{SS} \rho c_{SS}^{-\rho-1} (c_t - c_{SS}) \\
 \Leftrightarrow -\nu \zeta_m m_{SS}^{-\nu} \hat{m}_t &= \frac{1}{1 + i_{SS}} c_{SS}^{-\rho} (\widehat{1 + i_{t+1}}) - \dot{I}_{SS} \rho c_{SS}^{-\rho} \hat{c}_t;
 \end{aligned}$$

dividing through by the steady-state version $\zeta_m m_{SS}^{-\nu} = \dot{I}_{SS} c_{SS}^{-\rho}$ (Equation (F.17)) then leads to

$$\begin{aligned}
 -\nu \hat{m}_t &= \frac{1}{i_{SS}} (\widehat{1 + i_{t+1}}) - \rho \hat{c}_t \\
 \Leftrightarrow \hat{m}_t &= \frac{\rho}{\nu} \hat{c}_t - \frac{1}{\nu} \frac{1}{i_{SS}} (i_{t+1} - i_{SS}). \tag{F.41}
 \end{aligned}$$

F.3.1.8. Consolidated-Government Liabilities

Linearizing consolidated-government liabilities (2.28) yields

$$\check{z}_t = \frac{|b_{SS}|}{|z_{SS}|} \check{b}_t + \frac{m_{SS}}{|z_{SS}|} \hat{m}_t. \tag{F.42}$$

F.3.2. Dynamic IS Curve

Specific Functional Form Rearrange the consumption Euler equation (2.21) to

$$1 = \beta (1 + r_{t+1}) \frac{\mathbb{E}_t u_c(c_{t+1})}{u_c(c_t)},$$

where the assumption of additively separable utility allows to drop real money balances m from the marginal-utility terms, and insert the specific utility function (7.3) to get

$$1 = \beta (1 + r_{t+1}) \frac{c_t^\rho}{\mathbb{E}_t c_{t+1}^\rho}.$$

Log-Linearization Starting to log-linearize using the second replacement rule (E.4) and, in the subsequent step, noting that the Euler equation (2.21) also implies $(1 + r_{ss}) = \beta^{-1}$, the above equation can be rewritten as

$$\begin{aligned} 1 &= \beta (1 + r_{ss}) \exp(\widehat{1 + r_{t+1}}) \frac{c_{ss}^\rho \exp(\rho \hat{c}_t)}{c_{ss}^\rho \exp(\rho \mathbb{E}_t \hat{c}_{t+1})} \\ &= \exp(\widehat{1 + r_{t+1}}) \exp(\rho \hat{c}_t) \exp(-\rho \mathbb{E}_t \hat{c}_{t+1}). \end{aligned}$$

By Equation (E.5), each of the exponential functions can be substituted out, giving

$$\begin{aligned} 1 &= \left[1 + \widehat{1 + r_{t+1}} \right] (1 + \rho \hat{c}_t) (1 - \rho \mathbb{E}_t \hat{c}_{t+1}) \\ &= \left[1 - \rho \mathbb{E}_t \hat{c}_{t+1} + \rho \hat{c}_t - \rho^2 \hat{c}_t \mathbb{E}_t \hat{c}_{t+1} + \widehat{1 + r_{t+1}} - \widehat{1 + r_{t+1}} \rho \mathbb{E}_t \hat{c}_{t+1} \right. \\ &\quad \left. + \widehat{1 + r_{t+1}} \rho \hat{c}_t - \widehat{1 + r_{t+1}} \rho^2 \hat{c}_t \mathbb{E}_t \hat{c}_{t+1} \right]. \end{aligned}$$

Products of Log Deviations (p. 231) can be ignored because they are very small, simplifying the above equation to

$$\begin{aligned}
 1 &= \left[1 - \rho \mathbb{E}_t \hat{c}_{t+1} + \rho \hat{c}_t + \widehat{(1 + r_{t+1})} \right] \\
 \Leftrightarrow \hat{c}_t &= \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\rho} \widehat{(1 + r_{t+1})} \\
 &= \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\rho} [\ln(1 + r_{t+1}) - \ln(1 + r_{ss})] \\
 &\approx \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\rho} (r_{t+1} - r_{ss}).
 \end{aligned}$$

Combining this with linearized goods-market clearing ⟨F.28⟩ gives

$$\begin{aligned}
 \frac{y_{ss}}{c_{ss}} \hat{y}_t - \frac{g_{ss}}{c_{ss}} \hat{g}_t &= \mathbb{E}_t \left(\frac{y_{ss}}{c_{ss}} \hat{y}_{t+1} - \frac{g_{ss}}{c_{ss}} \hat{g}_{t+1} \right) - \frac{1}{\rho} (r_{t+1} - r_{ss}) \\
 \Leftrightarrow \hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \frac{c_{ss}}{y_{ss}} \frac{1}{\rho} (r_{t+1} - r_{ss}) - \frac{g_{ss}}{y_{ss}} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t) \\
 &= \mathbb{E}_t \hat{y}_{t+1} - \frac{c_{ss}}{y_{ss}} \frac{1}{\rho} (i_{t+1} - \mathbb{E}_t \pi_{t+1} - r_{ss}) - \frac{g_{ss}}{y_{ss}} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t),
 \end{aligned}$$

where the last rearrangement uses an approximate version of the Fisher equation ⟨2.7⟩ ($r_{t+1} = i_{t+1} - \mathbb{E}_t \pi_{t+1}$). Use Equation ⟨F.10⟩ to express the equation in terms of the output gap \tilde{y} :

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{c_{ss}}{y_{ss}} \frac{1}{\rho} (i_{t+1} - \mathbb{E}_t \pi_{t+1} - r_{ss}) - \frac{g_{ss}}{y_{ss}} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t) + (\mathbb{E}_t \hat{y}_{t+1}^{\text{nat}} - \hat{y}_t^{\text{nat}})$$

Equation ⟨F.36⟩ implies that deviations of natural output from its steady-state level depend on shocks to technology a and to fiscal policy (Equation ⟨7.36⟩ defines the shorthand Θ that is also used in the following). Using it to eliminate \hat{y}^{nat} finally gives the dynamic IS equation in terms of deviations of actual from natural output ⟨7.32⟩:

$$\begin{aligned}
 \tilde{y}_t &= \mathbb{E}_t \tilde{y}_{t+1} - \frac{c_{ss}}{y_{ss}} \frac{1}{\rho} (i_{t+1} - \mathbb{E}_t \pi_{t+1} - r_{ss}) \\
 &\quad + \frac{1}{\Theta} \left[\frac{1 + \eta}{\zeta} (\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t) + \rho \frac{g_{ss}}{c_{ss}} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t) \right] - \frac{g_{ss}}{y_{ss}} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t). \\
 &= \mathbb{E}_t \tilde{y}_{t+1} - \Xi_r (i_{t+1} - \mathbb{E}_t \pi_{t+1} - r_{ss}) + \Xi_a (\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t) + \Xi_g (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t).
 \end{aligned}$$

The steady-state ratios in the shorthands $\langle 7.33 \rangle$ - $\langle 7.36 \rangle$ can be eliminated via Equations $\langle 7.22 \rangle$ and $\langle F.15 \rangle$ - $\langle F.16 \rangle$.

F.3.3. New-Keynesian Phillips Curve

F.3.3.1. Groundwork

To make linearization easier, rearrange Equation $\langle F.7 \rangle$ as follows: Divide by P_{t-1} and substitute in the desired mark-up ϱ from Equation $\langle 7.16 \rangle$. Substitute out the *Stochastic Discount Factors* (cf. p. 26), using the specific utility function $\langle 7.3 \rangle$ as well as aggregate goods-market clearing $\langle 7.27 \rangle$ in the process. Simplify. Formally:

$$\begin{aligned}
 \frac{P_t^o(j)}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} \left(\beta^{T-t} \frac{c_T^{-\rho}}{c_t^{-\rho}} \frac{P_t}{P_T} \right) y_T (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
 &= \frac{\varrho}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} \left(\beta^{T-t} \frac{c_T^{-\rho}}{c_t^{-\rho}} \frac{P_t}{P_T} \right) MC_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
 \Leftrightarrow \frac{P_t^o(j)}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} \left(\beta^{T-t} \frac{c_T^{-\rho}}{P_T} \right) y_T (\Pi^*)^{T-t} \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
 &= \frac{\varrho}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} \left(\beta^{T-t} \frac{c_T^{-\rho}}{P_T} \right) MC_T(j) y_T \left[\frac{(\Pi^*)^{T-t}}{P_T} \right]^{-\theta} \\
 \Leftrightarrow \frac{P_t^o(j)}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_T^{1-\rho} P_T^{\theta-1} \left[(\Pi^*)^{T-t} \right]^{1-\theta} \\
 &= \frac{\varrho}{P_{t-1}} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_T^{1-\rho} P_T^{\theta} MC_T(j) \left[(\Pi^*)^{T-t} \right]^{-\theta}
 \end{aligned} \tag{F.43}$$

F.3.3.2. Step-By-Step Linearization

To make the exposition more comprehensible, log-linearization of Equation $\langle F.43 \rangle$ is carried out in several steps:

1. Taylor expansion of the left-hand side of Equation $\langle F.43 \rangle$ (p. 260)
2. Taylor expansion of the right-hand side of Equation $\langle F.43 \rangle$ (p. 260)
3. Merging the results for both sides (p. 261)
4. Relating the resulting expression to marginal costs (p. 262)
5. Producing a difference equation in inflation (p. 263)
6. Relating inflation to the output gap (p. 265)

Step 1 First is a Taylor expansion of the left-hand side of Equation (F.43) around steady state. The expression is divided into the steady state as well as four derivative terms pertaining to $P_t^o(j)$, P_{t-1} , $\mathbb{E}_t P_T$ and $\mathbb{E}_t y_T$, respectively:

$$\begin{aligned}
& \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} \\
& + \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} [P_t^o(j) - P_{SS}^o(j, t)] \\
& - \frac{P_{SS}^o(j)}{[P_{SS}(t-1)]^2} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} [P_{t-1} - P_{SS}(t-1)] \\
& + \Pi^* \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} (\theta-1) [P_{SS}(T)]^{\theta-2} [(\Pi^*)^{T-t}]^{1-\theta} [P_T - P_{SS}(T)] \\
& + \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\rho) y_{SS}^{-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} (y_T - y_{SS})
\end{aligned}$$

The following step of rearrangements uses the steady-state relationship between prices (F.11) to build terms of the form $(\square_t - \square_{SS})/\square_{SS}$ at the end of the lines containing derivative parts (not shown), which are approximately equivalent to log deviations because of Equations (7.29) (the definition of log differences) and (E.2) (the first replacement rule):

$$\begin{aligned}
& \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} \\
& + \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} [\ln P_t^o(j) - \ln P_{SS}^o(j, t)] \\
& - \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} [\ln P_{t-1} - \ln P_{SS}(t-1)] \\
& + \Pi^* \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} (\theta-1) [\Pi^* P_{SS}(t-1)]^{\theta-1} [\ln P_T - \ln P_{SS}(T)] \\
& + \Pi^* \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\rho) y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^{\theta-1} (\ln y_T - \ln y_{SS}) \\
& = (\Pi^*)^{\theta} [P_{SS}(t-1)]^{\theta-1} y_{SS}^{1-\rho} \\
& \quad \times \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \{1 + [\ln P_t^o(j) - \ln P_{SS}^o(j, t)] - [\ln P_{t-1} - \ln P_{SS}(t-1)] \\
& \quad + (\theta-1) [\ln P_T - \ln P_{SS}(T)] + (1-\rho) (\ln y_T - \ln y_{SS})\}
\end{aligned}$$

Step 2 The next step is a similar approximation of the right-hand side of Equation (F.43):

$$\begin{aligned}
& \frac{\varrho}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta mc_{SS}(j) \\
& - \frac{\varrho}{[P_{SS}(t-1)]^2} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta mc_{SS}(j) [P_{t-1} - P_{SS}(t-1)] \\
& + \frac{\varrho}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} \theta [\Pi^* P_{SS}(T)]^{\theta-1} mc_{SS}(j) [P_T - P_{SS}(T)] \\
& + \frac{\varrho}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\rho) y_{SS}^{-\rho} [\Pi^* P_{SS}(t-1)]^\theta mc_{SS}(j) (y_T - y_{SS}) \\
& + \frac{\varrho}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta [mc_T(j) - mc_{SS}(j)]
\end{aligned}$$

Rearrangements include utilization of the relationship between marginal costs in steady state $mc_{SS}(j)$ and the frictionless markup ϱ (F.9) as well as generating log deviations similarly to \nearrow Step 1 (p. 260):

$$\begin{aligned}
& \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta \\
& - \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta [\ln P_{t-1} - \ln P_{SS}(t-1)] \\
& + \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} \theta [\Pi^* P_{SS}(T)]^\theta [\ln P_T - \ln P_{SS}(T)] \\
& + \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (1-\rho) y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta (\ln y_T - \ln y_{SS}) \\
& + \frac{1}{P_{SS}(t-1)} \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} y_{SS}^{1-\rho} [\Pi^* P_{SS}(t-1)]^\theta [\ln mc_T(j) - \ln mc_{SS}(j)] \\
& = (\Pi^*)^\theta [P_{SS}(t-1)]^{\theta-1} y_{SS}^{1-\rho} \\
& \quad \times \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \{1 - [\ln P_{t-1} - \ln P_{SS}(t-1)] + \theta [\ln P_T - \ln P_{SS}(T)] \\
& \quad + (1-\rho) (\ln y_T - \ln y_{SS}) + [\ln mc_T(j) - \ln mc_{SS}(j)]\}
\end{aligned}$$

Step 3 Finally, both parts are reunited. With some simplifications (terms cancelling out) already incorporated in the first line, we have:

$$\begin{aligned}
& \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \{ [\ln P_t^o(j) - \ln P_{SS}^o(j, t)] - [\ln P_T - \ln P_{SS}(T)] \} \\
& \quad = \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\ln mc_T(j) - \ln mc_{SS}(j)] \\
& \Leftrightarrow \frac{1}{1-\alpha\beta} [\ln P_t^o(j) - \ln \Pi^* - \ln P_{t-1}] \\
& \quad = \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \ln mc_T(j) - \ln(\varrho^{-1}) + \ln P_T - \ln [P_{t-1} (\Pi^*)^{T-t+1}] \right\} \\
& \Leftrightarrow \ln P_t^o(j) = \ln \varrho + (1-\alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\ln mc_T(j) + \ln P_T] \\
& \quad \quad + \ln \Pi^* - (1-\alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (T-t+1) \ln \Pi^* \\
& \hspace{15em} \langle \text{F.44} \rangle
\end{aligned}$$

At this point, it is important that the last term on the right-hand side is finite (the derivation is relegated to Section F.3.3.3 in order to not clutter this section any further), so the equation can be simplified further to

$$\ln P_t^o(j) = \ln \varrho + (1-\alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\ln mc_T(j) + \ln P_T] - \frac{\alpha\beta}{1-\alpha\beta} \ln \Pi^*. \quad \langle \text{F.45} \rangle$$

Step 4 Combine the log expressions $\langle \text{F.45} \rangle$ (from directly above) and $\langle \text{F.33} \rangle$ (the logarithm of marginal costs) to

$$\begin{aligned}
\ln P_t^o(j) = (1-\alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} & \left\{ \ln mc_T + \frac{\theta(1-\zeta)}{\zeta} [\ln P_T - \ln P_T(j)] \right. \\
& \left. + \ln \varrho + \ln P_T \right\} - \frac{\alpha\beta}{1-\alpha\beta} \ln \Pi^*.
\end{aligned}$$

Let it be noted again that Equation $\langle \text{F.45} \rangle$ stems from the firm's *Maximization Problem* (p. 242) so $P_T(j)$ can be expressed in terms of $P_t^o(j)$ and target inflation Π^* . Further,

Equation (F.9) is applied to $\ln \varrho$:

$$\begin{aligned}
 \ln P_t^o(j) &= (1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \widehat{mc}_T + \left[1 + \frac{\theta(1-\zeta)}{\zeta} \right] \ln P_T \right. \\
 &\quad \left. - \frac{\theta(1-\zeta)}{\zeta} [\ln P_t^o(j) + (T-t) \ln \Pi^*] \right\} - \frac{\alpha\beta}{1-\alpha\beta} \ln \Pi^* \\
 \Leftrightarrow \left[1 + \frac{\theta(1-\zeta)}{\zeta} \right] \ln P_t^o(j) &= (1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \widehat{mc}_T \right. \\
 &\quad \left. + \left[1 + \frac{\theta(1-\zeta)}{\zeta} \right] \ln P_T \right\} - \frac{\alpha\beta}{1-\alpha\beta} \left[1 + \frac{\theta(1-\zeta)}{\zeta} \right] \ln \Pi^* \\
 \Leftrightarrow \ln P_t^o(j) &= (1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left\{ \left[\frac{\zeta}{\zeta + \theta(1-\zeta)} \right] \widehat{mc}_T + \ln P_T \right\} - \frac{\alpha\beta}{1-\alpha\beta} \ln \Pi^*
 \end{aligned}$$

Let

$$\Theta' \equiv \frac{\zeta}{\zeta + \theta(1-\zeta)} \quad \langle \text{F.46} \rangle$$

and rearrange further by subtracting $\ln P_{t-1}$ from both sides:

$$\ln P_t^o(j) - \ln P_{t-1} = (1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\Theta' \widehat{mc}_T + \ln P_T - \ln P_{t-1}) - \frac{\alpha\beta}{1-\alpha\beta} \ln \Pi^* \quad \langle \text{F.47} \rangle$$

Step 5 Equation (F.47) can be rearranged into a difference equation in inflation. To do so, start with the part of the sum term on the right-hand side that contains the price levels:

$$\begin{aligned}
 &(1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (\ln P_T - \ln P_{t-1}) \\
 &= (1 - \alpha\beta) \mathbb{E}_t [(\ln P_t - \ln P_{t-1}) + (\alpha\beta) (\ln P_{t+1} - \ln P_{t-1}) \\
 &\quad + (\alpha\beta)^2 (\ln P_{t+2} - \ln P_{t-1}) + \dots] \\
 &= (1 - \alpha\beta) \mathbb{E}_t [(\ln P_t - \ln P_{t-1}) + (\alpha\beta) (\ln P_{t+1} - \ln P_t + \ln P_t - \ln P_{t-1}) \\
 &\quad + (\alpha\beta)^2 (\ln P_{t+2} - \ln P_{t+1} + \ln P_{t+1} - \ln P_t + \ln P_t - \ln P_{t-1}) + \dots] \\
 &\approx (1 - \alpha\beta) \mathbb{E}_t [\pi_t + (\alpha\beta) (\pi_{t+1} + \pi_t) + (\alpha\beta)^2 (\pi_{t+2} + \pi_{t+1} + \pi_t) + \dots]
 \end{aligned}$$

$$\begin{aligned}
&= \mathbb{E}_t \left[\pi_t + (\alpha\beta) (\pi_{t+1} + \pi_t) + (\alpha\beta)^2 (\pi_{t+2} + \pi_{t+1} + \pi_t) + \dots \right] \\
&\quad - \mathbb{E}_t \left[\alpha\beta\pi_t + (\alpha\beta)^2 (\pi_{t+1} + \pi_t) + (\alpha\beta)^3 (\pi_{t+2} + \pi_{t+1} + \pi_t) + \dots \right] \\
&= \mathbb{E}_t \left[\pi_t + \alpha\beta\pi_{t+1} + (\alpha\beta)^2 \pi_{t+2} + \dots \right] \\
&= \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \pi_T
\end{aligned}$$

Plugging this back into Equation (F.47) gives

$$\ln P_t^o(j) - \ln P_{t-1} = \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 - \alpha\beta) \Theta' \widehat{m}c_T + \pi_T] - \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^*, \quad \langle \text{F.48} \rangle$$

which can be rearranged in two ways. For one, it can be shifted forward once, yielding

$$\begin{aligned}
\mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] &= \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 - \alpha\beta) \Theta' \widehat{m}c_{T+1} + \pi_{T+1}] - \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^* \\
\Leftrightarrow \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 - \alpha\beta) \Theta' \widehat{m}c_{T+1} + \pi_{T+1}] &= \mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] + \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^*.
\end{aligned} \quad \langle \text{F.49} \rangle$$

On the other hand, one can rearrange the sum term (using Equation (F.49) in the second step already):

$$\begin{aligned}
\ln P_t^o(j) - \ln P_{t-1} &= \alpha\beta \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(1 - \alpha\beta) \Theta' \widehat{m}c_{T+1} + \pi_{T+1}] \\
&\quad + (1 - \alpha\beta) \Theta' \widehat{m}c_t + \pi_t - \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^* \\
&= \alpha\beta \left\{ \mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] + \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^* \right\} \\
&\quad + (1 - \alpha\beta) \Theta' \widehat{m}c_t + \pi_t - \frac{\alpha\beta}{1 - \alpha\beta} \ln \Pi^* \\
&= \alpha\beta \mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] + (1 - \alpha\beta) \Theta' \widehat{m}c_t + \pi_t - \alpha\beta \ln \Pi^*
\end{aligned}$$

To conclude this step, first substitute out π_t via Equation (F.40) and later substitute it in again (twice):

$$\alpha [\ln P_t^o(j) - \ln P_{t-1}] = \alpha\beta \mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] + (1 - \alpha\beta) \Theta' \widehat{m}c_t + \alpha (1 - \beta) \pi^*$$

$$\begin{aligned}
&\Leftrightarrow (1 - \alpha) [\ln P_t^o(j) - \ln P_{t-1}] \\
&= \beta (1 - \alpha) \mathbb{E}_t [\ln P_{t+1}^o(j) - \ln P_t] + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \Theta' \widehat{mc}_t + (1 - \alpha)(1 - \beta) \pi^* \\
&\Leftrightarrow \pi_t - \alpha \pi^* = \beta (\mathbb{E}_t \pi_{t+1} - \alpha \pi^*) + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \Theta' \widehat{mc}_t + (1 - \alpha - \beta + \alpha\beta) \pi^*
\end{aligned}$$

To make the equation more legible, introduce the shorthand

$$\Theta'' \equiv \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \Theta' \quad \langle \text{F.50} \rangle$$

and rewrite the last equation as

$$\pi_t - \pi^* = \beta (\mathbb{E}_t \pi_{t+1} - \pi^*) + \Theta'' \widehat{mc}_t. \quad \langle \text{F.51} \rangle$$

Step 6 In this final step, replace the marginal-cost expression via Equation $\langle \text{F.37} \rangle$ and use the shorthand κ as defined in Equation $\langle 7.38 \rangle$ (see also Equations $\langle 7.36 \rangle$, $\langle \text{F.46} \rangle$ and $\langle \text{F.50} \rangle$) to arrive at the New-Keynesian Phillips Curve $\langle 7.37 \rangle$:

$$\pi_t - \pi^* = \beta (\mathbb{E}_t \pi_{t+1} - \pi^*) + \kappa \tilde{y}_t$$

This relatively general form with possibly decreasing returns to scale in the production function (if $\zeta < 1$) can also be found in Galí, Gertler, and López-Salido (2001) and Sbordone (2002), for instance.

F.3.3.3. A Relegated Derivation

In \nearrow Step 3 of the previous subsection, it is argued that the last term on the right-hand side of Equation $\langle \text{F.44} \rangle$,

$$-(1 - \alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (T - t + 1) \ln \Pi^*,$$

is finite. To see this, consider the following series (in a general notation), where $|\square| < 1$:

$$\begin{aligned}
\bigcirc &\equiv \sum_{j=0}^J (j+1) \square^j = 1 + 2\square + 3\square^2 + \dots + (J+1) \square^J \\
&\Rightarrow \square \bigcirc = \square + 2\square^2 + \dots + J\square^J + (J+1) \square^{J+1} \\
&\Rightarrow (1 - \square) \bigcirc = 1 + \square + \square^2 + \dots + \square^J - (J+1) \square^{J+1}
\end{aligned}$$

$$\Rightarrow \square(1-\square)\bigcirc = \square + \square^2 + \dots + \square^J + \square^{J+1} - (J+1)\square^{J+2}$$

$$\Rightarrow (1-\square)(1-\square)\bigcirc = 1 - (J+2)\square^{J+1} + (J+1)\square^{J+2}$$

With regard to the last two terms on the right-hand side of the last equation, taking limits as $J \rightarrow \infty$ gives expressions of the form ' $\infty \cdot 0$.' Rearranging and applying de l'Hôpital's rule, we have (for the last term, but the same applies to the second-to-last term)

$$\lim_{J \rightarrow \infty} \frac{J+1}{\square^{J+2}} \stackrel{\text{r.H.}}{=} \lim_{J \rightarrow \infty} \frac{1}{-\left(\frac{1}{\square^{J+2}}\right)^2 \square^{J+2} \ln \square} = \lim_{J \rightarrow \infty} \frac{\square^{J+2}}{-\ln \square} = 0.$$

Therefore,

$$\bigcirc \equiv \sum_{j=0}^{\infty} (j+1) \square^j = \frac{1}{(1-\square)^2}.$$

Applying this to the term originally under consideration, with $(T-t) \triangleq j$ and $\alpha\beta \triangleq \square$, we arrive at

$$-(1-\alpha\beta) \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} (T-t+1) = -(1-\alpha\beta) \ln \Pi^* \frac{1}{(1-\alpha\beta)^2} = -\frac{\ln \Pi^*}{(1-\alpha\beta)},$$

which is then used in the rearrangement leading to Equation (F.45).

F.3.4. Approximate Consolidated-Government Budget Constraint

The first step is to linearize the law of motion for consolidated-government liabilities (F.18) around steady state:

$$z_t - z_{SS} = \left(\frac{z_{SS}}{\Pi^*} - s_{SS} - m_{SS} \right) (i_{t+1} - i_{SS}) + \frac{1+i_{SS}}{\Pi^*} (z_{t-1} - z_{SS})$$

$$- \frac{1+i_{SS}}{(\Pi^*)^2} z_{SS} (\Pi_t - \Pi^*) - (1+i_{SS}) (s_t - s_{SS}) - (m_t - m_{SS})$$

Dividing through by $|z_{SS}|$ (in order to reach the form of Equation (7.31)) and using steady-state renditions of Equation (F.18) as well as definition (2.23) on the first term in

parentheses on the right-hand side, this becomes

$$\begin{aligned}\check{z}_t = & \frac{z_{SS} - m_{SS}}{|z_{SS}|} \frac{(1 + i_{t+1}) - (1 + i_{SS})}{1 + i_{SS}} + \beta^{-1} \check{z}_{t-1} - \beta^{-1} \frac{z_{SS}}{|z_{SS}|} \hat{\Pi}_t \\ & - (1 + i_{SS}) \frac{|s_{SS}|}{|z_{SS}|} \check{s}_t - \frac{i_{SS}}{|z_{SS}|} (m_t - m_{SS}).\end{aligned}$$

Some more rearrangements affect the log-deviations of the nominal interest rate and inflation rate (making use of the relationship between *Gross and Net Rates*, p. 231), the definition of consolidated-government liabilities (2.28), and the inclusion of \hat{m}_t . The final result is Equation (7.39):

$$\check{z}_t = \beta^{-1} \check{z}_{t-1} + \frac{b_{SS}}{|z_{SS}|} (i_{t+1} - i_{SS}) - \beta^{-1} \frac{z_{SS}}{|z_{SS}|} (\pi_t - \pi^*) - (1 + i_{SS}) \frac{|s_{SS}|}{|z_{SS}|} \check{s}_t - \frac{i_{SS} m_{SS}}{|z_{SS}|} \hat{m}_t$$

F.3.5. Policy in the Log-Linear Model

Taylor Rule The interest-rate rule (7.18) can be linearized using the *Simple Method*, which results in

$$\begin{aligned}i_{t+1} - i_{SS} &= (1 + i_{SS}) \gamma_{\pi}^C (\pi_t - \pi^*) + (1 + i_{SS}) \gamma_y^C (\hat{y}_t - \hat{y}_t^{\text{nat}}) + (1 + i_{SS}) \varepsilon_t^i \\ \Leftrightarrow \frac{1 + i_{t+1} - (1 + i_{SS})}{1 + i_{SS}} &= \gamma_{\pi}^C (\pi_t - \pi^*) + \gamma_y^C \tilde{y}_t + \varepsilon_t^i,\end{aligned}$$

where Equation (F.10) is used to receive \tilde{y} . Using the relationship between *Gross and Net Rates* (E.3) on the left-hand side, we arrive at Equation (7.41):

$$i_{t+1} = i_{SS} + \gamma_{\pi}^C (\pi_t - \pi^*) + \gamma_y^C \tilde{y}_t + \varepsilon_t^i$$

Surplus Rule Similarly, linearizing Equation (7.23) produces

$$\begin{aligned}s_t - s_{SS} &= \gamma_y^T [(y_t - y_{SS}) - (y_t^{\text{nat}} - y_{SS}^{\text{nat}})] + \gamma_z^T \left[\frac{1}{\Pi^*} (z_{t-1} - z_{SS}) - \frac{z_{SS}}{(\Pi^*)^2} (\Pi_t - \Pi^*) \right] \\ &\quad - \gamma_m^T \left[\frac{m_{SS}}{(1 + i_{SS})^2} (i_{t+1} - i_{SS}) + i_{SS} (m_t - m_{SS}) \right] + (\varepsilon_t^s - \varepsilon_{SS}^s),\end{aligned}$$

where $\partial \dot{I} / \partial i = (1 + i)^{-2}$ is used. Utilizing the relationships between *Natural vs. Steady-State Output* (F.10) (p. 246, for \tilde{y}) and *Gross and Net Rates* (p. 231), the above

equation can be rearranged to

$$\begin{aligned}\check{s}_t &= \gamma_y^T \frac{y_{SS}}{|s_{SS}|} [\hat{y}_t - \hat{y}_t^{\text{nat}}] + \gamma_z^T \frac{1}{\Pi^*} \frac{1}{|s_{SS}|} (|z_{SS}| \check{z}_{t-1} - z_{SS} \hat{\Pi}_t) \\ &\quad - \gamma_m^T \frac{1}{1+i_{SS}} \frac{m_{SS}}{|s_{SS}|} \left[\widehat{(1+i_{t+1})} + i_{SS} \hat{m}_t \right] + \frac{1}{|s_{SS}|} \varepsilon_t^s \\ &= \gamma_y^T \frac{y_{SS}}{|s_{SS}|} \tilde{y}_t + \gamma_z^T \frac{1}{\Pi^*} \frac{1}{|s_{SS}|} [|z_{SS}| \check{z}_{t-1} - z_{SS} (\pi_t - \pi^*)] \\ &\quad - \gamma_m^T \frac{1}{1+i_{SS}} \frac{m_{SS}}{|s_{SS}|} [(i_{t+1} - i_{SS}) + i_{SS} \hat{m}_t] + \frac{1}{|s_{SS}|} \varepsilon_t^s.\end{aligned}$$

Finally, inserting the Taylor rule [\(7.41\)](#) for i_{t+1} leads to Equation [\(7.42\)](#):

$$\begin{aligned}\check{s}_t &= \gamma_y^T \frac{y_{SS}}{|s_{SS}|} \tilde{y}_t + \gamma_z^T \frac{1}{\Pi^*} \frac{1}{|s_{SS}|} [|z_{SS}| \check{z}_{t-1} - z_{SS} (\pi_t - \pi^*)] \\ &\quad - \gamma_m^T \frac{1}{1+i_{SS}} \frac{m_{SS}}{|s_{SS}|} \left[\gamma_\pi^C (\pi_t - \pi^*) + \gamma_y^C \tilde{y}_t + \varepsilon_t^i + i_{SS} \hat{m}_t \right] + \frac{1}{|s_{SS}|} \varepsilon_t^s \\ &= \frac{1}{|s_{SS}|} \left(\gamma_y^T y_{SS} - \gamma_m^T \frac{m_{SS}}{1+i_{SS}} \gamma_y^C \right) \tilde{y}_t - \frac{1}{|s_{SS}|} \left(\gamma_z^T \frac{z_{SS}}{\Pi^*} + \gamma_m^T \frac{m_{SS}}{1+i_{SS}} \gamma_\pi^C \right) (\pi_t - \pi^*) \\ &\quad + \gamma_z^T \frac{1}{\Pi^*} \frac{|z_{SS}|}{|s_{SS}|} \check{z}_{t-1} - \gamma_m^T \frac{m_{SS}}{|s_{SS}|} \hat{m}_t - \gamma_m^T \frac{1}{1+i_{SS}} \frac{m_{SS}}{|s_{SS}|} \varepsilon_t^i + \frac{1}{|s_{SS}|} \varepsilon_t^s\end{aligned}$$

F.4. Calibration and Robustness

Introductory Note The following points should be understood as *caveats* rather than full-on robustness checks. On the one hand, this acknowledges that the present model, like most modern DSGE models, strongly depends on the parameters it is fed (aside from policy parameters of course, the influence of which is one of the main concerns of this thesis). On the other hand, the baseline calibrations chosen here are common in the literature so I (warily) rely on them to analyze the monetary-fiscal interactions instead of ‘including another thesis’ on sensitivity analysis.

Utility Weights The weight of utility from real money in the utility function [\(7.3\)](#) is set to $\xi_m = 0.0013$. Along with the calibrations of the other relevant parameters, this can be plugged into the steady-state money-demand equation [\(F.17\)](#) and then leads to m_{SS}/y_{SS} ratios between 5.5% and 7.8%. This is in line with the 5.0% average ratio of base money to GDP in the United States from 1980 to 2007 (average M1/GDP over the same time

period is 12.1%).³⁵

It is already expressed in the main text that the details of modeling the supply side of the model are not the main focus of this thesis (see Footnote 16, for instance). Accordingly, the weight of labor disutility is set to unity for simplicity.

Autoregression Coefficients Mostly, the impulse-response functions of the two baseline calibrations (following Galí 2008 with $\gamma_a = \gamma_\varepsilon^C = \gamma_\varepsilon^T = 0.5$ and Kim 2003 with $\gamma_a = \gamma_\varepsilon^C = \gamma_\varepsilon^T = 0.8$, respectively) resemble each other with the expectable gradual differences in amplitudes and speeds of adjustment. It has to be noted, however, that setting the persistence parameters is not entirely inconsequential: Switching the parameters (from 0.5 to 0.8 in the Galí 2008 calibration and from 0.8 to 0.5 in the Kim 2003 calibration) leads to sign changes in the responses of the nominal interest rate i , real money m , and real treasury debt b to an interest-rate shock. By contrast, these sign changes are not observed when autoregressive parameters are switched in the analyses of technology or surplus shocks.

Surpluses The fixed component of surpluses s^{fix} is set to 0.005 in the baseline calibrations. This is guided somewhat by its repercussions on steady-state consolidated-government liabilities z_{SS} , which are also affected by other parameters, however (see Equation (F.19) in Appendix F.2.2), so that a definitive calibration of s^{fix} cannot be given.

While $s^{\text{fix}} = 0$ is not permitted in the current state of the model (see *Non-Zero Steady States*, p. 235), both positive and negative values sufficiently far away from zero are possible. Further, because of the additive structure of the equations depending on the constant s^{fix} (again, see Equation (F.19)), zero is typically not the demarcation line for sign switches in the impulse-response functions.

Public Relative to Private Spending The ratio of treasury expenditure to private consumption Γ is set to 2/3, but (unreported) robustness checks show that setting it to different values only causes gradual changes in the impulse-response functions.

³⁵ Data taken from $\nearrow\text{FRED}$ (cf. Footnote 27, p. 159). Data series: $\nearrow\text{MBCURRCIR}$, $\nearrow\text{M1}$, $\nearrow\text{GDP}$.

F.5. Figures and Tables for the Sticky-Price Model

no.	variable	type	description
1	\hat{a}	log-lin	technology
2	\check{b}	lin	treasury debt
3	\hat{c}	log-lin	consumption
4	\hat{g}	log-lin	treasury expenditure
5	i	level	nominal interest rate
6	$\hat{\ell}$	log-lin	labor
7	\hat{m}	log-lin	real money
8	\hat{r}	log-lin	real interest rate
9	\check{s}	lin	budget surpluses of the treasury
10	\hat{t}^{TH}	lin	transfers to households
11	\hat{w}	log-lin	real wage
12	\hat{y}	log-lin	output
13	\tilde{y}	log-lin	output gap
14	\hat{y}^{nat}	log-lin	natural output
15	\check{z}	lin	consolidated-government liabilities
16	ε^i	level	AR(1) interest-rate shock
17	ε^s	level	AR(1) surplus shock
18	π	level	inflation rate

Table F.1a: *Overview of Variables in the Linearized New-Keynesian Model.* \diamond
Explanations: Variables of type “log-lin” are linearized via Equation (7.29), variables of type “lin” are linearized via Equation (7.31), and variables of type “level” emerge from linearization in levels (see Section 7.2.1 of the main text and Appendix E).

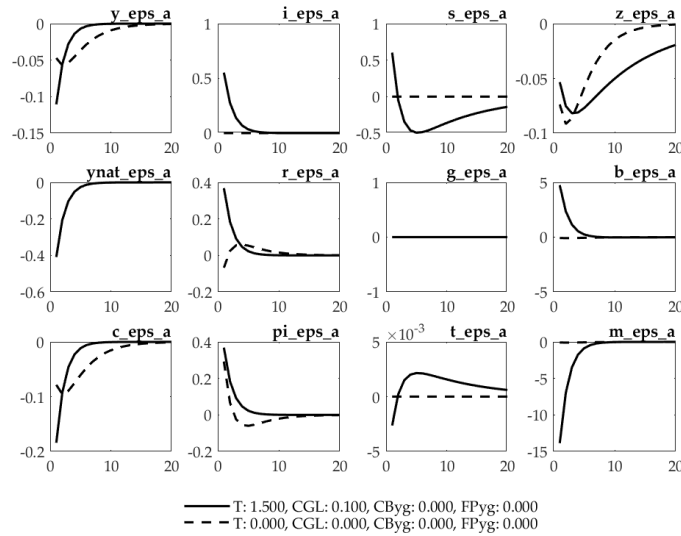


Figure F.2a: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

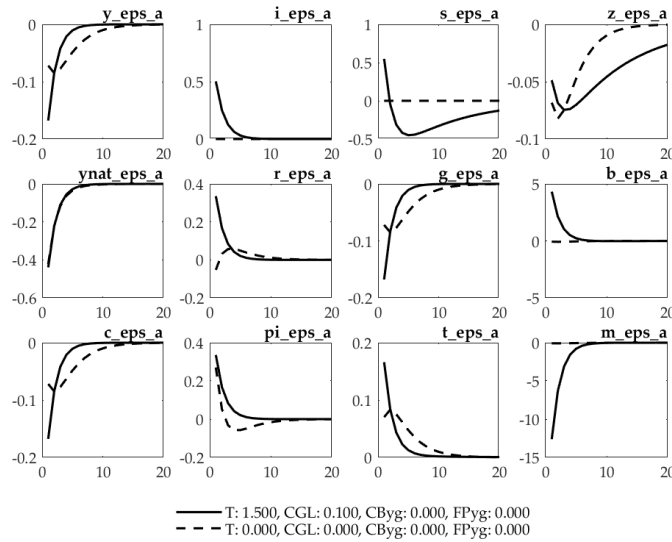


Figure F.2b: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

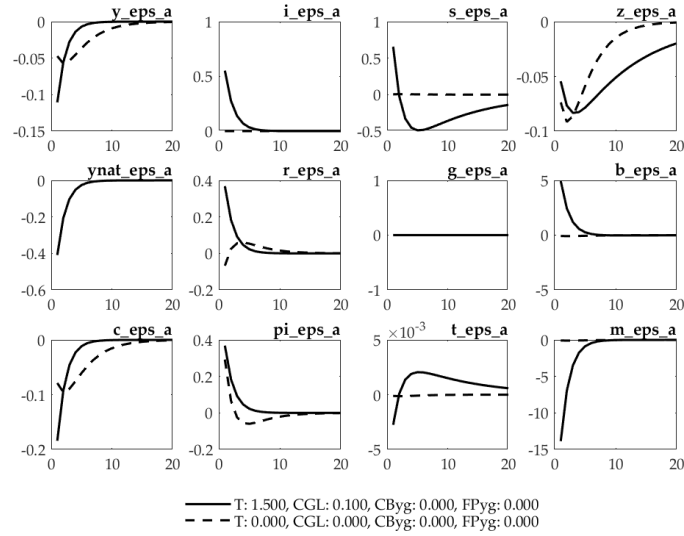


Figure F.2c: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

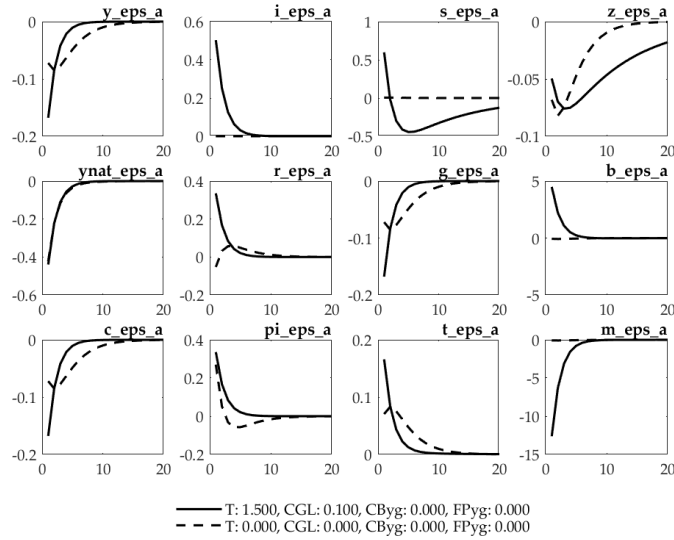


Figure F.2d: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

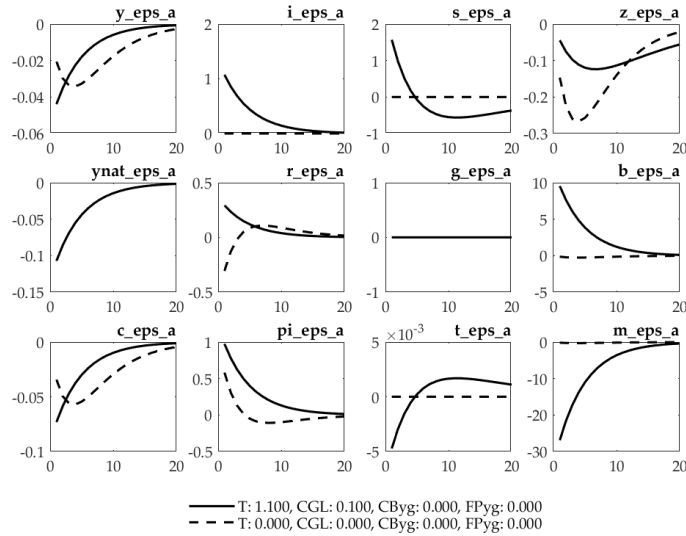


Figure F.2e: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

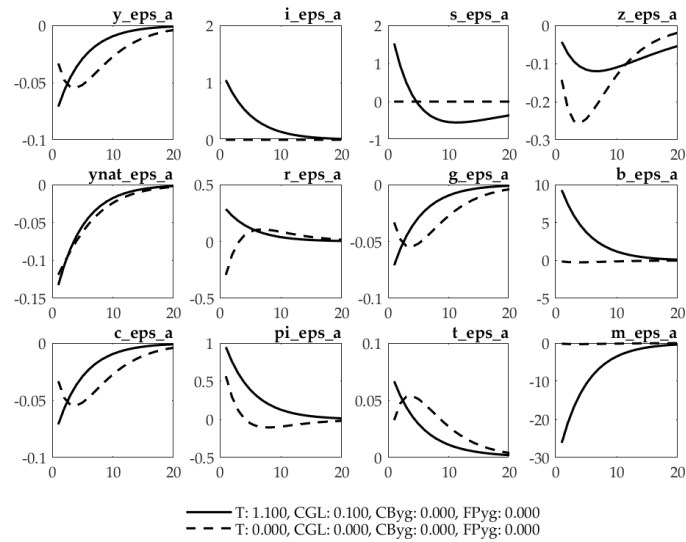


Figure F.2f: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

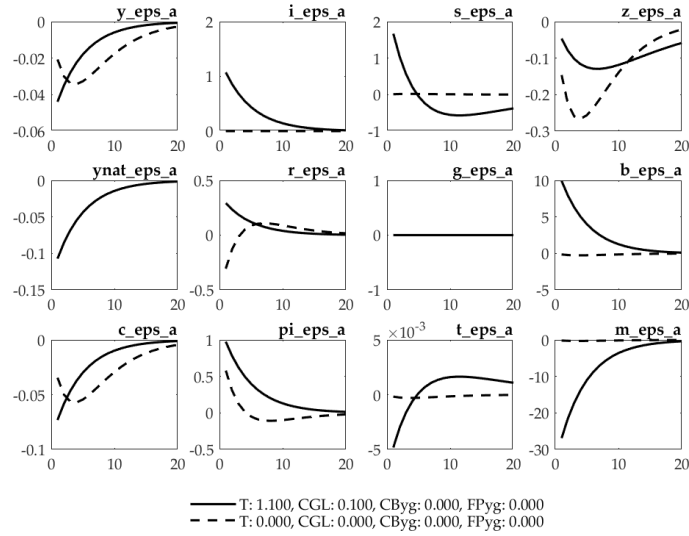


Figure F.2g: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

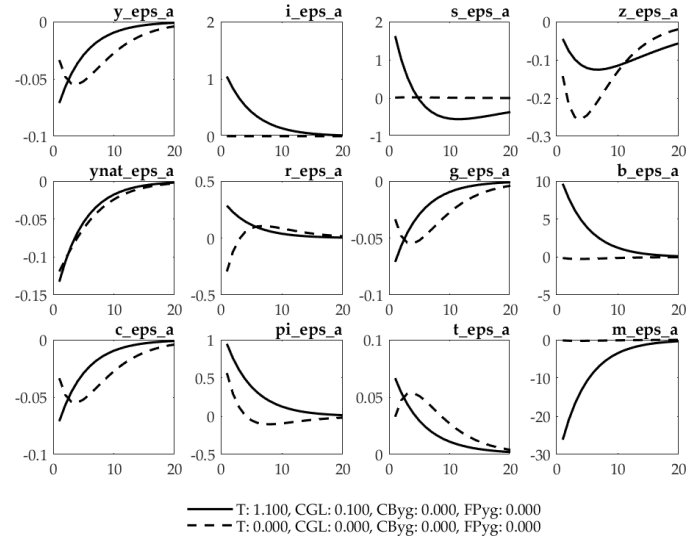


Figure F.2h: *Simulation of the Sticky-Price Model: Technology Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^a = -0.25$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.1.

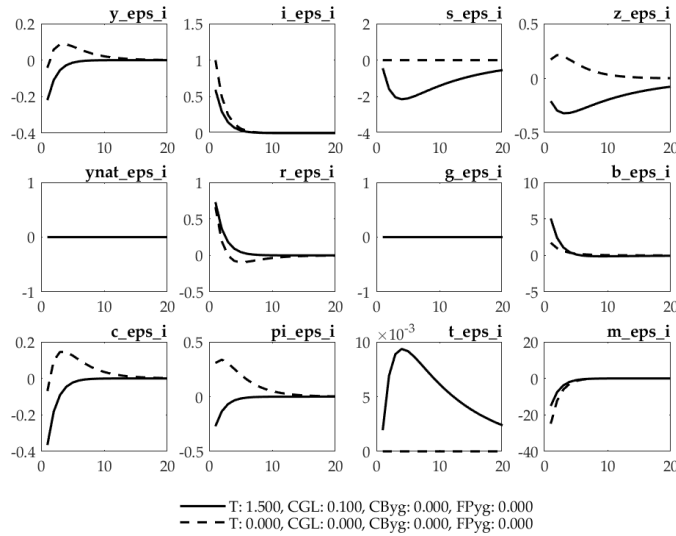


Figure F.3a: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

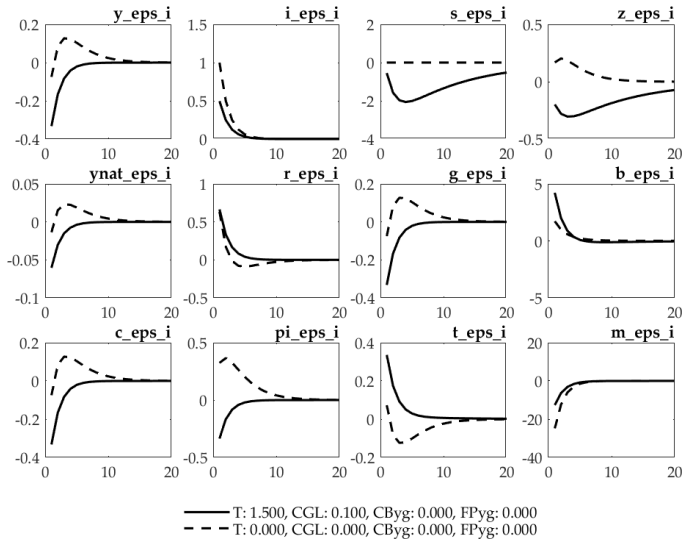


Figure F.3b: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

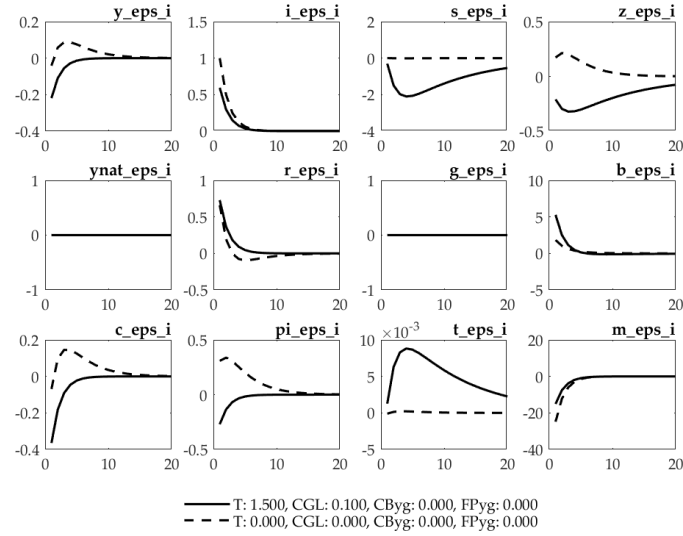


Figure F.3c: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

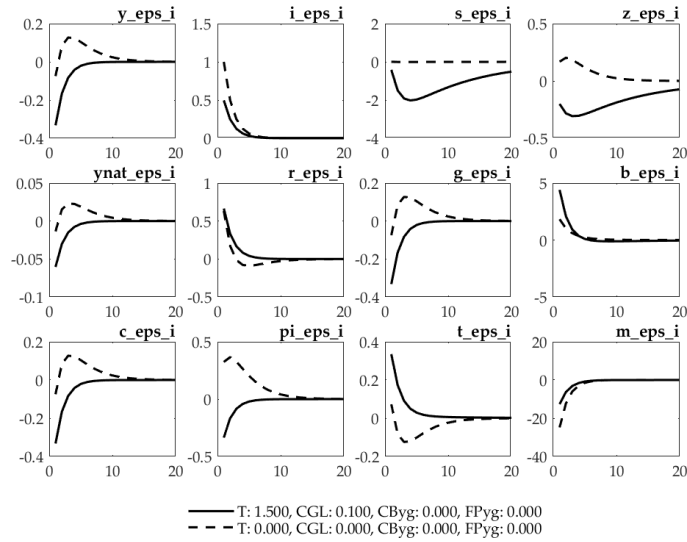


Figure F.3d: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

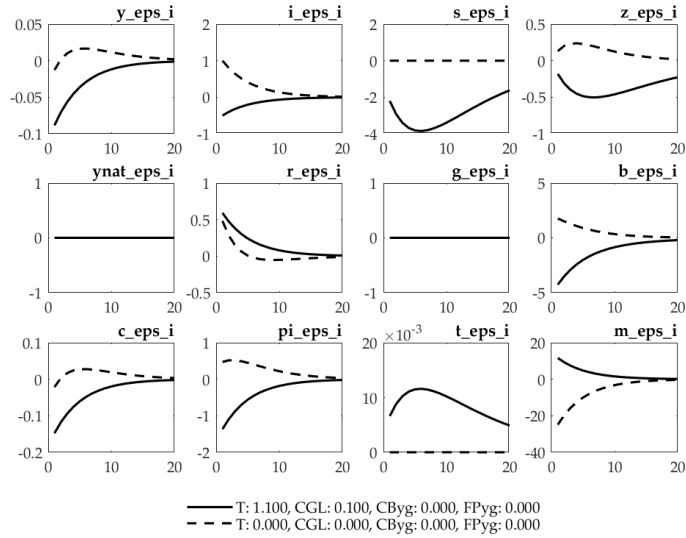


Figure F.3e: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^i = 1$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule $\langle 7.25a \rangle$. See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

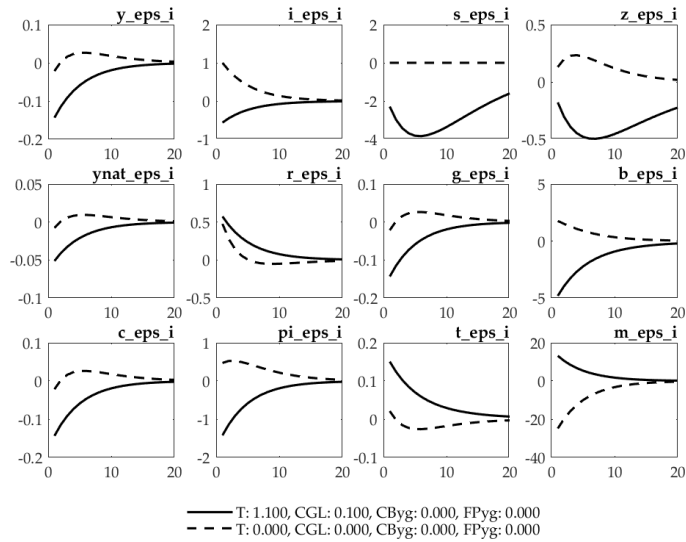


Figure F.3f: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^i = 1$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule $\langle 7.25b \rangle$. See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

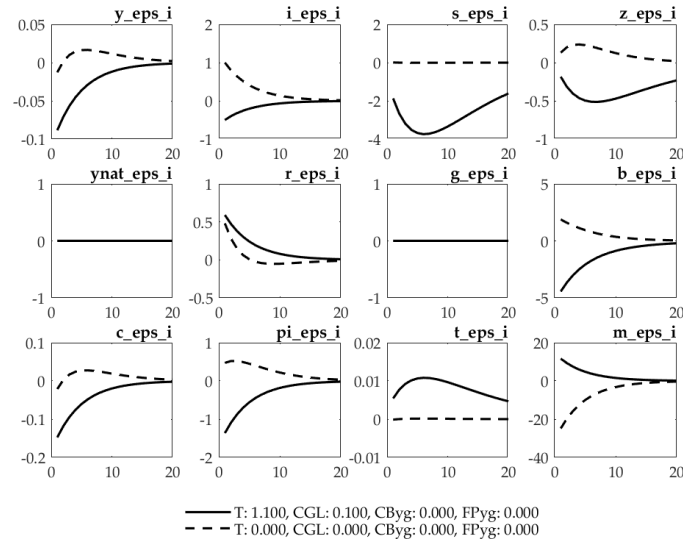


Figure F.3g: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

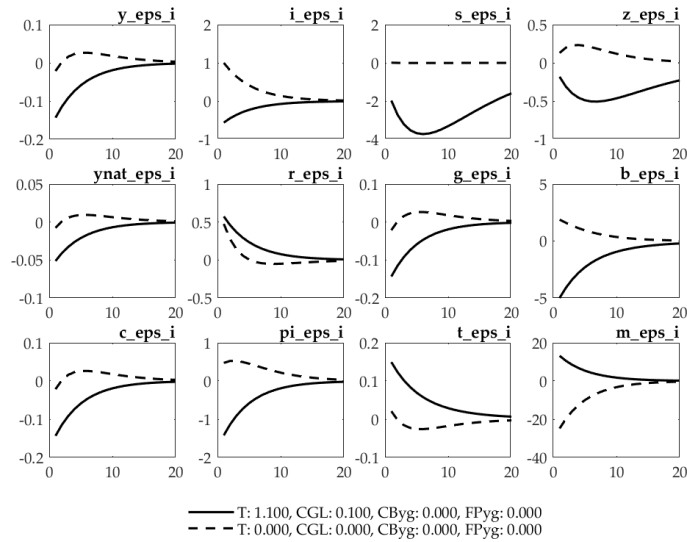


Figure F.3h: *Simulation of the Sticky-Price Model: Interest-Rate Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.2.

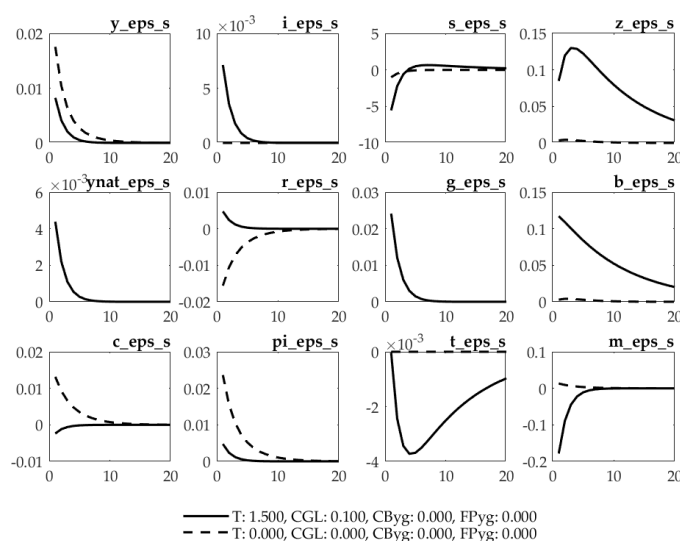


Figure F.4a: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

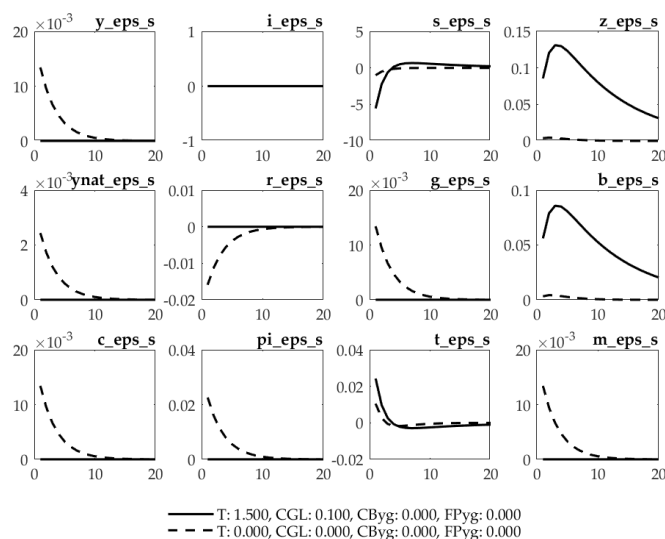


Figure F.4b: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

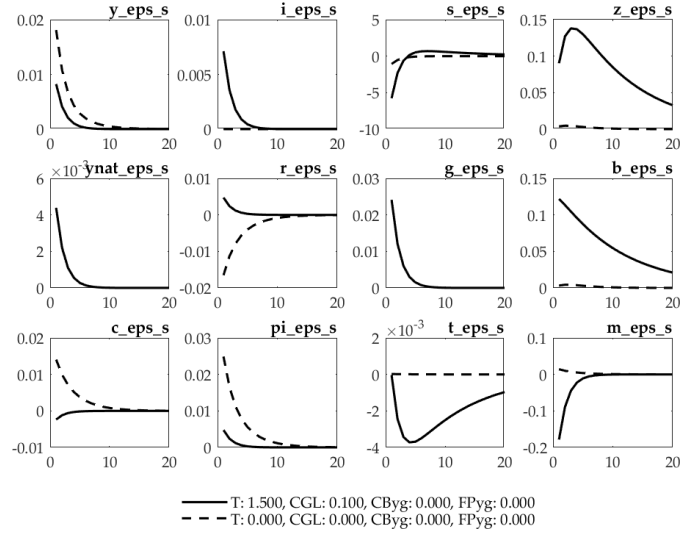


Figure F.4c: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^s = -0.01$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

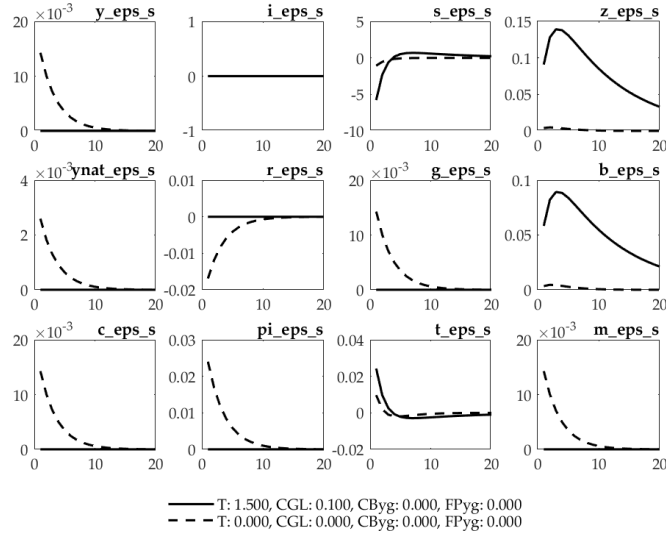


Figure F.4d: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^s = -0.01$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

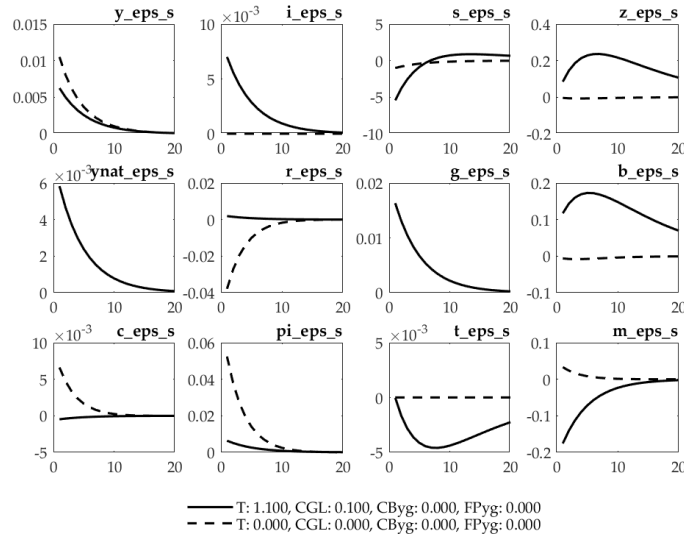


Figure F.4e: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule $\langle 7.25a \rangle$. See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

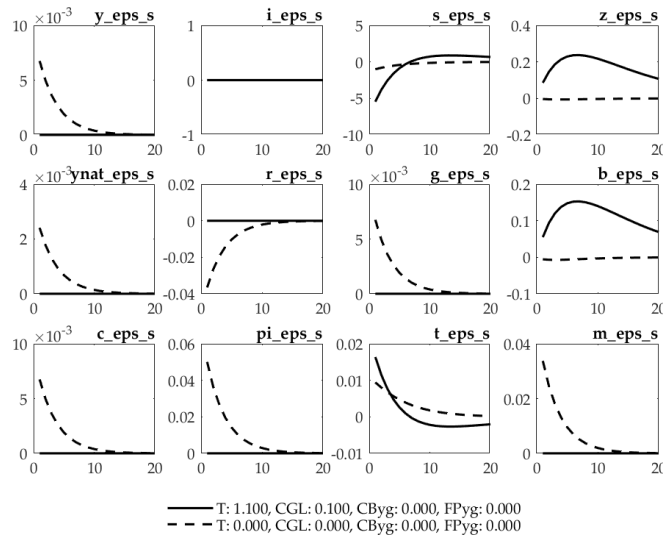


Figure F.4f: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule $\langle 7.25b \rangle$. See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

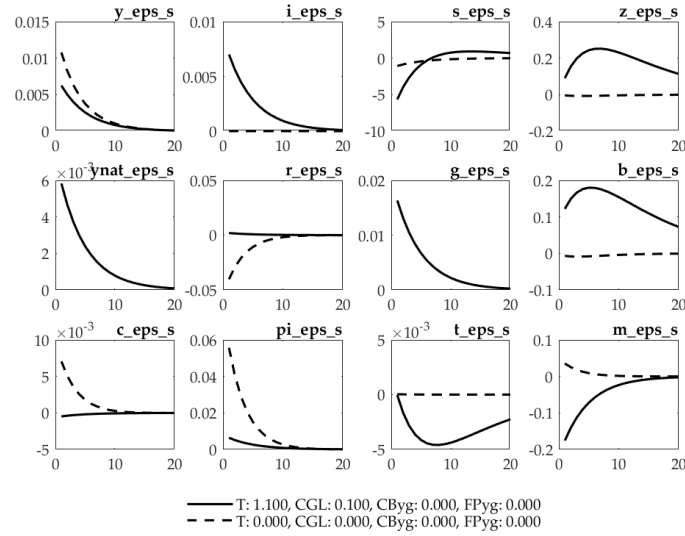


Figure F.4g: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^s = -0.01$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

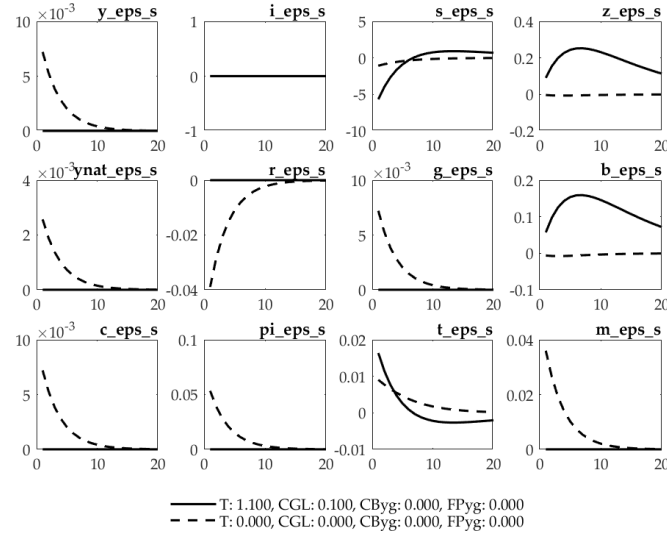


Figure F.4h: *Simulation of the Sticky-Price Model: Surplus Shock.* \diamond Source: Own illustration. \diamond Explanations: $\varepsilon^s = -0.01$, Kim (2003) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.2.3.

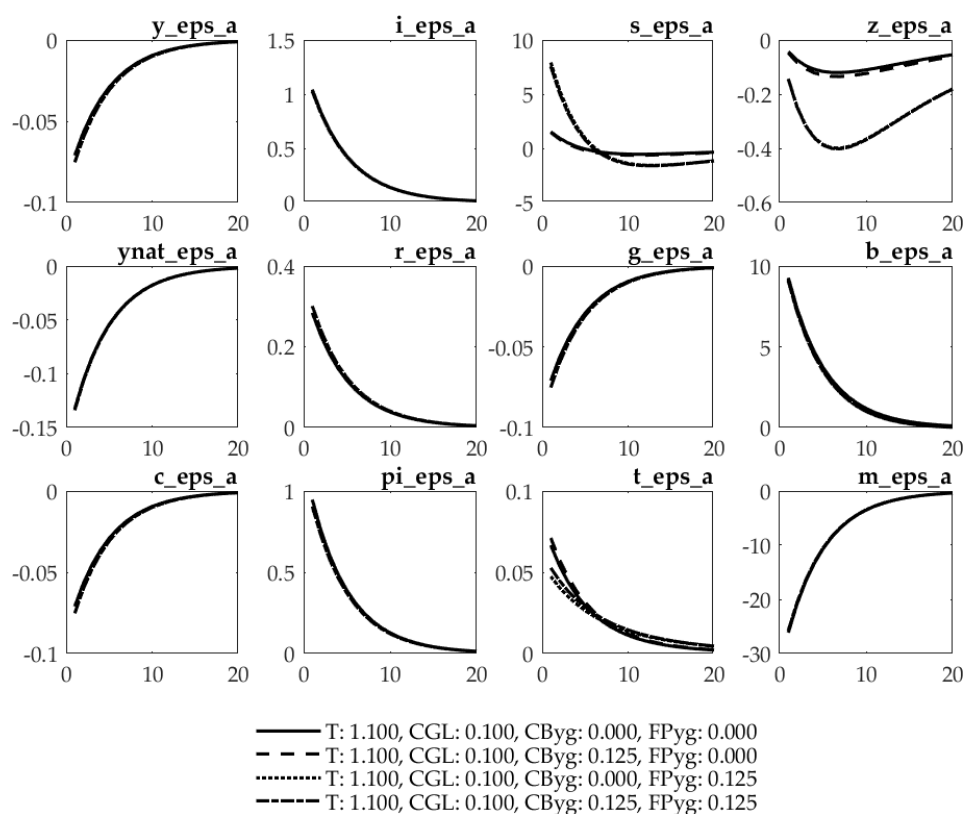


Figure F.5a: *Simulation of the Sticky-Price Model: Technology Shock with Different Policy Reactions to the Output Gap.* ♦ Source: Own illustration. ♦ Explanations: $\varepsilon^a = -0.25$, Kim (2003) calibration, no seigniorage rebate, treasury expenditure rule (7.25b). See also ↗ *Impulse-Response Functions* on p. 137 and Section 7.3.3.4.

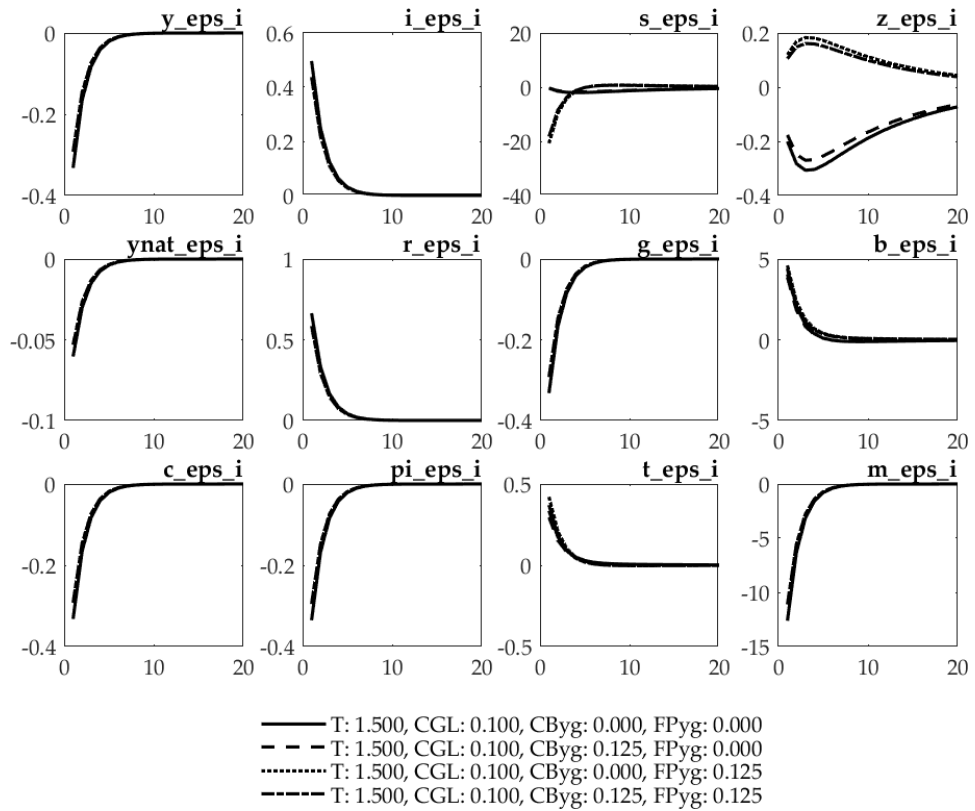


Figure F.5b: *Simulation of the Sticky-Price Model: Interest-Rate Shock with Different Policy Reactions to the Output Gap.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^i = 1$, Galí (2008) calibration, seigniorage rebate granted, treasury expenditure rule (7.25b). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.3.4.

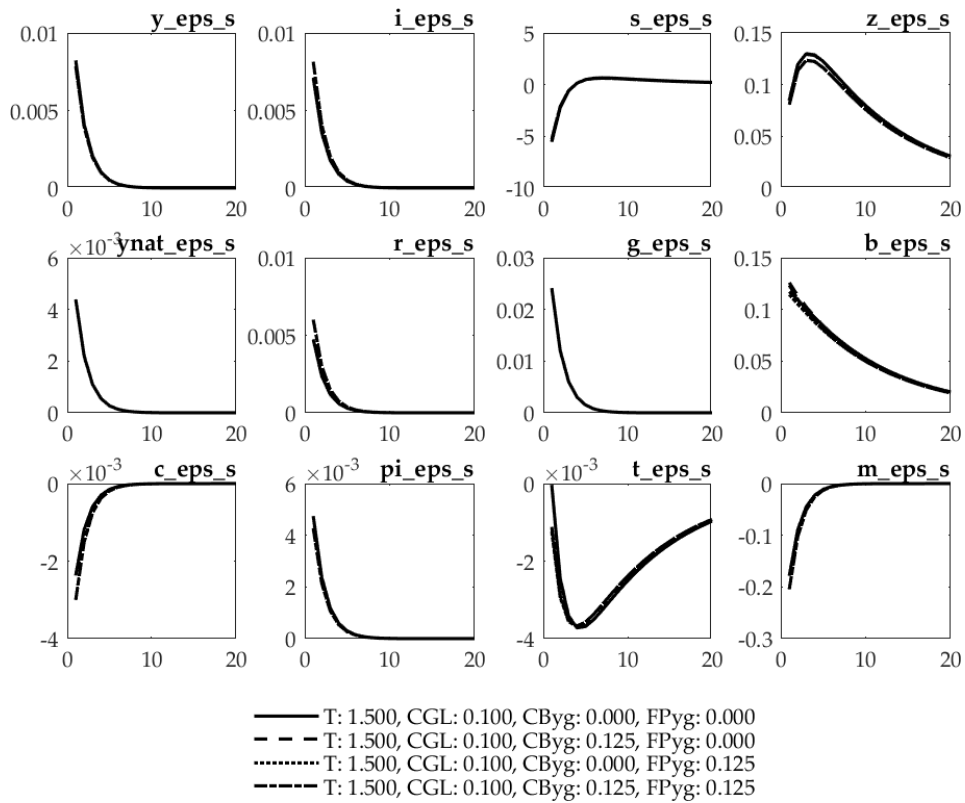


Figure F.5c: *Simulation of the Sticky-Price Model: Surplus Shock with Different Policy Reactions to the Output Gap.* \diamond Source: Own illustration. \diamond Explanations: $\epsilon^s = -0.01$, Galí (2008) calibration, no seigniorage rebate, treasury expenditure rule (7.25a). See also \nearrow Impulse-Response Functions on p. 137 and Section 7.3.3.4.

no.	equation	description
1	⟨7.19⟩	AR(1) interest-rate shock
2	⟨7.24⟩	AR(1) surplus shock
3	⟨7.32⟩	dynamic IS
4	⟨7.37⟩	New-Keynesian Phillips Curve
5	⟨7.39⟩	law of motion for consolidated-government liabilities
6	⟨7.40⟩	technology
7	⟨7.41⟩	Taylor rule
8	⟨7.42⟩	surplus rule of the treasury
9	⟨7.43⟩	treasury surplus (linearized definition)
10	⟨7.44⟩	treasury expenditure rule
11	⟨F.10⟩	output gap
12	⟨F.20⟩	Fisher equation
13	⟨F.21⟩	labor supply
14	⟨F.27⟩	aggregate production function
15	⟨F.28⟩	goods-market clearing
16	⟨F.36⟩	natural output
17	⟨F.41⟩	money demand
18	⟨F.42⟩	consolidated-government liabilities

Table F.1b: *Equations in the Linearized New-Keynesian Model.*

calibration:		Galí (2008)		Kim (2003)	
seigniorage rebate:		not granted	granted	not granted	granted
var.	regime				
b_{SS}	AMPF	-0.1802	-0.1733	-0.1837	-0.1764
	PMAF	1.0000	0.9379	1.0000	0.9347
c_{SS}	AMPF	0.6214	0.6214	0.9184	0.9184
	PMAF	0.6214	0.6214	0.9184	0.9184
g_{SS}	AMPF	0.4143	0.4143	0.6123	0.6123
	PMAF	0.4143	0.4143	0.6123	0.6123
i_{SS}	AMPF	0.0101	0.0101	0.0101	0.0101
	PMAF	0.0101	0.0101	0.0101	0.0101
ℓ_{SS}	AMPF	1.0540	1.0540	1.5310	1.5310
	PMAF	1.0540	1.0540	1.5310	1.5310
m_{SS}	AMPF	0.0621	0.0621	0.0653	0.0653
	PMAF	0.0621	0.0621	0.0653	0.0653
r_{SS}	AMPF	0.0101	0.0101	0.0101	0.0101
	PMAF	0.0101	0.0101	0.0101	0.0101
s_{SS}	AMPF	-0.0018	-0.0017	-0.0018	-0.0018
	PMAF	0.0100	0.0094	0.0100	0.0093
t_{SS}^{TH}	AMPF	-0.4125	-0.4126	-0.6104	-0.6105
	PMAF	-0.4243	-0.4237	-0.6223	-0.6216
w_{SS}	AMPF	0.6551	0.6551	1.0000	1.0000
	PMAF	0.6551	0.6551	1.0000	1.0000
y_{SS}	AMPF	1.0360	1.0360	1.5310	1.5310
	PMAF	1.0360	1.0360	1.5310	1.5310
z_{SS}	AMPF	-0.1180	-0.1111	-0.1184	-0.1111
	PMAF	1.0620	1.0000	1.0650	1.0000

Table F.2: *Steady-State Values of the Sticky-Price Model.* \diamond *Explanations:*
Some of these values enter the linearized model as parts of steady-state coefficient ratios. The active-monetary/passive-fiscal regime is denoted by “AMPF” and the passive-monetary/active-fiscal regime is denoted by “PMAF” in this table.

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